

Nominal and Real Rigidities in  
Monetary Stochastic Dynamic  
General Equilibrium Models of the  
Business Cycle

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# Chapter 1

## Introduction

Macroeconomists have for a long time tried to explore which mechanisms are important to explain actual business cycles. On the one hand, this led to a large amount of literature on theoretical models that can account for the business cycle. This literature focuses on the impulses – the shocks which are the sources of the cycles – and the propagation mechanisms – the ways these shocks are transmitted in the economy. On the other hand, there has been an immense effort to describe the business cycle empirically using various sophisticated econometric methods.

### 1.1 Advances in Macroeconomic Theory

In the 80s of the last century there were two main research agendas on the theoretical side: The New Keynesian Macroeconomics (NKM) and the Real Business Cycle (RBC) school.

The Real Business Cycle literature argues that technology shocks are the driving source of economic fluctuations.<sup>1</sup> The theory is based on the neoclassical growth model. RBC models are stochastic dynamic general equilibrium models in which a representative household maximizes its life-time utility while a representative firm optimizes its profits under rational expectations.<sup>2</sup> Markets always clear because prices are fully flexible. Households and firms react optimally to shocks to the total factor productivity that is modeled as a stochastic process. Business cycle are thus pareto optimal responses of the agents to these technology shocks. Households vol-

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<sup>1</sup>The RBC literature is immense. For an introduction see Gail (1998), Chapter 2. King, Plosser and Rebelo (1988) is a classic of RBC research. For an excellent overview considering all facets of the approach see Stadler (1994).

<sup>2</sup>The basic model considers a Robinson Crusoe economy where there is no distinction between households and firms.

untarily substitute leisure for labor intertemporally but also consumption for leisure intratemporally. Fiscal or monetary policy are suboptimal and would destabilize the economy. The RBC approach has been generalized to suboptimal equilibria allowing for unemployment or fiscal shocks.<sup>3</sup> RBC models have been criticized mainly for two reasons: First, critics question the assumption that technology shocks are the driving impulse of the business cycle. Second, the validity of intertemporal substitution as the main propagation mechanism to explain economic fluctuations, especially those of the labor force, is criticized.

The NKM stresses the role of monetary impulses for the business cycle in contrast to the RBC approach where money does not play an important role because money reacts to changes in the real variables (reverse causation).<sup>4</sup> Additionally NKM economists stress the importance of sticky prices and wages for the propagation of monetary shocks. The NKM is characterized by very heterogeneous models, there is no common framework, no workhorse, as in the RBC research agenda (see also Illing (1992)). According to Mankiw and Romer (1991) NKM models are typically characterized by the assumption that in the short run the classical dichotomy does not hold. Nominal variables can have an influence on real quantities. In the economy sticky prices and wages as well incomplete markets, coordination failures and real rigidities are important propagation mechanisms. The collection of papers in Mankiw and Romer demonstrates the variety of different approaches of the NKM which can also be considered as a weakness. Many models only focus on specific markets of the economy such as the labor or goods market. They do not consider the economy as whole as is standard in the RBC research agenda. Additionally NKM authors often consider static models which are not appropriate for the study of business cycles. Finally most models in the NKM tradition cannot ‘be taken to the data’ since there is no direct empirical link.<sup>5</sup>

With the end of the 1980s some authors began to include money in a standard RBC model. Cooley and Hansen (1989) were the first to incorporate money via a cash-in-advance (CIA) constraint where the inflation tax operated as a propagation mechanism. In a related paper they consider the role of monetary shocks as a source of economic fluctuations and conclude that such shocks could not account for actual business cycles, see Cooley and Hansen (1995). This result does not change when real money balances are included in the utility function (MIU) to motivate money demand as in Walsh (1998), Chapter 3. Gerke (2003) confirms these results in

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<sup>3</sup>The book of Cooley (1995) is a representative collection of such work.

<sup>4</sup>See King and Plosser (1984) for an exposition of this argument.

<sup>5</sup>See the discussion in Marchlewitz (1995), p. 264.

his Chapter C. If prices are flexible there is no transmission mechanism for money growth shocks in an augmented RBC model.

By the mid of the 1990s RBC theorists were able to include important building blocks of the NKM agenda. These are nominal rigidities such as sticky prices and wages (as e.g. in Fischer (1977) and Taylor (1980)) as well as monopolistic competition (as in Blanchard and Kiyotaki (1987)). This new research agenda has been labeled the *New Neoclassical Synthesis* by Goodfriend and King (1997). It can be viewed as the new workhorse which has been intensively used in the last years.<sup>6</sup> Today the notion ‘*real* business cycle’ is no longer justified since the literature has moved away from considering technology shocks as only the driving source of business cycles. The framework makes it possible to consider various exogenous shocks such as money supply, money demand, interest rate, taste, energy and capital utilization shocks. As the focus of this work is on monetary shocks the notion ‘monetary stochastic dynamic general equilibrium models’ is used.<sup>7</sup>

## 1.2 The Empirics of the Business Cycle

Concerning the empirical measurement of the business cycle the RBC approach had a very important influence. Beginning with the seminal paper of Kydland and Prescott (1982) RBC economists tried to reconcile their models with the data. Since they are computable general equilibrium models they imply specific dynamic processes for macroeconomic aggregates such as output, consumption, investment or labor. With the help of other empirical studies one can calibrate these models which means that one can insert numerical values for the model parameters (e.g. labor’s share or the depreciation rate) and for the exogenous process of the technology shock (e.g. by estimating Solow residuals from an aggregate production function). Then the model can be simulated many times and one can calculate moments such as standard deviations and cross correlations with output to explore the business cycle properties of the model.<sup>8</sup> The same can be done with the data:

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<sup>6</sup>There are some very early contributions of King (1991) and Hairault and Portier (1993) which date before 1997.

<sup>7</sup>This label goes back to an initiative of Christian Zimmermann, the founder of the RePEc archive (a collection of research papers in economics available online). In the late 1990s he put to the vote many different labels for this research agenda. The result is given above.

<sup>8</sup>Depending on the structure of the model and the solution method it is also possible to calculate moments without simulating the processes and taking averages afterwards by using spectral analysis methods. This is done in this book.

After removing a time trend<sup>9</sup> – using the HP-filter e.g. – standard deviations as a measure for the volatility and cross correlations as a measure for the comovement with the cycle can be calculated. There is a large empirical literature employing the HP-filter to extract the cyclical component and to calculate moments, see for example Kydland and Prescott (1990) for the US, Brandner and Neusser (1992) for Germany and Austria or Christodoulakis, Demelis and Kollintzas (1995) for the EU. Afterwards the results of the model can be compared with the empirical moments to evaluate the performance of the model. This method has been extensively used in the early RBC literature, see e.g. King, Plosser and Rebelo (1988). Critics argue that this procedure lacks formal statistical testing of the validity of the model. The ability of the model to match empirical moments of the data is judged informally by the researcher and thus depends on her or his subjective opinion.<sup>10</sup> There are some authors who propose econometric estimation procedures to evaluate the performance of the model. For example Altug (1989) estimates the model of Kydland and Prescott (1982) using the maximum likelihood (ML) method. Others like Burnside, Eichenbaum and Rebelo (1993) use the ‘Generalized Method of Moments’ (GMM) proposed by Hansen (1982). Watson (1993) suggests another metric to measure the fit of a calibrated model. But formal econometric estimation is not (yet) common in the literature. Many researchers still use the calibration technique. It has to be seriously questioned whether formal testing actually does make sense. Prescott (1986), p.12, concludes that the models are necessarily false because they are highly abstract so that statistical hypothesis testing would reject them immediately. This view is strengthened by Hoover (1995) who strictly separates the calibration method from econometric estimation techniques. In his view there is a different understanding of how to develop a theory. The econometric approach is accordingly based on a competitive strategy. There are competing theories and the econometric estimation guides the researcher in deciding which theory has to be discarded and which theory has to be accepted because it fits the facts best. The calibration technique starts with an overwhelmingly simple model which serves as a benchmark. When confronted with the data the weaknesses of the model will clearly show up and guide the researcher where she or he has to modify the theory in order to improve the fit. Thus a bad empirical performance does not lead to a rejection of the theory but to further efforts to improve the model by plugging in or removing parts of the setup

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<sup>9</sup>The best way to measure the business cycle is quite controversial as the study of Canova (1998) impressively demonstrates. See also Gail (1998), Chapter 3.

<sup>10</sup>See e.g. Fair (1991) and Andersen (1991).

which are responsible for this result. This adaptive strategy is also used in this study as it is considered to be the most adequate approach to develop business cycle models.<sup>11</sup>

There is another way to compare the model results with the data. One can subject the model to a one percent technological shock and study the reaction of the variables using impulse response functions. This technique has been as widely used to explain the implications of RBC models as the calculation of moments. But it was not employed to study *empirically* the effects of technological shocks on macroeconomic aggregates. The situation changed when in the mid of the 1990s researchers started to include money in their models. Economists tried to explore the effects of a monetary policy shock on various macroeconomic variables in the data. Ideally they wanted to conduct the same analysis as in the model: to subject the economy to a monetary shock and to compare the theoretical impulse responses with the empirical ones. This agenda has stimulated a growing empirical literature on the transmission of monetary disturbances in an economy. The paper of Christiano, Eichenbaum and Evans (1999) is an excellent survey of this literature. It discusses in depth the use of vector autoregressive models (VARs) to estimate the impact of money on the economy.<sup>12</sup> How do VARs work and what are their main results? Since it is not intended to estimate a VAR in this work the discussion will be very brief and will concentrate on the most important issues.<sup>13</sup>

A VAR is a system of dynamic stochastic equations in which lagged values of every variable can potentially have an influence on every other variable. A VAR of order  $q$  can be written as follows

$$\vec{y}_t = A_1 \vec{y}_{t-1} + A_2 \vec{y}_{t-2} + \dots + A_q \vec{y}_{t-q} + \mu_t \quad (1.1)$$

where  $\vec{y}_t$  is the vector of the  $n$  macroeconomic aggregates  $y_{n,t}$ ,  $A_i$  are  $(n \times n)$  matrices

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<sup>11</sup>As monetary dynamic stochastic general equilibrium models are still in their infancy the calibration and the empirical evaluation of the models cannot be conducted as precisely as in RBC models. This justifies the neglect of a rigorous analysis of the German business cycle (by calculating standard deviations and cross correlations) and of the models' ability to match these stylized facts here.

<sup>12</sup>There are of course other approaches to explore the short run effects of money on output such as the concept of Granger causality (Sims (1972)) or the Friedman-Meiselman equation (Friedman and Meiselman (1963)) that served as the basis for the famous St. Louis equations. But these concepts cannot answer questions like those that are posed above. For a brief overview of this literature see Walsh (1998).

<sup>13</sup>Gerke (2003) provides a VAR analysis for Germany. Most other work in this area is concerned with the US.



of coefficients ( $i = 1, \dots, q$ ) and  $\vec{\mu}_t$  is given by

$$\vec{\mu}_t = B\vec{\epsilon}_t \quad (1.2)$$

where  $\vec{\epsilon}_t$  is a vector of stochastic disturbances  $\epsilon_{n,t}$  with  $E(\vec{\epsilon}_t) = 0$  and a positive definite variance-covariance matrix and  $B$  is an  $(n \times n)$  coefficient matrix. The disturbances are not serially correlated. There can also be a vector of constants and a time trend which are dropped here for simplicity. To illustrate the problem of extracting a monetary shock out of a VAR assume the simplest case where there are only two variables in  $\vec{y}_t$ : Output  $Y_t$  and the money supply  $M_t$  as the policy variable (M1 e.g.).<sup>14</sup> This system reads

$$\begin{bmatrix} Y_t \\ M_t \end{bmatrix} = \begin{bmatrix} a_Y & a_M \\ 0 & 0 \end{bmatrix} \begin{bmatrix} Y_{t-1} \\ M_{t-1} \end{bmatrix} + \begin{bmatrix} \mu_{Y,t} \\ \mu_{M,t} \end{bmatrix} \quad (1.3)$$

with  $0 < a_Y < 1$ . Suppose that the innovations are related according to

$$\begin{bmatrix} \mu_{Y,t} \\ \mu_{M,t} \end{bmatrix} = \begin{bmatrix} 1 & b_M \\ b_Y & 1 \end{bmatrix} \begin{bmatrix} \epsilon_{Y,t} \\ \epsilon_{M,t} \end{bmatrix} \quad (1.4)$$

so that the one-period ahead forecasting error  $\mu_{M,t}$  of  $M_t$  depends on the exogenous shock to output  $\epsilon_{Y,t}$  and the exogenous shock to money  $\epsilon_{M,t}$ :  $\mu_{M,t} = b_Y\epsilon_{Y,t} + \epsilon_{M,t}$ . As long as  $b_Y \neq 0$  the shock to money  $M_t$  will depend on *both* exogenous shocks so that an estimate of  $\mu_{M,t}$  does in general not provide a measure for the policy shock. To be more explicit the simple structure of (1.3) allows to write  $Y_t$  as  $Y_t = a_Y Y_{t-1} + \mu_{Y,t} + a_M \mu_{M,t-1}$  so that the moving average representation of  $Y_t$  can be written as<sup>15</sup>

$$Y_t = \sum_{i=0}^{\infty} a_Y^i \mu_{Y,t-i} + \sum_{i=0}^{\infty} a_Y^i a_M \mu_{M,t-i-1}$$

This equation does not tell us what the effect of a policy shock  $\epsilon_{M,t}$  will be. For that purpose  $\mu_{Y,t}$  and  $\mu_{M,t}$  have to be inserted. This yields

$$\begin{aligned} Y_t &= \sum_{i=0}^{\infty} a_Y^i (\epsilon_{Y,t-i} + b_M \epsilon_{M,t-i}) + \sum_{i=0}^{\infty} a_Y^i a_M (\epsilon_{M,t-i-1} + b_Y \epsilon_{Y,t-i-1}) \\ &= \epsilon_{Y,t} + \sum_{i=0}^{\infty} a_Y^i (a_Y + a_M b_Y) \epsilon_{Y,t-i-1} + b_M \epsilon_{M,t} + \sum_{i=0}^{\infty} a_Y^i (a_Y b_M + a_M) \epsilon_{M,t-i-1} \end{aligned}$$

The impulse response (in periods  $t, t+1, t+2, \dots$ ) to a unit shock  $\epsilon_{M,t} = 1$  is then given by

$$b_M, \quad a_Y b_M + a_M, \quad a_Y (a_Y b_M + a_M), \quad a_Y^2 (a_Y b_M + a_M), \dots$$

<sup>14</sup>The exposition follows Walsh (2003), p. 24-27.

<sup>15</sup>One can calculate this by inserting  $Y_{t-i}$  recursively.

