Abstract

Transport of Physical Quantities: More-Dimensional Quantile Motion and Flux Lines in Phase Space

The transport of physical quantities such as energy, probability, like-sign charges is studied. Conserved physical quantities described by a density ϱ and a current density $\vec{j}$ satisfy a continuity equation $\partial\varrho/\partial t + \vec{\nabla} \cdot \vec{j} = 0$. For positive density, $\varrho \geq 0$, quantile trajectories $\vec{r}_{\mathrm{c}}(t)$ can be defined. They satisfy the differential equation $\mathrm{d}\vec{r}_{\mathrm{c}}(t)/\mathrm{d}t = \vec{j}/\varrho$. In this context Bohm's trajectories obtain a probabilistic interpretation. The concepts developed here are applied to the probability density and energy density, respectively, and the corresponding current density of (a) the two-dimensional Gaussian wave packet and (b) the electromagnetic dipole radiation.

The Wigner distribution in phase space of a quantum-mechanical wave function is not positive everywhere, however, the domains of negative values cover small phase-space volumes of the size of Planck's quantum h for a one-dimensional problem. The current density associated with the Wigner distribution is chosen such that $\dot{x} = p/m$ is maintained as one of the equations of motion. The phase-space flux lines (trajectories) of a number of quantum-mechanical systems are computed and graphically displayed.