This sequence represents the values  $Y_t, Y_{t+1}, Y_{t+2}, \dots$ . Note that they depend not only on  $a_M, a_Y$  but also on  $b_M$ . This implies that the impulse responses cannot be identified without further restrictions because  $b_M$  is not known when estimating (1.3). There are three main identification schemes:<sup>16</sup> One way to solve the problem is to assume that  $b_M = 0$ . In this case the money shock influences output only with a lag since  $b_M \epsilon_{M,t}$  drops out. There is no contemporaneous impact of the policy shock on output. The output shock is exogenous to the money shock because  $\mu_{Y,t}$  is not influenced by  $\epsilon_{M,t}$ .<sup>17</sup> The impulse responses then depend only on  $a_M$  and  $a_Y$ :

$$0, \quad a_M, \quad a_Y a_M, \quad a_Y^2 a_M, \dots$$

A second possibility is to assume that  $b_Y = 0$  which would imply that the money shock is exogenous to the output shock.<sup>18</sup> This is because  $\mu_{M,t} = \epsilon_{M,t}$  and the VAR residuals  $\mu_{Y,t} = b_M \mu_{M,t} + \epsilon_{Y,t}$  can be estimated from a regression of  $\mu_{Y,t}$  on the VAR residuals  $\mu_{M,t}$  which will give an estimate for  $b_M$ . In this case the policy variable  $M_t$  does not respond contemporaneously to output shocks, perhaps because there are information lags in specifying monetary policy. A third way to solve the problem is to impose restrictions on the long-run effects of the policy shock on output. An example can be the assumption of long-run monetary neutrality. This would imply that a monetary shock  $\epsilon_{M,t}$  has no long-run effect on output. Formally this is achieved when all shocks add up to zero so that  $b_M + (a_Y b_M + a_M) \sum_{i=0}^{\infty} a_Y^i = 0$ . This condition is equivalent to  $b_M = -a_M$ . The impulse responses of output will then be equal to

$$-a_M, \quad a_M(1 - a_Y), \quad a_Y a_M(1 - a_Y), \quad a_Y^2 a_M(1 - a_Y), \dots$$

Again an unknown coefficient of the  $B$ -matrix is eliminated using a known one of the  $A$ -matrix.

This discussion demonstrates the difficulty of estimating the impact of a money shock on output. There are three identifying assumptions and accordingly three different results for the impulse responses of output and thus for the specific effects of a money supply shock. Which assumption is correct? There is no final answer to this question. All assumptions do not follow out of the model but are mainly based on informal plausibility considerations. Is it more plausible that the money shock has no contemporaneous impact on output than vice versa? Using annual

<sup>16</sup>See also Uhlig (2004), p. 5.

<sup>17</sup>This is a Choleski decomposition with output (the non-policy variable) ordered first.

<sup>18</sup>This is a Choleski decomposition with money (the policy variable) ordered first.

data money should have an impact on output but not in monthly data. But if there are information lags money should not react directly to output shocks. How can the problem be solved? There is a tendency in the literature to use the estimated impulse responses to check the validity of the assumptions. If the impulse response seems implausible this is interpreted as a misspecification of the VAR and would lead to a revision of the setup (see Leeper, Sims and Zha (1996), p. 29.)

The example above neglects the fact that macroeconomists are usually interested in the effects of policy on several other variables such as unemployment, consumption and inflation so output should be replaced by a vector of non-policy variables. In addition the choice of money as the policy variable can be controversial. In general there are several candidate variables for monetary policy such as either some short-term interest rate or a money aggregate such as the monetary base, M1 or reserves. Each will depend in various degrees on both policy and non-policy disturbances. In turn identifying restrictions corresponding to  $b_Y = 0$  or  $b_M = 0$  will be more complicated and less easily justified. This problem is discussed at length in Christiano, Eichenbaum and Evans (1999). The following exposition will briefly summarize the evidence on monetary policy shocks for Germany as described in Gerke (2003).

Gerke considers three different measures for money (reserves, the monetary base and M1) and a 3-month money market rate as the policy variable. He includes real consumption  $C$ , real gross domestic product  $Y$  (GDP) and the GDP-deflator  $P$  in his VAR. He conducts a detailed analysis on the seasonality and the stationarity of the variables. Since he finds at least one cointegrating vector he generalizes the setup to account for the non-stationarity (in fact, he estimates a Vector-Error-Correction-Model and uses the resulting estimates to determine the coefficients of the VAR). This procedure is new since the literature does not focus on the issue of stationarity in the context of VARs so far.<sup>19</sup> In order to calculate confidence intervals he uses a bootstrap technique. The identification scheme corresponds to the second case described above so that non-policy shocks do not have a contemporaneous effect on the monetary policy variable.<sup>20</sup> The result of the VAR with M1 as the policy variable is presented in Figure 1.1 where  $r$  is the short term money market rate. An expansionary shock to money  $M$  leads to a hump-shaped rise in output and consumption which is statistically significant and which lasts for about 8 quarters

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<sup>19</sup>Very recently Altig et al. (2003) also began to study the implications of cointegration in a VAR for the US.

<sup>20</sup>He reports that the impulse responses are not sensitive with respect to this assumption.

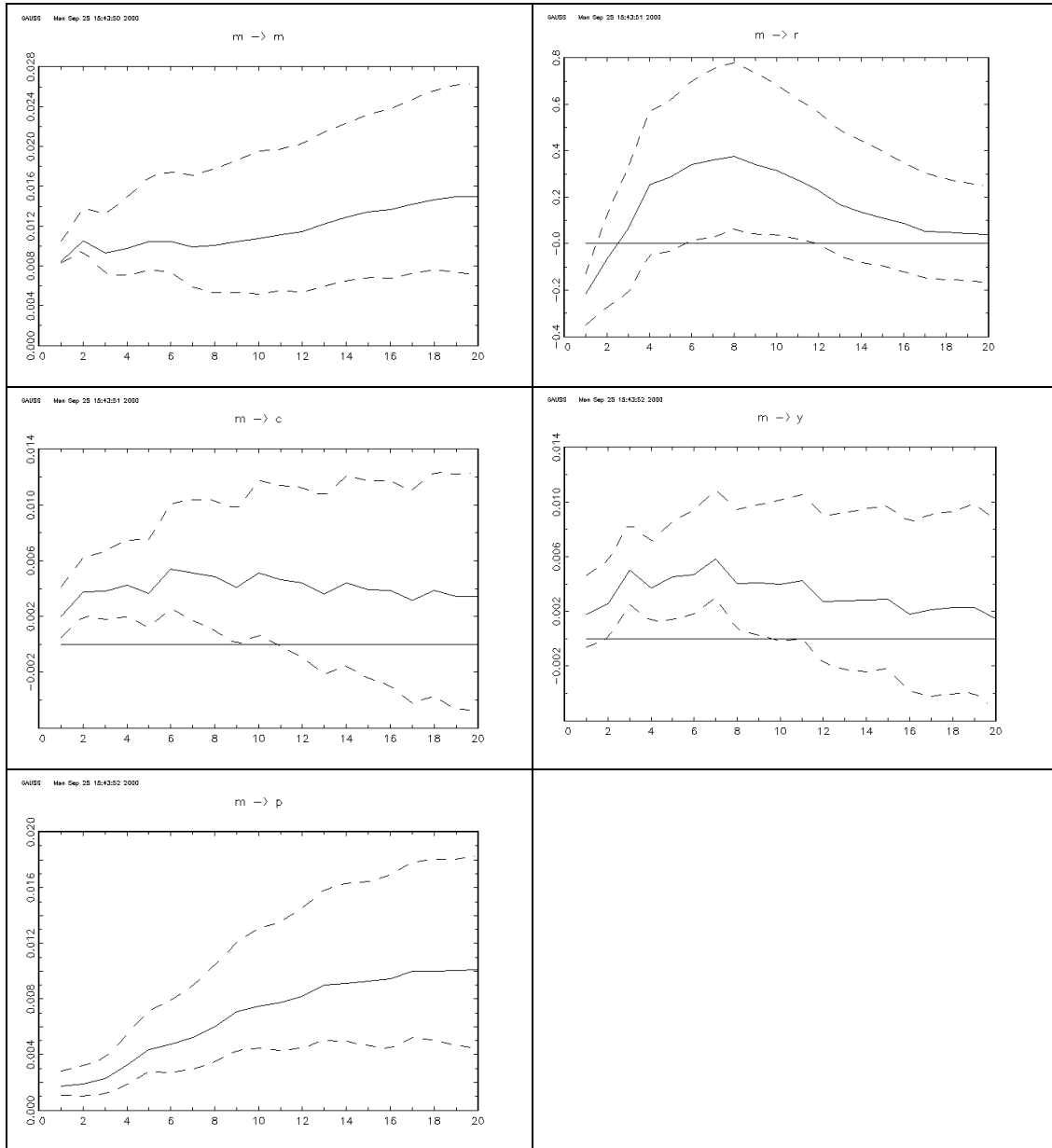


Figure 1.1: Empirical Impulse Responses due to a Monetary Shock, Order of Variables:  $M, r, C, Y, P$ ; Source: Gerke (2003), p. 56, Figure 2.2

(see the confidence intervals that are given by the dotted lines). The interest rate falls while the deflator rises slowly. In successive periods the interest rate rises as well. This is a result one would also get qualitatively out of a typical IS-LM model using comparative static analysis. Using instead the monetary base (Gbasis) as the policy variable the hump-shaped responses of consumption and output are no longer significant, see Figure 1.2. The interest rate rises from the beginning, a result known as the *liquidity puzzle*. This could be explained when interpreting a shock to the monetary base as a demand shock, not a supply shock. The rise in the price level is also not significant any more. When using the interest rate as the policy variable the results change a little. Figure 1.3 reveals that a contractionary interest rate shock, i.e. a rise in the interest rate (which corresponds to a contractionary shock to money) leads to a rise in the price level as well. This behavior is known as the *price puzzle*.<sup>21</sup> Consumption and output fall significantly and never approach their initial values. The initial rise in output is not statistically significant as the confidence intervals indicate.

These results overall confirm the evidence in Christiano, Eichenbaum and Evans (1999) for the US. In the paper of Christiano, Eichenbaum and Evans (2003) the authors consider a VAR with many more variables. They do so by partitioning the vector  $\vec{y}_t$  in (1.1) as follows:

$$\vec{y}_t = \begin{bmatrix} \vec{y}_{1t} \\ r_t \\ \vec{y}_{2t} \end{bmatrix}$$

where  $\vec{y}_{1t}$  contains real GDP, real consumption, the GDP deflator, real investment, the real wage and labor productivity.  $\vec{y}_{2t}$  is given by real profits and the growth rate of M2.  $r_t$  is the Federal Funds rate and operates as the policy variable. Note that they use a different identification scheme in that they order  $\vec{y}_{1t}$  first so that interest rate shocks do not contemporaneously influence those aggregates.<sup>22</sup> They find a significant and hump-shaped increase in investment and labor productivity.

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<sup>21</sup>One can solve the problem by introducing an additional commodity price index as in Christiano, Eichenbaum and Evans (1999) in order to dampen the reaction of the price level. Another way out is to argue that a rise in the interest rate acts like a positive cost shock because it raises the cost of holding inventories. So it can be interpreted as a negative supply effect that raises prices and lowers output. It is accordingly labeled the cost channel of monetary policy, see Barth and Ramey (2001).

<sup>22</sup>It corresponds to the first scheme discussed above. The authors justify this identification scheme as reflecting ‘a long-standing view that macroeconomic variables do not respond instantaneously to policy shocks (see Friedman (1968))’. See Christiano, Eichenbaum and Evans (2003), p. 4.

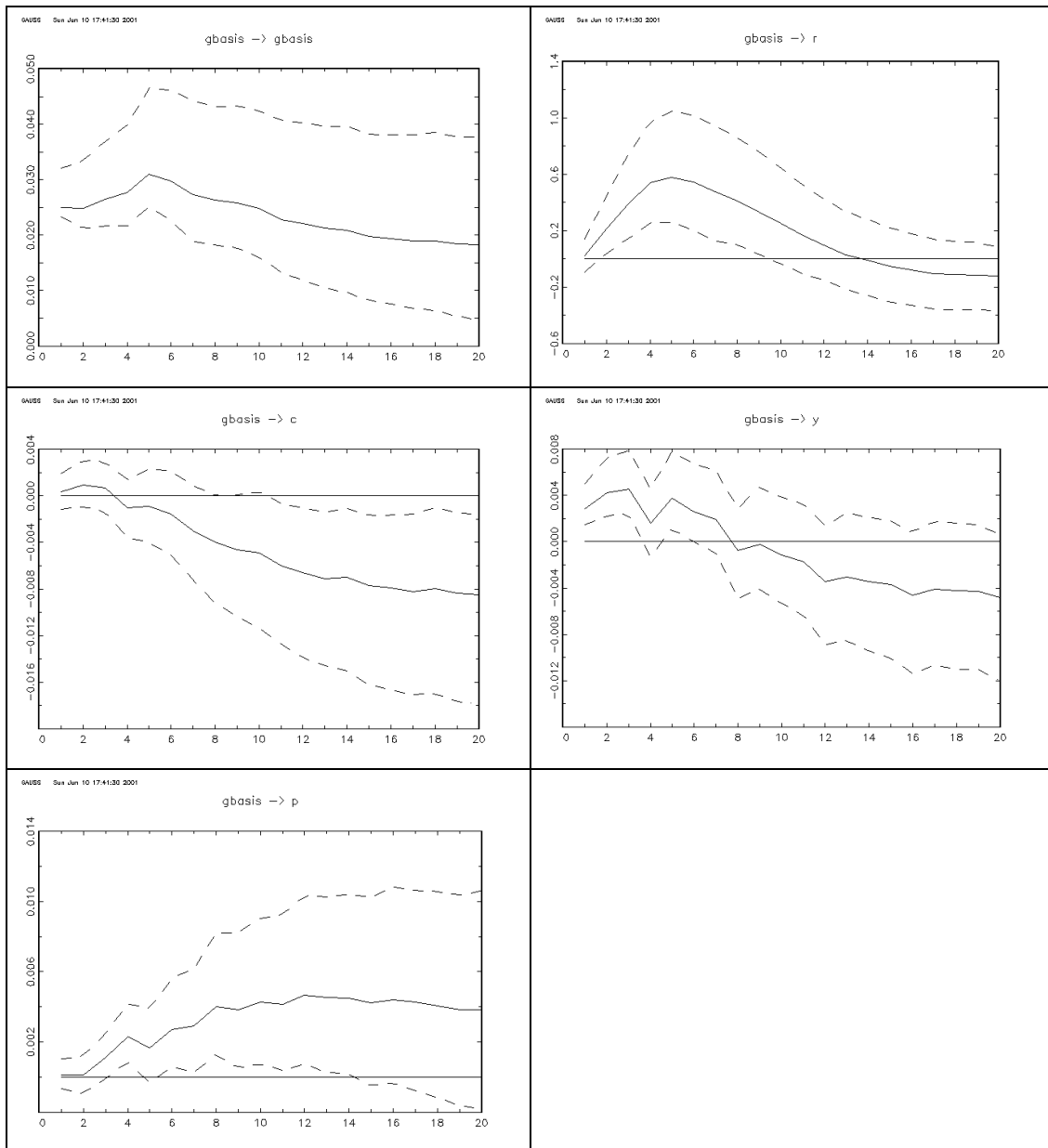


Figure 1.2: Empirical Impulse Responses due to a Monetary Shock, Order of Variables:  $Gbasis, r, C, Y, P$ ; Source: Gerke (2003), p. 58, Figure 2.4

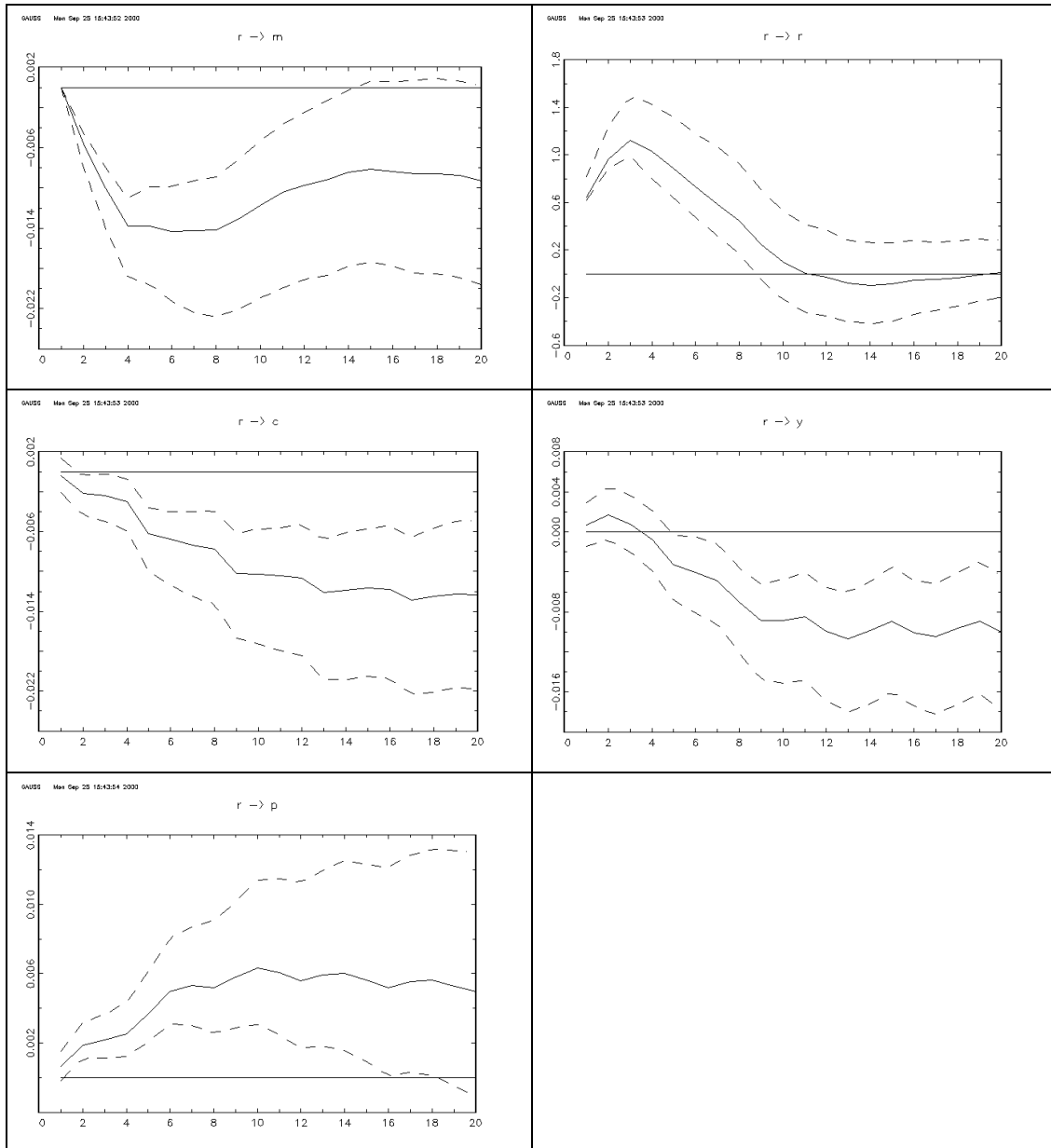


Figure 1.3: Empirical Impulse Responses due to a Monetary Shock, Order of Variables:  $r, M, C, Y, P$ ; Source: Gerke (2003), p. 57, Figure 2.3

Real profits as well as the real wage also rise but not significantly. The inflation rate shows a cyclical reaction which they interpret as a hump-shaped response. The interest rate falls whereas money growth rises significantly. Figure 1.4 reproduces their results together with the outcome of their benchmark model given by the solid lines (see the legend in the figure). The impulse responses are invariant to the ordering of the variables within  $\vec{y}_{1t}$  and  $\vec{y}_{2t}$  but it is not clear whether they are also invariant to the general ordering with the aggregates in  $\vec{y}_{1t}$  ordered first as Gerke (2003) claimed for his results.

VARs have been criticized on various grounds. Besides the problem of identification there is a fundamental disadvantage: In every VAR monetary policy is described only by random disturbances. In reality however monetary policy is endogenous in the sense that it reacts to changes in the development in the economy. If there were a feedback rule that completely characterized monetary policy so that there were no exogenous shocks to policy, then the VAR approach would conclude that monetary policy does not matter. In this case there would be no movements in output. But this does not imply that monetary policy is unimportant. The reaction of the aggregates to non-policy shocks may be influenced to a great extent by the endogenous adjustment of the policy.<sup>23</sup> A related point concerns the question whether anticipated or unanticipated monetary policy matters. If it is anticipated policy that matters then the hump-shaped output response can mainly be caused by persistent systematic policy actions and not by the initial policy shock.<sup>24</sup>

A last interesting question is whether monetary shocks can contribute significantly to the observed variability of output and inflation in the data. Christiano, Eichenbaum and Evans (1999) conclude that when monetary policy is described by the Federal Funds rate policy shocks account for 21% of the four-quarter ahead forecast error variance for quarterly real GDP. At the 12-quarter horizon this value rises up to 38%. The effects are smaller when a money aggregate is considered to be the policy instrument. Monetary policy shocks – on the other hand – account for only a small fraction of the forecast error variance of the price level. However Altig et al. (2003) find that the variance decompositions are sensitive with respect to the ordering of the variables in the VAR and have therefore to be interpreted with caution. But overall monetary shocks seem to contribute significantly to the explanation of the fluctuations in real macroeconomic aggregates.

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<sup>23</sup>Sims (1998) uses the VAR framework to assess the systematic effects of monetary policy.

<sup>24</sup>For a further discussion see Cochrane (1998).



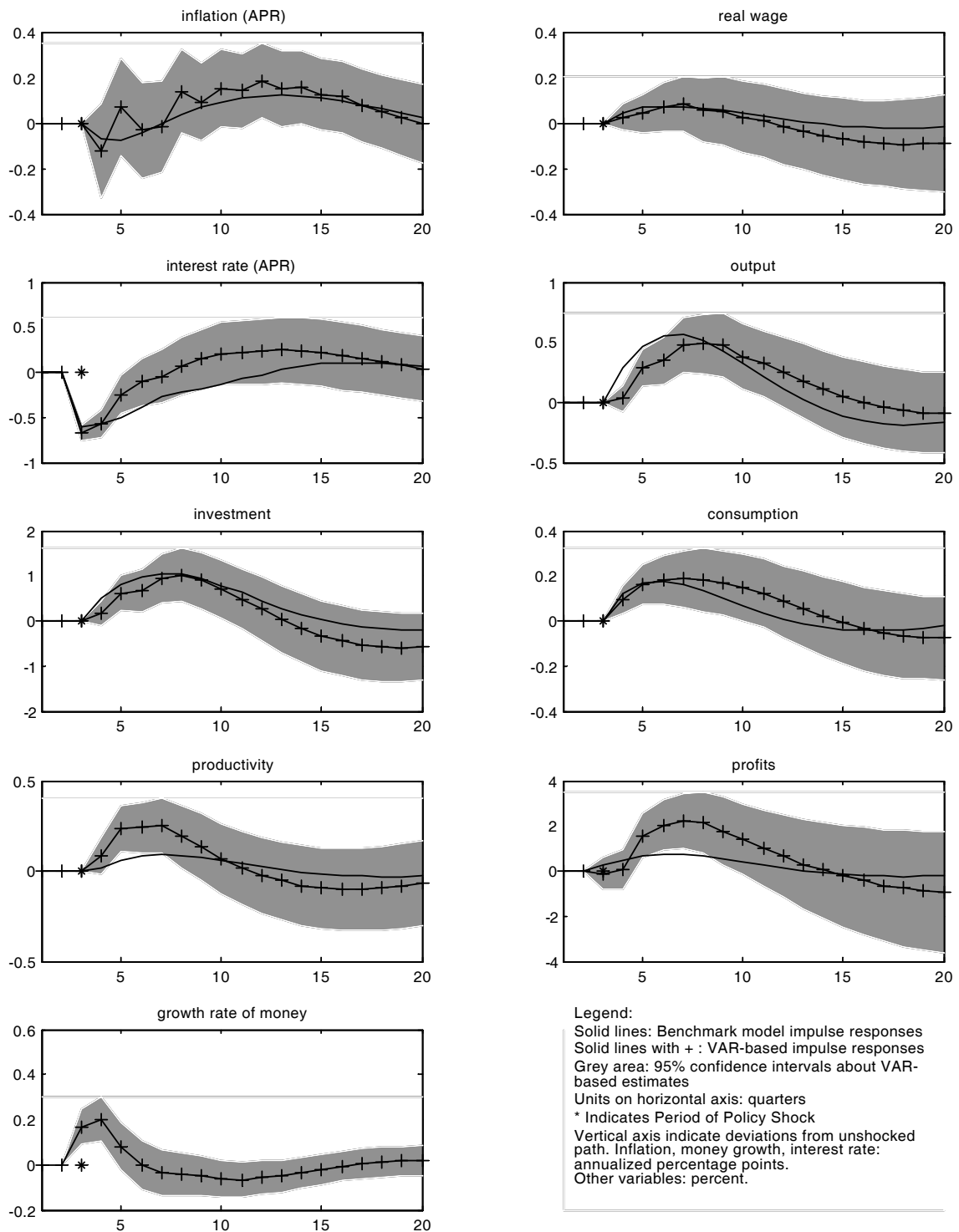


Figure 1.4: Model- and VAR-Based Impulse Responses;

Source: Christiano, Eichenbaum and Evans (2003), p. 38, Figure 1

### 1.3 Nominal and Real Rigidities and the Propagation of Monetary Policy Shocks

While the existence of monetary policy shocks and their effects on real macroeconomic aggregates have been documented empirically (as the above discussion has demonstrated) it is not (yet) clear how the shocks are propagated in an economy. The VAR literature is – at least to a great extent – atheoretical. But economists try to understand through which channels monetary policy is transmitted. There are a lot of transmission mechanisms which are surveyed in Mishkin (1995). Accordingly the transmission can be interpreted as a black box as it consists of a mixture of various different channels the details of which are not exactly known yet. Mishkin distinguishes the interest rate, the exchange rate, the credit and the relative price channel. These approaches seek to explain how a monetary shock influences aggregate demand through reactions of households and firms assuming that the price level adjusts only slowly.

Monetary stochastic dynamic general equilibrium (DGE) models do not try to explain *why* the price level reacts only moderately to monetary shocks. Instead they try to implement certain nominal frictions that have been found to be of empirical importance and for which a microeconomic foundation can be offered (see the survey of Taylor (1999)) and to show *that* prices react slowly. While in static models the assumption of constant or fixed prices is justified it is necessary to assume some sort of sluggish prices in dynamic models. There are two candidate avenues to follow: first, one can analyze the consequences of reduced price flexibility of firms (sticky prices), second one can explore the effects of a reduced flexibility of wages of the households (sticky wages). These are the most important nominal rigidities analyzed in the literature and also in this work.

But these nominal rigidities do not guarantee that prices will react sluggishly for a long period of time as is observed empirically. Therefore real rigidities are added to the models in order to strengthen the effects of monetary shocks without having to assume an implausible degree of nominal rigidities. Real rigidities are e.g. capital adjustment costs, habit persistence in consumption, variable capital utilization, multiple stages of production or adjustment costs of employment. In order to motivate the focus of the present study it is necessary to review some important contributions in the literature.<sup>25</sup>

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<sup>25</sup>The exposition is in no way a complete description of the literature. This task would require to write a second book.

The question whether money growth shocks can contribute significantly to the explanation of observed business cycle fluctuations is explored intensively in the seminal paper of Chari, Kehoe and McGrattan (2000). They consider a benchmark model with price staggering as in Taylor (1980) in which firms can set prices for a fixed period of time. They study a model with a money-in-the-utility (MIU) function and a production function where labor is the only productive input. They find that output is not persistent in response to a money growth shock. Especially there is only a positive deviation of output from its steady state as long as prices are fixed. In turn they consider various changes in the setup of their model to find out whether these changes can improve upon the empirical fit. Among these extensions are some real rigidities such as the assumption of specific factors of production or convex demand functions. The latter implies that the elasticity of demand increases as prices rise. Although this helps in dampening the rise of the markup and thus marginal costs to a money growth shock the effect is not strong enough to increase output persistence. Chari, Kehoe and McGrattan (2000) also vary the utility function in a way to make consumption and leisure nearly perfect substitutes. While this raises persistence in a model with only labor as the productive input it does not help in a model with intertemporal links such as capital accumulation.<sup>26</sup> A major finding is that once such intertemporal links are added to the model the persistence of output is even smaller and thus the contract multiplier – defined as the ratio of the half-life of output deviations after a monetary shock with staggered price setting to the corresponding half-life with synchronized price setting – is even smaller.

This paper has stimulated a large body of research.<sup>27</sup> Huang and Liu have contributed extensively to this literature. They study closed as well as open economy models giving special attention to the role of capital accumulation. In Huang and Liu (2000) they consider a two-country model with capital and find that multiple stages of production are important in propagating money growth shocks through time. The input-output structure gives rise to significant cross-country correlations in aggregate output and to persistent deviations of real exchange rates from purchasing power parity. The paper of Huang and Liu (2001*a*) confirms this finding. In closed economies without capital such an input-output structure is equally important, see Huang and Liu (2001*b*).<sup>28</sup> The higher the number of stages of production the more

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<sup>26</sup>Also a high labor supply elasticity can only enhance persistence in a model without capital.

<sup>27</sup>Note that the working paper version was already published in 1996. Yun (1996) also found a strong recognition in the literature.

<sup>28</sup>Interestingly in Huang and Liu (1999) – the working paper version – they show that the results also hold in a model with capital accumulation.

persistent the output response. With a sufficient number of stages the response can even be arbitrarily large, given that the share of intermediates is one at all stages of production. In Huang and Liu (2002) they explore the role of wage staggering in comparison to price staggering and conclude that sticky wages have a higher potential to create a persistent response of output than sticky prices.

Christiano, Eichenbaum and Evans (2003) develop a DGE model that is capable of generating the observed persistence of monetary shocks in US data. These authors estimate their model using a limited information econometric strategy that is not yet common in the literature so that the results are difficult to compare to other existing studies (see also Figure 1.4 for the results of their benchmark model). The aim of their approach is to enable the model to mimic the empirical impulse responses as close as possible. They try to achieve this by incorporating various nominal and real rigidities that have the potential to strengthen the persistence in output and inflation. They show that wage contracts with an average duration of two to three quarters are the critical nominal friction, not price contracts. If inertia in inflation and output persistence is the main goal to match then they show that variable capital utilization is most important. To explain the reaction of all variables they include habit persistence in consumption as well as adjustment costs in investment.

Dotsey and King (2001) stress the importance of variable capital utilization as well. They demonstrate that persistence is possible even in a sticky price model that features labor supply variability through changes in employment and incorporates produced inputs as intermediate goods. All these three ingredients together produce a flat reaction of real marginal costs to a money growth shock. In turn this reduces the extent of price adjustments of the firms. Unfortunately this gradual adjustment of the price level is responsible for the rise in the nominal interest rate: the model does not display the liquidity effect.

Bergin and Feenstra (2000) use a modified DGE model with intermediate goods and so called translog preferences which is essentially a non-CES aggregator for intermediate goods that replaces the Dixit and Stiglitz (1977) aggregator. They show that intermediates in production are very important to generate persistent output responses but they also find a strengthening role for the translog preferences: The higher the share of intermediates in production the higher the persistence.

Intermediates also play an important role in the work of Huang, Liu and Phaneuf (2001). They evaluate the performance of staggered wage models in relation to staggered price models. They show that only a model with intermediates, staggered price and staggered wage setting can explain persistent responses of output and,

depending on the share of intermediates in production, a weak but slightly positive response of the real wage to a monetary shock, as is observed empirically in the postwar period.

Maußner (2002) has proposed a model with wage staggering augmented by adjustment costs of employment and prices at the firm level. This model delivers the best results in a variant with small adjustment costs of labor while otherwise responses are even too strong.

Dib and Phaneuf (2001) discuss a similar model as Maußner but with price staggering instead of wage staggering. In a variant of the model with a nominal rigidity through costly price adjustment and a real rigidity through adjusting the labor input output, hours and real wages show a persistent reaction to a monetary shock. Moreover, the model can explain the decline in hours worked after a productivity shock, as observed in US postwar data.

Kiley (1997*a*) explores the role of efficiency wages as a real rigidity that could probably enhance persistence. He develops a model along the lines of Taylor (1980) without explicit intertemporal optimizing behavior. He finds no important role for efficiency wages in generating endogenous price stickiness (which is equal to persistence in prices) because they do not lower the sensitivity of marginal cost to output. In Kiley (1997*b*) several additional real rigidities are implemented in a fully specified dynamic general equilibrium model. In a one-sector model with increasing returns to scale and an interest rate rule à la Taylor (see Taylor (1993)) output is only persistent when there are implausibly high increasing returns. But in a two-sector model with intermediate goods for consumption and investment a smaller degree of increasing returns is sufficient to generate flat marginal costs and thus persistence in output and inflation. Finally he considers the role of countercyclical markups and concludes that they work together with increasing returns and allow to create persistence if the markup rate is smaller in the investment sector than in the consumption sector. In Kiley (2002) he compares two different ways to model price staggering in a setup with money growth shocks: Taylor and Calvo pricing.<sup>29</sup> Under Calvo (1983) pricing the firms face a specific probability to be able to adjust their price. He can show that both approaches yield fundamentally different results. Output is only persistent under Calvo pricing. In an extension he examines the implications of technology shocks when the central bank follows an interest rate rule. He finds that output persistence in response to a productivity shock is smaller

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<sup>29</sup>It should be noted that he uses the notion ‘partial adjustment’ for Calvo price staggering and ‘staggered price staggering’ only for Taylor pricing.

with Taylor staggering.

Andersen has also contributed significantly to the research on price rigidities. In Andersen (1994) he surveys the progress made by partial equilibrium models in explaining causes as well as consequences of price rigidity. Andersen (1998) compares price and wage staggering in a setup that is similar to Taylor (1980) but generalized to a general equilibrium environment. But he considers a stripped down economy without explicit optimizing behavior of households and firms. He can obtain analytical solutions and finds that price staggering models need to assume a high labor supply elasticity to generate output persistence. On the contrary wage staggering models are very well able to create a persistent response of output to a money growth shock. In Andersen (2004) he can confirm this finding in a fully specified DGE model. He stresses that the interaction of capital accumulation and nominal wage contracts can contribute significantly to the observed persistence in output and inflation.

Ascari has extensively examined the role of staggered wages in monetary DGE models. The basic framework of his approach is presented in Rankin (1998) for one-period wage contracts. Ascari and Rankin (2002) study the output costs of a reduction in the money growth rate under two-period wage contracts. They are able to resolve the puzzle of a boom in output due to a disinflation in ad hoc models with staggered prices. In their setup output falls in a disinflation. In Ascari (1998) he studies the long run implications of a positive steady state money growth rate in a DGE model with two-period wage staggering and shows that output and welfare depend to large extent on this growth rate. This is a very fundamental result which he extends to the dynamics in the context of price staggering in Ascari (2003*b*). He finds that a positive steady state inflation rate has dramatic consequences for the dynamic evolution of output under Calvo pricing. Interestingly this is not the case under Taylor pricing. Ascari (2000) considers Taylor type wage staggering in a fully specified DGE model in the framework of Rankin (1998) and concludes that high persistence is an unlikely outcome. In Ascari (2003*a*) he provides a general common framework for the analysis of Taylor wage and price staggering. He finds that, first, labor immobility plays a major role in generating persistence, and second, this result does not depend on a high labor supply elasticity, as claimed by Andersen (1998).

Bénassy has also concentrated in his work on models with explicit closed form solutions. In Bénassy (2003*b*) he develops a special variant of wage contracts in the spirit of Calvo (1983) and can show that – in contrast to results in Ascari (2000) – output and employment can display a hump-shaped behavior. In Bénassy (2003*a*) he

combines wage and price contracts and finds that one needs both rigidities to obtain hump-shaped responses of both inflation and output to a money growth shock.

Ireland uses the monetary DGE model to study its ability to explain postwar US business cycles. In Ireland (1997) he develops a model with sticky prices through adjustment costs of prices as in Rotemberg (1982). He estimates his model using the maximum likelihood method and finds that most of the empirical variation in output is due to supply side shocks which are either technology or money growth shocks. In Ireland (2001) he extends this approach along three dimensions: First, he lets the data decide whether adjustment costs of prices or of inflation matter. Second, he considers an interest rate rule à la Taylor as the monetary policy instrument. Third, he incorporates adjustment costs of capital. Through formal hypothesis testing he finds instability in the estimated parameters and concludes that further efforts are needed to explain postwar US business cycles with a DGE model.

Carlstrom and Fuerst have highlighted another important aspect of these models: the problem of real indeterminacy or sunspots. Although they do not (always) study models with nominal rigidities the results have important implications for such frameworks as well and are thus briefly summarized here. In Carlstrom and Fuerst (2003) they show that it is very likely to get indeterminacy of the equilibrium when operating with a CIA-constraint combined with a high degree of risk aversion and low (or zero) interest elasticities of money demand in models with money growth rules. Indeterminacy is even more likely in models with Taylor type interest rate rules. In Carlstrom and Fuerst (2001*a*) the authors demonstrate that – again in a flexible price model and a CIA-constraint – real indeterminacy arises whenever the interest moves too closely with current or expected inflation. But if the interest rate responds to past inflation there will be determinacy. Finally, in Carlstrom and Fuerst (2001*b*) they show that the timing of the real balances which enter the utility function in a MIU-setup is of crucial importance for both money growth and interest rate rules. If money enters into the preference function under CIA-timing then this will lead to indeterminacy.

## 1.4 Plan of the Book

The purpose of the present study is to explore the specific contributions of various nominal and real rigidities in monetary DGE models in a systematic way and in a common framework. I will concentrate on a quite simple model setup in order to find out the important transmission mechanisms at work. The focus will be on

exogenous money growth shocks as the driving force of the business cycle and not on interest rate shocks.

The book contains five main chapters. All these chapters are presented in a way that allows the reader to study them separately. Therefore the building blocks of the models will be repeated in every chapter.<sup>30</sup> The first four chapters are concerned with the question which rigidities are essential to explain actual business cycles while Chapter 6 analyzes optimal monetary policy in a stochastic DGE model.

In Chapter 2 the basic model is presented. Prices are set in a staggered way as in Taylor (1980). The chapter addresses two questions that have not yet been answered in the literature. First: Is there a difference between money introduced via a CIA-constraint or via a MIU-specification? Second: Does it matter how the household's preferences look like? The answers are yes in both cases. It turns out that the CIA-model with a standard CRRA utility function can better account for the business cycle. Thus in Chapter 3 the MIU-setup as well as GHH preferences will be discarded. But the model will be augmented by capital accumulation considerations. The chapter considers instead the implications of the price setting scheme: Taylor pricing is compared to Calvo pricing. It turns out that the failure of the basic model to generate persistent output responses is due to Taylor type price staggering. The model version with Calvo pricing can account quite well for the empirical impulse responses, confirming the results of Kiley (2002) in a more general setup.

Chapter 4 considers the role of habits in consumption. While this feature has already been analyzed by others, e.g. Christiano, Eichenbaum and Evans (2003), there is no study that tries to figure out the specific effects of habit formation in isolation. In addition, related studies use Calvo pricing. Here the MIU-model with Taylor price staggering will again be considered in order to examine whether this can improve the model with respect to its ability to create persistence in output. Unfortunately only the response of consumption to a money growth shock can be improved. For a high enough value of the habit persistence parameter consumption can even be hump-shaped, as it is empirically (see Figure 1.4).

Chapter 5 presents a model with Taylor wage staggering and adjustment costs of price changes as in Rotemberg (1982). It turns out that this specific combination is important to get persistent output responses to a money growth shock. When using also Taylor *price* staggering the result breaks down and output and prices will not be persistent. Sticky prices through adjustment costs of prices operate similar as sticky prices under Calvo pricing. When they interact with adjustment costs of

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<sup>30</sup>This procedure is also used in Gerke (2003).



capital they can even strengthen the persistence in output. In the absence of the costs for adjusting the capital stock there are only very moderate effects on output.

Chapter 6 goes a step further. Here the question is not whether a monetary stochastic DGE model can generate persistence but whether a central bank can stabilize the price level as claimed by King and Wolman (1999). The analysis builds upon the framework used before: the household maximizes life-time utility and firms maximize profits. The central bank acts as a social planner that takes into account the optimizing behavior of the household and the firms. Maximizing welfare is then equivalent to maximizing utility of the representative household. It is shown that the result of King and Wolman does not hold under a different specification of the preference function so that in general the monetary authority will not be successful in completely stabilizing the price level, as is also observed empirically.

## Chapter 2

# Price Staggering in a Monetary Stochastic Dynamic General Equilibrium Model with Labor

### 2.1 Introduction

This chapter develops the basic model which will be used as a framework in the following chapters. First, I will describe in detail the problem of maximizing lifetime utility which the household faces. Second, the economy is assumed to consist of intermediate as well as finished goods producing firms. The finished goods producing firm acts as a ‘bundler’ using the intermediate goods as inputs. The final good will be consumed by the household. The intermediate goods producing firms operate under a technology that is linear in labor. Price staggering will be implemented as in Taylor (1980). Finally, money growth is assumed to follow a stochastic process and it is the source of disturbance to which the economy reacts optimally. Business cycles thus arise as optimal responses of households and firms to this nominal shock.<sup>1</sup>

Special attention is given to the way money is introduced and to the form of the utility function to account for persistence. To do so CIA- as well as MIU-models are proposed. The importance of the way money demand is modeled in a dynamic general equilibrium model has not yet been recognized in the literature. There is also no detailed analysis of the role played by the utility function. The results obtained here speak in favor of the setup although the quantitative difference is of minor importance. First, it turns out that the specific form of the utility function has important (qualitative and – for some variables – also quantitative) effects on

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<sup>1</sup>Interest rate shocks will not be considered as the driving force of the business cycle.

the model outcomes. In the CIA-setup a CRRA utility function generates more persistence than GHH preferences. Second, persistent output and inflation responses depend only in part on the value of the elasticity of labor supply with respect to the real wage (as found by Andersen (1998) as well as Chari, Kehoe and McGrattan (2000)). Third, persistence depends also crucially upon the implied money demand function. Persistent output reactions emerge only in the MIU-model with GHH preferences *and* a high value for the elasticity of labor with respect to the real wage. In a CIA-model this result does not hold. Forth, CIA-models generally create more persistence than MIU-models.

These results make clear that it matters how money is introduced. The equivalence result for CIA- and MIU-models in Feenstra (1986) cannot be generalized to a broader setup where utility depends also on leisure and where prices are set in a staggered way. In addition the chapter shows that the contract multiplier in Chari, Kehoe and McGrattan (2000) has to be interpreted carefully as these authors only analyze a MIU-model. The multiplier seems to be different in a CIA-economy. To uncover the different reactions of labor inputs and firms' outputs I do not study a symmetric equilibrium. Instead, I look at firm specific labor inputs and outputs, as in King and Wolman (1999).

The chapter is organized as follows: Section 2.2 describes in detail the different models and the calibration. In Section 2.3 impulse responses are discussed for the CIA- and the MIU-model while in Section 2.4 the business cycle implications will be presented. Section 2.5 concludes and gives some suggestions for future research.

## 2.2 The Models

### 2.2.1 The Household

The representative household is assumed to have preferences over consumption ( $c_t$ ) and leisure ( $1 - n_t$ ). I consider two different sets of functions under two different setups. In the one setup, CIA-models are considered while in the other MIU-models are evaluated. Both will be calculated through for special utility functions. Since they differ for the setups they will be discussed separately below. The first momentary utility function considered under CIA is the one used by King and Wolman (1999) and is given by

$$u(c_t, n_t, a_t) = \frac{\left[ c_t - \frac{a_t \theta}{1+\gamma} n_t^{1+\gamma} \right]^{1-\sigma} - 1}{1 - \sigma} \quad (2.1)$$

Here  $a_t$  is a preference shock that also acts like a productivity shock.  $\theta$  and  $\gamma$  are positive parameters,  $\sigma$  governs the degree of risk aversion. This function is familiar from the analysis of Greenwood, Hercowitz and Huffman (1988) and accordingly labeled GHH preferences. It has the special property that hours worked only depend upon the real wage and not upon consumption (no wealth effects).

The second utility function analyzed under CIA is the standard constant relative risk aversion function (CRRA) used in many Real Business Cycle models.  $\zeta$  measures the relative weight of consumption for the representative agent.

$$u(c_t, n_t, a_t) = \frac{\left[ a_t c_t^\zeta (1 - n_t)^{1-\zeta} \right]^{1-\sigma} - 1}{1 - \sigma} \quad (2.2)$$

It should be noted that in contrast to the standard use of this utility function there is a disturbance  $a_t$  acting like a preference shock.<sup>2</sup>

Under a MIU-specification the corresponding GHH function to (2.1) is given by

$$u\left(c_t, \frac{M_t}{P_t}, n_t, a_t\right) = \frac{\left[ \left( \eta c_t^\nu + (1 - \eta) \left( \frac{M_t}{P_t} \right)^\nu \right)^{\frac{1}{\nu}} - \frac{a_t \theta}{1 + \gamma} n_t^{1 + \gamma} \right]^{1-\sigma} - 1}{1 - \sigma} \quad (2.3)$$

The MIU-specification was - among others - proposed by Sidrauski (1967). Consumers are supposed to have preferences over real money balances  $M_t/P_t$  since they facilitate transactions. They are introduced using a CES function together with consumption. This expression replaces the consumption term in (2.1).  $\eta$  is a share parameter and  $\nu$  will be shown to determine the interest elasticity of the implied money demand function. In case of CRRA preferences the specification in the CES form is embedded in a Cobb-Douglas structure with labor where  $\zeta$  again acts as a weighting parameter.

$$u\left(c_t, \frac{M_t}{P_t}, n_t, a_t\right) = \frac{\left[ a_t \left( \eta c_t^\nu + (1 - \eta) \left( \frac{M_t}{P_t} \right)^\nu \right)^{\frac{\zeta}{\nu}} (1 - n_t)^{1-\zeta} \right]^{1-\sigma} - 1}{1 - \sigma} \quad (2.4)$$

Note that for  $\nu = \eta = 1$  both specifications collapse to their CIA-counterparts. The nonseparability allows to consider the influence of the money demand distortions on the dynamic evolution of consumption and labor because the variables will influence each other as cross derivatives will be non zero.

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<sup>2</sup>King and Wolman (1999) argue that it is necessary in (2.1) to have  $a_t$  affecting equally production and preferences in order to achieve balanced growth. This is doubtful because the model does not explicitly account for growth aspects as, e.g., in King, Plosser and Rebelo (1988).

The intertemporal optimization problem for the household is to maximize lifetime utility subject to an intertemporal budget constraint. In the case of utility function (2.1) and (2.2) it also faces a CIA-constraint. The household is assumed to have access to a bond market and to hold money. Its budget constraint is therefore given by

$$P_t c_t + M_t + B_t = P_t w_t n_t + M_{t-1} + (1 + R_{t-1}) B_{t-1} + \Xi_t + M_t^s \quad (2.5)$$

where

$$\Xi_t = \int_0^1 \Xi_{j,t} dj \quad (2.6)$$

are the nominal profits of the intermediate goods producing firms. The uses of wealth are nominal consumption  $P_t c_t$ , holdings of money balances  $M_t$  and bonds  $B_t$ . The household has several sources of its wealth. It earns money working in the market at the real wage rate  $w_t$  ( $P_t w_t n_t$ ) and can spend its money holdings carried over from the previous period ( $M_{t-1}$ ). There are also previous period bond holdings including the interest on them  $(1 + R_{t-1}) (B_{t-1})$ . Finally, the household receives a monetary transfer  $M_t^s$  from the monetary authority and the profits from the intermediate goods firms  $\Xi_t$ , respectively. This transfer is equal to the change in money balances, i.e.

$$M_t^s = M_t - M_{t-1} \quad (2.7)$$

For utility functions (2.1) and (2.2) the household faces a CIA-constraint. It can consume only out of cash balances it has received before. This condition is therefore given by<sup>3</sup>

$$P_t c_t \leq M_{t-1} + M_t^s \quad (2.8)$$

The Lagrangian for the household in case of utility function (2.1) and (2.2) (index  $H1$ ) (CIA-model) can then be written as follows:

$$\begin{aligned} L_{H1} = & E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c_t, n_t, a_t) \right. \\ & + \sum_{t=0}^{\infty} \beta^t \lambda_t \left( w_t n_t + m_{t-1} \frac{P_{t-1}}{P_t} + \frac{\Xi_t}{P_t} + m_t^s \right. \\ & \quad \left. \left. + (1 + R_{t-1}) b_{t-1} \frac{P_{t-1}}{P_t} - c_t - m_t - b_t \right) \right. \\ & \left. + \sum_{t=0}^{\infty} \beta^t \Omega_t \left( m_{t-1} \frac{P_{t-1}}{P_t} + m_t^s - c_t \right) \right] \quad (2.9) \end{aligned}$$

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<sup>3</sup>The formulation of the CIA-constraint, the monetary transfer and the intertemporal budget constraint is consistent with the timing in Walsh (1998), pp. 100-102.

Here small variables indicate real quantities, i.e. for example  $m_t = M_t/P_t$ . Households optimize over  $c_t, n_t, m_t$  and  $b_t$  taking prices and the initial values of the price level  $P_0$  as well as the outstanding stocks of money  $M_0$  and bonds  $B_0$  as given. The first order conditions for an interior solution are reported below.

$$\frac{\partial L_{H1}}{\partial c_t} = \beta^t \frac{\partial u(c_t, n_t, a_t)}{\partial c_t} - \beta^t \lambda_t - \beta^t \Omega_t = 0 \quad (2.10)$$

$$\frac{\partial L_{H1}}{\partial n_t} = \beta^t \frac{\partial u(c_t, n_t, a_t)}{\partial n_t} + \beta^t \lambda_t w_t = 0 \quad (2.11)$$

$$\frac{\partial L_{H1}}{\partial m_t} = -\beta^t \lambda_t + E_t \beta^{t+1} \lambda_{t+1} \frac{P_t}{P_{t+1}} + E_t \beta^{t+1} \Omega_{t+1} \frac{P_t}{P_{t+1}} = 0 \quad (2.12)$$

$$\frac{\partial L_{H1}}{\partial b_t} = -\beta^t \lambda_t + E_t \beta^{t+1} \lambda_{t+1} (1 + R_t) \frac{P_t}{P_{t+1}} = 0 \quad (2.13)$$

The derivatives with respect to  $\lambda_t$  and  $\Omega_t$  are omitted since they are equal to the budget constraint and the CIA-constraint, respectively. It should be noted that these conditions result from the more general Kuhn-Tucker conditions assuming that all variables and multipliers are strictly positive. This implies especially that - given  $\Omega_t > 0$  - the CIA-constraint is always binding and that the nominal interest rate  $R_t$  is positive. Otherwise (2.12) and (2.13) will not be compatible. In addition the household's optimal choices must also satisfy the transversality conditions:

$$\lim_{t \rightarrow \infty} \beta^t \lambda_t x_t = 0 \quad \text{for } x = m, b \quad (2.14)$$

The familiar result that the first two efficiency conditions imply the equality of the marginal rate of substitution between consumption and labor and the real wage does not hold here because of the CIA-constraint. Instead one gets

$$w_t = -\frac{1}{\beta} E_t \left( \frac{\frac{\partial u(c_t, n_t, a_t)}{\partial n_t}}{\frac{\partial u(c_{t+1}, n_{t+1}, a_{t+1})}{\partial c_{t+1}}} \frac{P_{t+1}}{P_t} \right) \quad (2.15)$$

This equation can be derived by eliminating  $\Omega_t$  in the efficiency condition for consumption using the efficiency condition for money. There is a different timing of the marginal utility of consumption and labor which alters the dynamics of the real wage. In addition there is a direct influence of inflation. The marginal utility of consumption is given by  $(1 + R_{t-1}) \lambda_t$  so that the nominal interest rate acts like a tax on consumption.

The efficiency condition for bond holdings establishes a relation between the nominal interest rate and the price level. Rearranging terms yields

$$(1 + R_t) = E_t \left( \frac{\lambda_t}{\lambda_{t+1}} \frac{1}{\beta} \frac{P_{t+1}}{P_t} \right) \quad (2.16)$$

Supposed the Fisher equation is valid the real interest rate  $r_t$  is implicitly defined as

$$(1 + r_t) = E_t \left( \frac{\lambda_t}{\lambda_{t+1}} \frac{1}{\beta} \right) \quad (2.17)$$

because  $P_{t+1}/P_t$  equals one plus the rate of expected inflation which is approximated by the ex-post-inflation rate.

In case of the MIU-model the CIA-constraint is dropped since money demand will be determined endogenously through the derivative with respect to  $m_t$ . In this case  $m_t$  shows up in the utility function, of course. So the Lagrangian (index  $H2$ ) will be given by

$$\begin{aligned} L_{H2} = & E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c_t, m_t, n_t, a_t) \right. \\ & + \sum_{t=0}^{\infty} \beta^t \lambda_t \left( w_t n_t + m_{t-1} \frac{P_{t-1}}{P_t} + \frac{\Xi_t}{P_t} + m_t^s \right. \\ & \left. \left. + (1 + R_{t-1}) b_{t-1} \frac{P_{t-1}}{P_t} - c_t - m_t - b_t \right) \right] \end{aligned} \quad (2.18)$$

In order to compare both setups the first order conditions are again reported.

$$\frac{\partial L_{H2}}{\partial c_t} = \beta^t \frac{\partial u(c_t, m_t, n_t, a_t)}{\partial c_t} - \beta^t \lambda_t = 0 \quad (2.19)$$

$$\frac{\partial L_{H2}}{\partial n_t} = \beta^t \frac{\partial u(c_t, m_t, n_t, a_t)}{\partial n_t} + \beta^t \lambda_t w_t = 0 \quad (2.20)$$

$$\frac{\partial L_{H2}}{\partial m_t} = \beta^t \frac{\partial u(c_t, m_t, n_t, a_t)}{\partial m_t} - \beta^t \lambda_t + E_t \beta^{t+1} \lambda_{t+1} \frac{P_t}{P_{t+1}} = 0 \quad (2.21)$$

$$\frac{\partial L_{H2}}{\partial b_t} = -\beta^t \lambda_t + E_t \beta^{t+1} \lambda_{t+1} (1 + R_t) \frac{P_t}{P_{t+1}} = 0 \quad (2.22)$$

The derivatives with respect to  $n_t$  and  $b_t$  are essentially the same as for  $H1$ . As before,  $P_0, M_0$  and  $B_0$  are given and the transversality conditions hold. In the consumption Euler equation the influence of the second Lagrange multiplier  $\Omega_t$  disappears eliminating the dynamics present in the CIA-model. Now the marginal utility of consumption is just equal to the shadow price  $\lambda_t$ , there is no consumption tax working through the nominal interest rate. But in the efficiency condition for money the marginal utility of real balances has to be considered. This derivative determines the endogenous money demand function. Combining the optimum conditions for consumption, bonds and money yields the following equation:

$$\frac{\partial u(c_t, m_t, n_t, a_t)}{\partial m_t} = \frac{\partial u(c_t, m_t, n_t, a_t)}{\partial c_t} \frac{R_t}{1 + R_t} \quad (2.23)$$

This specification allows to estimate an empirical money demand function. A detailed description will be presented in the calibration section. The Taylor approximations are given in Appendix A.

Two important implications come out right here. First, the real wage rate will be determined by the usual marginal rate of substitution between consumption and labor, in contrast to the additional dynamics in the CIA-model (see (2.15)).

$$w_t = -\frac{\frac{\partial u(c_t, n_t, a_t)}{\partial n_t}}{\frac{\partial u(c_t, n_t, a_t)}{\partial c_t}} \quad (2.24)$$

Second, the implied money demand function is independent of the specific form of the monetary transfer  $M_t^s$  and, in addition, it depends directly upon the nominal interest rate (see (2.23)).

## 2.2.2 The Finished Goods Producing Firm

The firm producing the final good  $c_t = y_t$  in the economy uses  $c_{j,t}$  units of each intermediate good  $j \in [0, 1]$  purchased at price  $P_{j,t}$  to produce  $c_t$  units of the finished good. The production function is assumed to be a CES aggregator as in Dixit and Stiglitz (1977) with  $\epsilon > 1$ .

$$c_t = \left( \int_0^1 c_{j,t}^{(\epsilon-1)/\epsilon} dj \right)^{\epsilon/(\epsilon-1)} \quad (2.25)$$

The firm maximizes its profits over  $c_{j,t}$  given the above production function and given the price  $P_t$ . So the problem can be written as

$$\max_{c_{j,t}} \left[ P_t c_t - \int_0^1 P_{j,t} c_{j,t} dj \right] \text{ s.t. } c_t = \left( \int_0^1 c_{j,t}^{(\epsilon-1)/\epsilon} dj \right)^{\epsilon/(\epsilon-1)} \quad (2.26)$$

The first order conditions for each good  $j$  imply

$$c_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{-\epsilon} c_t \quad (2.27)$$

where  $-\epsilon$  measures the constant price elasticity of demand for each good  $j$ . Since the firm operates under perfect competition it does not make any profits. Inserting the demand function into the profit function and imposing the zero profit condition reveals that the only price  $P_t$  that is consistent with this requirement is given by

$$P_t = \left( \int_0^1 P_{j,t}^{(1-\epsilon)} dj \right)^{1/(1-\epsilon)} \quad (2.28)$$



In case that prices are fixed for just two periods and assuming that all price adjusting producers in a given period choose the same price the consumption aggregate can be written as

$$c_t = c(c_{0,t}, c_{1,t}) = \left( \frac{1}{2}c_{0,t}^{(\epsilon-1)/\epsilon} + \frac{1}{2}c_{1,t}^{(\epsilon-1)/\epsilon} \right)^{\epsilon/(\epsilon-1)} \quad (2.29)$$

where  $c_{j,t}$  can then be interpreted as the quantity of a good consumed in period  $t$  whose price was set in period  $t - j$ . Similarly in the two period price setting case to be explored in detail in the next section the price equation simplifies. With prices set for two periods half of the firms adjust their price in period  $t$  and half do not. Moreover all adjusting firms choose the same price. Then  $P_{j,t}$  is the nominal price at time  $t$  of any good whose price was set  $j$  periods ago and  $P_t$  is the price index at time  $t$  and is given by

$$P_t = \left( \frac{1}{2}P_{0,t}^{1-\epsilon} + \frac{1}{2}P_{1,t}^{1-\epsilon} \right)^{1/(1-\epsilon)} \quad (2.30)$$

### 2.2.3 The Intermediate Goods Producing Firm

Intermediate good firms can be considered to consist of a producing and a pricing unit. The producing unit operates under a technology that is linear in labor  $n_{j,t}$  and subject to random productivity shocks  $a_t$ .<sup>4</sup>

$$y_{j,t} = c_{j,t} = a_t n_{j,t} \quad (2.31)$$

Here  $n_{j,t}$  is the labor input employed in period  $t$  by a firm who set the price in period  $t - j$ . Firms always meet the demand for their product, that is  $y_{j,t} = c_{j,t}$ . Those who do not adjust their prices in a given period can be interpreted as passive while those who do adjust do so optimally.

The pricing unit sets prices to maximize the present discounted value of profits whereas the producing unit chooses labor to minimize costs.<sup>5</sup> In case of the models considered here there is no capital so the costs are solely given by the wage bill. Thus minimizing  $P_t w_t n_{j,t}$  with respect to  $n_{j,t}$  subject to the production function implies for the total cost function  $TC_{j,t}$ <sup>6</sup>

$$TC_{j,t} = \frac{P_t w_t c_{j,t}}{a_t} \quad (2.32)$$

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<sup>4</sup>There are no diminishing returns to labor.

<sup>5</sup>The model deviates in this respect from the standard textbook model in which profits are maximized over the quantity.

<sup>6</sup>It should be noticed that the wage is perfectly flexible in a competitive input market. So there is no index  $j$  for  $w_t$  and  $P_t$  which means that they are not firm-specific.

With only one factor of production one can just express the labor input by manipulating the production function so that  $n_{j,t} = c_{j,t}/a_t$  and insert this into the wage bill equation. It is useful for further calculations to define nominal marginal cost as  $\Psi_t$  which is equal to  $(\partial TC_{j,t}/\partial c_{j,t}) = P_t w_t/a_t$ . Thus real marginal costs are given by  $\psi_t = w_t/a_t$ . With a relative price defined by  $p_{j,t} = P_{j,t}/P_t$  real profit  $\xi_{j,t} = \Xi_{j,t}/P_t$  for a firm of type  $j$  is equal to

$$\xi_{j,t} = p_{j,t}c_{j,t} - w_t n_{j,t} \quad (2.33)$$

Using the demand function for the intermediate goods ( $c_{j,t} = p_{j,t}^{-\epsilon} c_t = a_t n_{j,t}$ ) and the definition of real marginal costs given above the profit function can be rewritten as

$$\xi_{j,t} = \xi(p_{j,t}, c_t, \psi_t) = p_{j,t}c_{j,t} - \psi_t c_{j,t} = c_{j,t}(p_{j,t} - \psi_t) = p_{j,t}^{-\epsilon} c_t (p_{j,t} - \psi_t) \quad (2.34)$$

In the case in which prices are not sticky the firm can just set prices on a period by period basis optimizing the profit function (2.34) with respect to  $p_{j,t}$ . The result of this exercise would be that relative prices will have to be set according to

$$p_{j,t} = \frac{\epsilon}{\epsilon - 1} \psi_t \quad (2.35)$$

Thus the optimal price is just a constant markup over real marginal costs. But when prices are fixed for two periods the firm has to take into account the effect of the price chosen in period  $t$  on current and future profits. The price in period  $t + 1$  will be affected by the gross inflation rate  $\Pi_{t+1}$  between  $t$  and  $t + 1$  ( $\Pi_{t+1} = P_{t+1}/P_t$ ).

$$p_{1,t+1} = \frac{p_{0,t}}{\Pi_{t+1}} \quad (2.36)$$

If there is positive inflation,  $p_{1,t+1}$  will fall because nominal prices are fixed for two periods. As the nominal price in period  $t$  is defined by  $P_{0,t}$  and in period  $t + 1$  by  $P_{1,t+1}$ , one has  $P_{0,t} = P_{1,t+1}$ , so that  $p_{0,t} = P_{0,t}/P_t$  and  $p_{1,t+1} = P_{1,t+1}/P_{t+1} = (P_{0,t}/P_t)(P_t/P_{t+1})$  which is what is stated in (2.36). So the optimal relative price has to balance the effects due to inflation between profits today and tomorrow. This intertemporal maximization problem is formally given by

$$\begin{aligned} \max_{p_{0,t}} E_t \left[ \xi(p_{0,t}, c_t, \psi_t) + \beta \frac{\lambda_{t+1}}{\lambda_t} \xi(p_{1,t+1}, c_{t+1}, \psi_{t+1}) \right] \\ \text{s.t. } p_{1,t+1} = \frac{p_{0,t}}{\Pi_{t+1}} \end{aligned} \quad (2.37)$$

The term  $\lambda_{t+1}/\lambda_t$  is equal to the ratio of future to current marginal utility of labor and the respective real wage ratio (derived in the household's optimization problem) and considered to be - in conjunction with  $\beta$  - the appropriate discount factor for real

profits. This is a consequence of the assumption that households own the production factor labor and rent it to the firms. They also own a diversified portfolio of claims to the profits earned by the firms. Although there will be no asset accumulation in equilibrium  $\lambda_t$  can be used to determine the present value of profits.<sup>7</sup> The efficiency condition for this problem is given by

$$0 = \frac{\partial \xi(p_{0,t}, c_t, \psi_t)}{\partial p_{0,t}} + \beta E_t \left( \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial \xi(p_{1,t+1}, c_{t+1}, \psi_{t+1})}{\partial p_{1,t+1}} \frac{1}{\Pi_{t+1}} \right) \quad (2.38)$$

Multiplying this equation by  $p_{0,t}$  and  $\lambda_t$  produces a more symmetric form of the efficiency condition that will be more convenient to derive the model solution later.

$$0 = \lambda_t p_{0,t} \frac{\partial \xi(p_{0,t}, c_t, \psi_t)}{\partial p_{0,t}} + \beta E_t \left( \lambda_{t+1} p_{1,t+1} \frac{\partial \xi(p_{1,t+1}, c_{t+1}, \psi_{t+1})}{\partial p_{1,t+1}} \right) \quad (2.39)$$

Using (2.34) one can solve this condition for the optimal price to be set in period  $t$  which corresponds to the optimal price in case that prices are flexible derived before. This yields a forward-looking form of the price equation and is in that respect similar to the one in Taylor (1980).

$$p_{0,t} = \frac{\epsilon}{\epsilon - 1} \left( \frac{\lambda_t c_t \psi_t + \beta E_t \lambda_{t+1} (P_{t+1}/P_t)^\epsilon c_{t+1} \psi_{t+1}}{\lambda_t c_t + \beta E_t \lambda_{t+1} (P_{t+1}/P_t)^{\epsilon-1} c_{t+1}} \right) \quad (2.40)$$

The optimal relative price  $p_{0,t}$  depends upon the current and future real marginal costs, the gross inflation rate, current and future consumption as well as today's and tomorrow's interest rates (through the influence of the  $\lambda$ -terms). It is thus fundamentally different from the one derived under fully flexible prices on a period-by-period basis (see (2.35)). (2.40) can be manipulated in a way that yields a form which is exactly equal to the one studied in Walsh (1998), p. 197, when using (2.16) for the interest rate factor. To derive the Taylor approximation in Appendix A it is useful to write (2.40) as

$$P_{0,t} = \frac{\epsilon}{\epsilon - 1} \frac{\lambda_t P_t^\epsilon c_t \psi_t + \beta E_t \lambda_{t+1} P_{t+1}^\epsilon c_{t+1} \psi_{t+1}}{\lambda_t P_t^{\epsilon-1} c_t + \beta E_t \lambda_{t+1} P_{t+1}^{\epsilon-1} c_{t+1}} \quad (2.41)$$

Finally, aggregate labor demand must be equal to the aggregate labor supply of the household.<sup>8</sup>

$$n_t = \frac{1}{2} n_{0,t} + \frac{1}{2} n_{1,t} \quad (2.42)$$

<sup>7</sup>More details on this can be found in Dotsey, King and Wolman (1999), p. 659-665 as well as in Dotsey, King and Wolman (1997), p. 9-13.

<sup>8</sup>The factor 0.5 shows up because  $n_{j,t}$  is labor hired per  $j$ -type firm and half the firms are of each type.

### 2.2.4 Market Clearing Conditions and Other Equations

It is well known that models like the one at hand imply multiple equilibria and sunspots because bonds are not determined. Carlstrom and Fuerst (2001*b*) include bond holdings in their CIA-constraint. Their equation reads

$$P_t c_t \leq M_{t-1} + M_t^s + (1 + R_{t-1}) B_{t-1} - B_t \quad (2.43)$$

The monetary transfer then includes also new bonds

$$M_t^s = M_t - M_{t-1} + B_t - (1 + R_{t-1}) B_{t-1} \quad (2.44)$$

Carlstrom and Fuerst show that this specification leads to a model solution with multiple equilibria under interest rate rules. The same result holds when using money growth rules (see their footnote 10) and it also holds in this model. To escape this problem the household budget constraint is dropped and bonds are set to zero:  $b_t = 0$  for all  $t$ .<sup>9</sup> Note that due to Walras' law the intertemporal budget constraint will also hold in equilibrium.

In the CIA-model the implicit money demand function is derived by substituting  $M_t^s$  in the CIA-constraint - holding with equality. This implies:

$$M_t = P_t c_t \quad (2.45)$$

It is essentially a quantity theoretic type of money demand.

In the MIU-model the efficiency condition for money determines the money demand function, of course (see the discussion of (2.23)).

The markup  $\mu_t$  is just the reciprocal of real marginal cost so that

$$\mu_t = \frac{1}{\psi_t} \quad (2.46)$$

### 2.2.5 The Monetary Authority

The model is closed by adding a monetary policy rule. Therefore an exogenous process for the money growth rate is considered. To achieve persistent but non permanent effects the level of money follows an AR(2)-process. Assume that money grows at a factor  $g_t$ :

$$M_t = g_t M_{t-1} \quad (2.47)$$

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<sup>9</sup>See Flodén (2000), p. 1413. He argues that bonds are introduced to determine the nominal interest rate.

If  $\hat{g}_t$  follows an AR(1)-process  $\hat{g}_t = \rho_g \hat{g}_{t-1} + \epsilon_{g_t}$  then money will follow an AR(2)-process.<sup>10</sup> Note that inflation is zero at the steady state so also money growth is zero there ( $g = 1$ ).

There is another shock in the model, namely the productivity shock  $a_t$ . As is clear from the utility functions this shock can also act as a taste shock. So one can easily analyze the model's impulse responses to this productivity/ taste shock. Under these circumstances  $\hat{a}_t$  follows an AR(1)-process

$$\hat{a}_t = \rho_a \hat{a}_{t-1} + \epsilon_{a_t} \quad (2.48)$$

with  $\epsilon_{a_t}$  white noise and  $0 < \rho_a < 1$ .

## 2.2.6 The Steady State

Imposing the condition of constancy of the price level in the steady state ( $P_t = P_{t-1} = P$ ) on the nominal interest rate equation reveals the familiar condition from RBC models that  $\beta = 1/(1 + R)$ . In addition, as there is no steady state inflation,  $R = r$ . The two period price setting of the firms implies  $P_0 = P_1$ . Using this in the price index reveals that  $P_0 = P_1 = P$ . Then the demand functions for  $c_0$  and  $c_1$  (2.27) imply  $c_0 = c_1$ . Inserting this in the Dixit/Stiglitz-aggregator (2.29) one gets the result that all consumption levels are equal:  $c_0 = c_1 = c$ . For the markup it follows  $\mu = 1/\psi$  while  $\psi$  is determined by the steady state of the efficiency condition for maximizing profits, (2.41). This amounts to  $\psi = (\epsilon - 1)/\epsilon$ . Then the real wage is given by  $w = a\psi = a/\mu$ . Finally, the production functions for  $c_0$  and  $c_1$  imply that  $n_0 = n_1$ . In the aggregate this implies  $n = n_0 = n_1$  using equation (2.42) and also  $c = an$ . In case of the CIA-model (2.15) is used to pin down the preference parameter, which is either  $\theta$  or  $\zeta$ . This implies  $\theta = \beta(1/\mu)(1/n^\gamma)$  and  $\zeta = c/[\beta(w - wn) + c]$ .

For the MIU-model with CRRA preferences the marginal rate of substitution between consumption and labor can also be used to calculate the preference parameter  $\zeta$ .<sup>11</sup> Using (2.23) the ratio of  $m$  over  $c$  depends only upon  $\beta, \eta$  and  $\nu$ .

$$m = c \left[ \frac{\eta}{1 - \eta} (1 - \beta) \right]^{\frac{1}{\nu-1}} \quad (2.49)$$

<sup>10</sup>A hat ( $\hat{\cdot}$ ) represents the relative deviation of the respective variable from its steady state (see the Appendix).  $\rho_g$  lies between 0 and 1 and  $\epsilon_{g_t}$  is white noise.

<sup>11</sup>Remember that this ratio is not the same as (2.15) but the standard formula which results from combining the efficiency conditions for consumption and labor.

In turn  $\zeta$  can be determined as a function of these parameters and  $c, w$  and  $n$ .

$$\zeta = \frac{c}{(1-n)} \Theta \left[ w + \frac{c}{1-n} \Theta \right]^{-1} \quad (2.50)$$

with

$$\Theta = 1 + (1-\beta)^{\frac{\nu}{\nu-1}} \left( \frac{\eta}{1-\eta} \right)^{\frac{1}{\nu-1}} \quad (2.51)$$

In the MIU-model with GHH preferences  $m$  is also given by (2.49). Then  $\theta$  changes to

$$\theta = \frac{1}{\mu} \frac{1}{n^\gamma} \left[ c^\nu \left( \eta + (1-\eta) \left( (1-\beta) \frac{\eta}{1-\eta} \right)^{\frac{\nu}{\nu-1}} \right) \right]^{\frac{1-\nu}{\nu}} \eta c^{\nu-1} \quad (2.52)$$

### 2.2.7 Calibration

To compute impulse responses the parameters of the model have to be calibrated. Some parameters depend upon the specific utility function used so it is useful to look at first at the parameters which are independent of these.

It is possible to either specify  $\beta$  or  $r$  exogenously. Here  $\beta$  will be set to 0.99 implying a value of  $r$  of about 0.0101 per quarter which is in line with other values used for the real interest rate in the literature.  $\psi$  and  $\mu$  can be determined by fixing a value for the elasticity of the demand functions for the differentiated products. This elasticity being equal to 6 causes the static markup  $\mu = \epsilon/(\epsilon - 1)$  to be 1.2 which is the mean value found in the study of Linnemann (1999) about average markups. In order to determine the steady state real wage  $w$  the productivity shock  $a$  has to be specified. As there is no information available about that parameter it is arbitrarily set at 10.<sup>12</sup> Either  $n$  or  $c$  have to be set exogenously to calculate  $c = an$ . Because more information is available about hours worked,  $n$  is specified to be equal to 0.25 implying that agents work 25 % of their non-sleeping time.

In the benchmark case,  $\sigma$ , the parameter governing the degree of risk aversion, is set to 2 in all models. For GHH preferences  $\gamma$  has to be specified. To make results comparable to the CRRA utility function  $\gamma$  is set to  $\bar{1.3}$  which implies the same static steady state elasticity of labor supply with respect to the real wage. In the sensitivity analysis the value will be changed to 0.1. The implied value of  $\theta$  under CIA is 5.2384.

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<sup>12</sup>In contrast to the well known basic neoclassical model of King, Plosser and Rebelo (1988) there is no escape from specifying parameters such as  $a$  at the steady state. The system cannot be reduced until only deep parameters remain to be calibrated.

Using the CRRA preference specification under CIA the parameter  $\zeta$  can be calculated using equation (2.15) which implies  $\zeta = 0.2878$ , a value that is reasonably in line with other studies.

In the MIU-model, both for CRRA and GHH preferences, the parameters  $\nu$  and  $\eta$  are calibrated by estimating an empirical money demand function the form of which is implied by the efficiency conditions of the household. This functional form is obtained by solving (2.23) for  $m_t$  and taking logarithms:

$$\ln m_t = \frac{1}{\nu - 1} \ln \frac{\eta}{1 - \eta} + \frac{1}{\nu - 1} \ln \left( \frac{R_t}{1 + R_t} \right) + \ln c_t \quad (2.53)$$

Estimates of Chari, Kehoe and McGrattan (2000) reveal that  $\eta = 0.94$  and  $\nu = -1.56$ . They use US data from Citibase covering 1960:1-1995:4 regressing the log of consumption velocity  $\ln(m_t/c_t)$  on the log of the interest rate  $\ln(R_t/(1 + R_t))$ . Since the focus is on the qualitative results of the model the money demand function is not estimated for specific German or other data. For CRRA utility the implied value of  $\zeta$  changes slightly to 0.2899 while  $m/c$  is equal to 2.06. Under GHH preferences  $\theta = 5.3240$ .

For the exogenous money growth process  $\rho_g = 0.5$  is used. As the focus of the model is on the persistence effects of money growth shocks productivity shocks will not be considered. But they can be used to check whether the model displays reasonable impulse responses to technology shocks.

## 2.3 Impulse Response Functions

The solution is conducted using an extended version of the algorithm of King, Plosser and Rebelo (2002) which allows for singularities in the system matrix of the reduced model. This algorithm builds upon the Blanchard and Kahn (1980) approach for solving a system of linear stochastic difference equations. The theoretical background is developed in King and Watson (1999) whereas computational aspects and the implementation are discussed in King and Watson (2002).

### 2.3.1 CIA-Model

Because results differ it is useful to subdivide this subsection in two further sections containing results for the GHH preferences and for the standard CRRA utility function.

### 2.3.1.1 GHH Preferences

Here the impulse responses of the model variables to a 1% shock to the money growth rate will be discussed. Figures 2.1 – 2.2 display the reaction of selected variables to this shock in the benchmark calibration. The reaction of  $\widehat{c}_{0,t}$  and of the prices are the most persistent ones of the variables under observation. Using as a metric of persistence the ratio of the period  $t + 1$  reaction of a variable to the period  $t$  reaction as proposed by Andersen (2004) for two period contracts – defined as the contract multiplier in Huang and Liu (2002) – reveals a value of 0.08 for  $\widehat{c}_{0,t}$  which can be considered as very low compared with Andersen’s results.<sup>13</sup> Real marginal costs as well as consumption of non-adjusting firms show a cyclical reaction which is counterfactual. Aggregate consumption rises on impact and falls immediately below the steady state in the next period. There is some persistence after the initial positive impact, beginning in the second quarter. Unfortunately the persistence consists of a tendency of aggregate consumption to remain below its steady state level for several successive periods. This is a feature not empirically observed either. Real marginal costs display a strong increase which amounts to a quite strong rise in the price firms set when they are allowed to do so. But it takes some 7 or 8 periods for the price level to reach the new equilibrium value so one can conclude that prices show at least some persistence. Inflation shows a hump as it does empirically. The decline in the real interest rate is more than three times the rise in the nominal rate. As for many dynamic general equilibrium models with sticky prices also this one fails to generate the liquidity effect (a falling nominal interest rate). But the nominal rate reacts quite persistently with a contract multiplier of 0.63.

In the literature several authors argue in favor of models generating flat marginal cost curves because then there is little incentive for firms to raise prices. In turn money growth shocks can have persistent effects on output. In case of the GHH utility function the static steady state elasticity of real marginal cost with respect to output is constant and equal to  $\gamma$ .

$$\frac{\partial \psi}{\partial c} \frac{c}{\psi} = \gamma \quad (2.54)$$

In the benchmark case  $\gamma$  was calibrated to be 1.3. Changing this value to 0.1 would considerably reduce this elasticity and would probably enhance the persistence effects of money growth shocks in the model. But a low value for this elasticity

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<sup>13</sup>His values for output range between 0.55 and 0.87. A variable that is cyclical is not persistent at all in this definition. Note that Chari, Kehoe and McGrattan (2000) use a different definition of the contract multiplier.



implies at the same time a high static steady state elasticity of labor supply with respect to the real wage. Formally this elasticity is given by

$$\frac{\partial n}{\partial w} \frac{w}{n} = \frac{1}{\gamma} \quad (2.55)$$

and it is equal to 10 here. In light of empirical estimates of the labor supply elasticity this value must be regarded as too high. Does the intuition from above of a low real marginal cost elasticity hold in this model? Figures 2.3 – 2.4 show the results. There is a smoother reaction but again consumption is cyclical approaching the new steady state from below. But  $\widehat{c}_{0,t}$  displays considerably more persistence than before with a contract multiplier of 0.53. This is also true for real marginal costs  $\widehat{\psi}_t$  but they react stronger than 0.1% as could have been expected due to the low output elasticity. Note that the price level now overshoots its new equilibrium value of 2 quite strongly, contrasting the result in Figure 2.2 for a higher value of  $\gamma$ .

How is a monetary policy shock transmitted in this model? As real marginal costs are proportional to the real wage (see (A.16)) and the responsiveness of the optimal price of price setting firms is determined largely by the reaction of real marginal costs it is useful to examine (2.15) carefully. It is repeated here for convenience.

$$w_t = -\frac{1}{\beta} E_t \left( \frac{\frac{\partial u(c_t, n_t, a_t)}{\partial n_t}}{\frac{\partial u(c_{t+1}, n_{t+1}, a_{t+1})}{\partial c_{t+1}}} \frac{P_{t+1}}{P_t} \right) \quad (2.56)$$

There are two important aspects which are crucial for the transmission of a money growth shock. First, the time  $t$  reaction of  $w_t$  is not only determined by time  $t$  derivatives and variables, i.e. the marginal disutility of work  $\partial u(c_t, n_t, a_t) / \partial n_t$  and the price level  $P_t$ , but also by time  $t + 1$  variables. This is a direct consequence of the CIA-setup analyzed here. Second, the utility function itself determines the strength of the reaction of the real wage rate through the respective marginal utilities of consumption and leisure. This is the influence of the different utility functions which are considered here. Now for GHH preferences (2.56) has a very simple form given by

$$w_t = \frac{1}{\beta} E_t a_t \theta n_t^\gamma \frac{\frac{\partial u(c_t, n_t, a_t)}{\partial c_t}}{\frac{\partial u(c_{t+1}, n_{t+1}, a_{t+1})}{\partial c_{t+1}}} \frac{P_{t+1}}{P_t} \quad (2.57)$$

An expansionary money growth shock leads to an increase in real aggregate demand so that firms have to hire more workers. In turn labor  $n_t$  goes up. Consumption  $c_t$  rises leading to a fall in  $\partial u / \partial c_t$ . Moreover the price level rises due to the increase in money. In  $t + 1$  the price level will rise further. But consumption in  $t + 1$  will fall

leading to a rise in  $\partial u/\partial c_{t+1}$ .<sup>14</sup> So the effect on  $w_t$  is in general not definite since the numerator can rise or fall depending on the relative strength of the monetary shock on labor and consumption and on the increase in  $P_{t+1}$ . The denominator will definitely increase. As can be seen from the impulse responses quantitatively the rise in  $n$  dominates the fall in marginal utility of consumption, the increase in the price level and the rise of the denominator so  $w_t$  rises. Thus also  $\psi_t$  rises. The extent of this rise is also determined by  $\gamma$ . With a small value of the elasticity of real marginal costs with respect to output the increase in the real wage and thus real marginal costs will be smaller than with a higher value. This is quite clear from (2.57) since a low value of  $\gamma$  implies a small exponent on labor  $n_t$  and in turn a moderate reaction of  $w_t$  while with  $\gamma=1.3$  the response will be stronger. These results exactly correlate with the impulse responses. Real marginal costs react only with a 0.3% deviation from steady state for a low  $\gamma$  while otherwise there is a 0.9% deviation, three times as large.

The reason why the variant of the model with a low elasticity of real marginal costs with respect to output fails to generate a persistent output reaction is thus also related to the implied money demand function which is essentially of a quantity theoretic type here. The inclusion of a CIA-constraint alters significantly the dynamics of the model which becomes very obvious in the combined efficiency condition (2.56). This in turn leads to quite complicated dynamics of real marginal costs and the optimal reset price  $\hat{P}_{0,t}$ .

Before exploring this preference specification in the MIU-model let's turn to the CRRA utility function first.

### 2.3.1.2 CRRA Preferences

Figures 2.5 – 2.6 summarize the impulse responses in the model with CRRA preferences (see (2.2)). At first glance these graphs seem to be very similar to those under GHH preferences. But there are some small interesting differences. First, there is a reduced cyclicity of the real interest rate and real marginal costs. Nevertheless aggregate consumption rises only on impact and approaches the steady state from below. Second, the reaction of  $\hat{c}_{0,t}$  is smoother showing no kink as under GHH util-

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<sup>14</sup>This is already a result which is due to increased real marginal costs. Non-price adjusting firms can only react to these increased marginal costs by lowering their output and thus consumption because  $c = y$ . The same holds for the price level that rises because firms raise their optimal price in response to higher real marginal costs. Due to the forward-looking nature of the efficiency condition for wages in the CIA-setup one has to rely partly on a result that one just wants to derive.

ity. The same holds for prices and inflation (compare Figures 2.5 and 2.1 as well as 2.6 and 2.2).

This is an interesting result pointing out the role played by the utility function. For the CRRA utility function the static steady state elasticity of labor supply with respect to the real wage rate depends only on the value of hours worked at the steady state,  $n$ .<sup>15</sup>

$$\frac{\partial n}{\partial w} \frac{w}{n} = 1 - n \quad (2.58)$$

This implies a value of 0.75 which is the same as in case of benchmark GHH preferences. Similarly the elasticity of real marginal cost with respect to output can be shown to be given by

$$\frac{\partial \psi}{\partial c} \frac{c}{\psi} = \frac{1}{1 - n} \quad (2.59)$$

which is equal to  $1.\bar{3}$  in the stationary equilibrium and equal to  $\gamma$  under GHH. So both models have the same implications concerning these elasticities. But nevertheless this leads to overall a bit more persistent reactions under CRRA preferences than under GHH utility in the CIA-setup. The contract multiplier for  $\widehat{c}_{0,t}$  is now 0.14 compared to 0.08. Obviously it makes a difference which type of utility function is used in dynamic general equilibrium models with sticky prices. Preferences thus are at least partly responsible for the degree of persistence. Taking a look at (2.56) for CRRA preferences (2.2) reveals that it cannot be simplified to yield a similar expression to (2.57). There is no possibility to separate  $n$  from the marginal utility of consumption. Both marginal utilities are quite complicated functions of  $c_t$  and  $n_t$ . A CRRA function causes overall a slightly smaller increase in  $w_t$  as a GHH preference specification since the impulse response of  $\widehat{\psi}_t$  shows a smaller initial deviation from steady state of only 0.7% compared to 0.9% under GHH.

## 2.3.2 MIU-Model

Similar to the CIA-case results differ in the MIU-model so there will be two subsections to treat each utility function separately.

### 2.3.2.1 GHH Preferences

Figures 2.7 – 2.8 visualize the impulse responses for the MIU-model with GHH preferences in the benchmark case. A first inspection of the impulses reveals that now all variables but the nominal interest behave cyclical: a positive (negative) reaction

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<sup>15</sup>It is important to consider this elasticity at the steady state where  $c = an$ .

is followed by an immediate negative (positive) one which reverts to positive (negative) behavior again. This is certainly counterfactual and not observed empirically. A second important result is the complete absence of persistence in the reactions of the variables, with the exception of the nominal interest rate which rises persistently. Third, price adjusting firms react very strongly in the first period so that the price level overshoots considerably. Even the behavior of prices shows no persistence at all. Fourth, real money balances decline on impact and then approach the steady state from below, a reaction which is also not observed empirically. A very low value of the risk aversion parameter  $\sigma$  creates extremely cyclical impulse responses with humps and dips for several periods. On the other hand high values of  $\sigma$  dampen the peaks and troughs.<sup>16</sup>

Obviously it makes a difference how money is introduced in dynamic general equilibrium models. Since the benchmark models have been calibrated the same way the absence of persistence must be due to the different way money is modeled, thus to the implied money demand function. It seems that in a MIU-model where money demand is interest rate sensitive persistent reactions to money growth shocks cannot be explained at all. An implied quantity theoretic type of money demand appears to be a more appropriate formulation on the way to achieve persistent output reactions in a price staggering model taking in mind that the models in the previous sections were also not very successful in reaching the goal. Is there some intuition behind this result? Again it is useful to look at the real wage rate. The corresponding equation to (2.15) is (2.24) and is repeated for convenience:

$$w_t = - \frac{\frac{\partial u(c_t, n_t, a_t)}{\partial n_t}}{\frac{\partial u(c_t, n_t, a_t)}{\partial c_t}} \quad (2.60)$$

In the MIU-setup the dynamics are simpler than in the CIA-model: First, there is no time  $t + 1$  variable or derivative in this equation. Second, there is no direct influence of the price level. In case of GHH preferences this condition simplifies again considerably:

$$w_t = a_t \theta n_t^\gamma \quad (2.61)$$

Wealth effects are now completely absent and the wage rate is solely determined by labor  $n_t$ . Thus the dynamics of consumption do not have an influence on the evolution of the real wage. In addition the price level also does not have any impact. So the reaction of labor proportionately translates to the reaction of  $w_t$  determined by the elasticity of real marginal costs with respect to output  $\gamma$ . In the benchmark

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<sup>16</sup>This is not shown in the Figures.

case  $\gamma=1.\bar{3}$  so that the real wage response is a multitude of labor's reaction. This explains the strong initial deviation of  $\widehat{\psi}_t$  from steady state: 1.8%. In turn the firms increase their prices very intensively leading to the overshooting behavior.

So it comes at no surprise that for a low value of  $\gamma$  real marginal costs react moderately. This gives rise to persistent reactions of consumption and prices. The results of the experiment are shown in Figures 2.9 – 2.10. Now all variables display very strong persistence after a money growth shock. Results are completely different to the CIA-outcome. Intermediate as well as aggregate consumption react strongly and stay above (or below) the steady state value for more than 8 quarters after the shock. The contract multiplier of consumption is given by 0.55. Real marginal costs are flat, showing only a 0.12% deviation from the equilibrium value. Note that this is very close to  $\gamma = 0.1$  pointing to the influence of the real marginal cost elasticity here. Real money balances rise all the time, due to the smooth and moderate price level increase. Intermediate goods firms raise their prices accordingly very slowly. Inflation displays a hump as observed empirically. Unfortunately the nominal interest rate counterfactually rises again. Thus changing from a CIA-setup to a MIU-model leads to completely different model outcomes. A low marginal cost elasticity is obviously not enough to generate persistence in output. It must be combined with an interest rate sensitive money demand function which is implied by a MIU-model. Under GHH preferences this generates a real marginal cost function which is independent of consumption (or output). In turn money growth shocks do only have a reduced effect on  $\psi_t$ .

### 2.3.2.2 CRRA Preferences

Finally, Figures 2.11 – 2.12 show the results for the MIU-model with CRRA preferences. Compared to the GHH version the outcome does not differ very much. But as in the CIA-setup there are some small differences. First, the reactions are all weaker than under GHH preferences. Second, the strength of the cyclical behavior is less, i.e. the dips and humps are smaller in size. Lowering the value of  $\sigma$  leads to more pronounced dips and humps whereas a higher risk aversion makes them smaller.<sup>17</sup>

Again, the MIU-model version generates even less persistent reactions than the CIA-setup. This is especially the case for  $\widehat{c}_{0,t}$  as well as the prices. As the models are again calibrated the same way the loss of persistence is due to the different implied money demand functions. An intuition for this can be the following. (2.24) is now

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<sup>17</sup>These Figures are again not shown.

given by

$$w_t = \frac{1 - \zeta}{\zeta} \frac{c_t}{1 - n_t} \quad (2.62)$$

As  $n$  rises the denominator will fall. The rise in consumption will raise the numerator so that there are two mechanisms which will reinforce each other resulting in a strong rise in  $w$ . Accordingly real marginal cost will go up equally strong. Note that  $\hat{\psi}_t$  initially deviates 1.2% from steady state compared to 0.7% in the CIA-setup.

This leads to the conclusion that two conditions have to be fulfilled in order to enable a dynamic general equilibrium model with price staggering to generate persistent output and inflation responses: First, the static steady state elasticity of labor supply with respect to the real wage must be high, and second, the money demand function has to be interest rate sensitive. Only one of these ingredients is not enough to generate persistence. This refines results in the literature, for example of Ascari (2003a) who investigates only MIU-specifications and concludes that a high labor supply elasticity is crucial for persistent output reactions in a price staggering model. Similarly Chari, Kehoe and McGrattan (2000) study a MIU-model and use an additively separable utility function (in all arguments) in their sensitivity analysis. They also point out the role of a high labor supply elasticity for a persistent output reaction. In addition they find that a very low value of the risk aversion parameter  $\sigma$  (quasi linearity in consumption) is also important for creating a high contract multiplier, contrasting the results here.

## 2.4 Business Cycle Properties

In order to explore the implications for the business cycle properties one has to specify the standard deviation of the AR(1)-process for money growth. Here the value estimated in Cooley and Hansen (1995), p.201, is used.<sup>18</sup> It implies a value of 0.0000792 for the variance  $\sigma_g^2$ . Table 2.1 shows the results for the CIA-model with GHH preferences after HP-filtering with  $\lambda = 1600$ .<sup>19</sup>

$\sigma_{\hat{x}}$  denotes the percentage standard deviation of  $\hat{x}$  whereas  $\sigma_{\hat{x}}/\sigma_{\hat{y}}$  measures the respective standard deviation relative to that of output  $\hat{y}$ . The next two columns report the autocorrelations for one and two lags of the respective aggregate. The remaining columns display the cross correlations with output. A variable  $\hat{x}$  is leading

<sup>18</sup>It is not intended to take the model explicitly to the data because of its overwhelming simplicity. This justifies the use of Cooley and Hansen's parameter values.

<sup>19</sup>Keep in mind that all values in the tables have been rounded using the computer output. So it is possible that the relative standard deviations deliver a different value when using the values in the table.

Table 2.1: Moments in the CIA-Model with GHH Preferences

$\hat{x}_t$	$\sigma_{\hat{x}}$	$\sigma_{\hat{x}}/\sigma_{\hat{y}}$	autocorrelation cross correlation of $\hat{x}_t$ with $\hat{y}$ in						
			1	2	$t-2$	$t-1$	$t$	$t+1$	$t+2$
$\hat{c}_{0,t}$	4.11	17.44	-0.02	0.01	-0.07	-0.00	-0.61	0.77	0.19
$\hat{c}_{1,t}$	4.42	18.72	-0.09	-0.00	0.06	-0.03	0.67	-0.75	-0.18
$\hat{c}_t$	0.24	1.00	-0.30	-0.04	-0.04	-0.30	1.00	-0.30	-0.04
$\hat{n}_t$	0.24	1.00	-0.30	-0.04	-0.04	-0.30	1.00	-0.30	-0.04
$\hat{w}_t$	0.77	3.26	-0.17	-0.02	0.05	-0.07	0.74	-0.72	-0.17
$\hat{\mu}_t$	0.77	3.26	-0.17	-0.02	-0.05	0.07	-0.74	0.72	0.17
$\hat{R}_t$	17.78	75.37	0.46	0.09	0.11	0.19	0.14	-0.78	-0.21
$\hat{\psi}_t$	0.77	3.26	-0.17	-0.02	0.05	-0.07	0.74	-0.72	-0.17
$\hat{\Pi}_t$	0.98	4.13	0.47	-0.06	0.04	0.46	-0.09	-0.69	-0.20
$\hat{P}_t$	1.97	8.34	0.88	0.64	-0.11	-0.13	-0.36	-0.31	0.03

$\hat{y}$  if the absolute value of the correlation  $\rho(\hat{x}_t, \hat{y}_{t+i})$  is highest for  $i > 0$ . Accordingly a variable  $\hat{x}$  is lagging  $\hat{y}$  if the absolute value of the correlation  $\rho(\hat{x}_t, \hat{y}_{t+i})$  has a maximum for  $i < 0$ . In case that this correlation is positive one speaks of a procyclical variable while it is called anticyclical if it is negative. If the maximum correlation occurs at lag 0 ( $i = 0$ ) the variable is moving with the cycle. This table strenghtens the insights from the impulse response functions. First, the cyclical character of most variables is displayed in their negative autocorrelations, see e.g. consumption and real marginal cost. Second, consumption – which is equal to output here – and labor have exactly the same business cycle properties because consumption is proportional to labor via the production function. The same holds for the real wage and real marginal costs. Their correlations at leads and lags and their autocorrelations are negative. Third, the absolute variability of consumption (0.24%) is quite low so that the relative volatilities of the nominal rate and the disaggregated consumption levels are very high. In German data I found a percentage standard deviation of 1.42% for consumption and 1.55% for output, see Gail (1998), p. 52. The nominal variables such as the inflation rate and the price level are by far too volatile. Empirical estimates of Maufner (1994), p. 19, for Germany reveal a relative volatility of the price level of 0.70 using the consumer price index and 0.58 when employing the GDP deflator. His measure of a short term nominal interest rate displays a relative variability of 15.2. Fourth, the nominal interest rate, the price level and inflation are persistent since their autocorrelations are positive and above 0.40 at the first lag. The price level displays considerable persistence with an autocorrelation at the

first lag of 0.88. Since the inflation rate is hump-shaped the autocorrelation is quite high (0.47). Fifth, the cross correlation of consumption and the real wage rate is less than one (0.74) and in the neighborhood of empirical estimates. Maußner (1994) reports a value of 0.40 but he finds that the real wage lags with two or three quarters behind output.

The results for the CRRA utility function are not that much different from the GHH case concerning the business cycle properties so they will not be presented in a separate table here.  $\widehat{c}_{0,t}$  is now slightly positively autocorrelated at the first lag (0.05) and consumption has a slightly increased absolute variability of 0.25%. It is more interesting to investigate the results for the MIU-model.

Table 2.2 shows the results for the MIU-model with GHH preferences after HP-filtering with  $\lambda = 1600$ .

Table 2.2: Moments in the MIU-Model with GHH Preferences

		autocorrelation		cross correlation of $\widehat{x}_t$ with $\widehat{y}$ in					
$\widehat{x}_t$	$\sigma_{\widehat{x}}$	$\sigma_{\widehat{x}}/\sigma_{\widehat{y}}$	1	2	$t-2$	$t-1$	$t$	$t+1$	$t+2$
$\widehat{c}_{0,t}$	5.24	4.41	-0.27	0.00	-0.01	0.26	-1.00	0.29	0.00
$\widehat{c}_{1,t}$	7.62	6.41	-0.27	0.00	0.01	-0.27	1.00	-0.29	0.00
$\widehat{c}_t$	1.19	1.00	-0.28	0.00	0.00	-0.28	1.00	-0.28	0.00
$\widehat{n}_t$	1.19	1.00	-0.28	0.00	0.00	-0.28	1.00	-0.28	0.00
$\widehat{w}_t$	1.64	1.38	-0.27	0.00	0.01	-0.27	1.00	-0.29	0.00
$\widehat{\mu}_t$	1.64	1.38	-0.27	0.00	-0.01	0.27	-1.00	0.29	-0.00
$\widehat{R}_t$	3.40	2.86	0.13	0.05	0.08	0.11	0.90	-0.32	-0.04
$\widehat{\psi}_t$	1.64	1.38	-0.27	0.00	0.01	-0.27	1.00	-0.29	0.00
$\widehat{\Pi}_t$	1.30	1.09	0.32	-0.21	-0.22	0.61	0.59	-0.24	-0.04
$\widehat{P}_t$	2.18	1.84	0.82	0.54	0.22	0.35	-0.01	-0.36	-0.22

The absolute volatility of consumption rises considerably to 1.19%. Although the absolute variability of  $\widehat{c}_{0,t}$  and  $\widehat{c}_{1,t}$  is even higher than in the CIA-setup their relative volatilities go down due to the increased variation in consumption. The nominal interest rate is only 1/5 as variable as in the CIA-model. Because most aggregates are again cyclical the autocorrelations are overall negative. Exceptions are solely the price level and the nominal interest rate. The overshooting behavior of prices cannot be captured by the autocorrelation coefficient. The cyclical reactions are also responsible for the perfect correlation with consumption. There is a tendency for the price level leading anticyclically at one lag (-0.36). But it can also be interpreted to lag procyclically with one lag (0.35). In the CIA-model it was clearly



anticyclical (contemporaneously). Maußner (1994) finds that the price level leads countercyclically with 4 quarters (-0.58 for the consumer price index).

Results for CRRA preferences are not very much different from the GHH results. But the GHH function with an implied high labor supply elasticity is able to account quite well for the business cycle. So Table 2.3 shows the results for the MIU-model with GHH preferences and  $\gamma = 0.1$  after HP-filtering with  $\lambda = 1600$ .

Table 2.3: Moments in the MIU-Model with GHH Preferences,  $\gamma = 0.1$

$\hat{x}_t$	$\sigma_{\hat{x}}$	$\sigma_{\hat{x}}/\sigma_{\hat{y}}$	autocorrelation		cross correlation of $\hat{x}_t$ with $\hat{y}$ in				
			1	2	$t-2$	$t-1$	$t$	$t+1$	$t+2$
$\hat{c}_{0,t}$	1.51	1.45	0.40	0.10	-0.10	-0.40	-1.00	-0.42	-0.12
$\hat{c}_{1,t}$	3.58	3.45	0.41	0.11	0.11	0.41	1.00	0.42	0.11
$\hat{c}_t$	1.04	1.00	0.42	0.11	0.11	0.42	1.00	0.42	0.11
$\hat{n}_t$	1.04	1.00	0.42	0.11	0.11	0.42	1.00	0.42	0.11
$\hat{w}_t$	0.13	0.13	0.40	0.10	0.09	0.40	1.00	0.42	0.12
$\hat{\mu}_t$	0.13	0.13	0.40	0.10	-0.09	-0.40	-1.00	-0.42	-0.12
$\hat{R}_t$	1.32	1.28	0.35	0.04	0.02	0.32	0.99	0.43	0.14
$\hat{\psi}_t$	0.13	0.13	0.40	0.10	0.09	0.40	1.00	0.42	0.12
$\hat{\Pi}_t$	0.71	0.69	0.68	0.20	0.30	0.84	0.85	0.32	0.04
$\hat{P}_t$	1.77	1.71	0.92	0.73	0.47	0.34	0.01	-0.33	-0.46

Note that now all variables are positively autocorrelated. Real marginal costs, the real wage, the nominal interest rate and consumption of non-adjusting firms are – counterfactually – perfectly correlated with output (consumption). The volatility of the real wage is very low because it is orthogonal to real marginal costs. The price level, inflation and the nominal rate display a reduced volatility which is more in line with empirical estimates.<sup>20</sup> Overall this version performs quite good but still cannot account for actual business cycles. It should be kept in mind that the results depend on a high steady state elasticity of labor supply with respect to the real wage which is not observed empirically.

## 2.5 Conclusions

In light of the main question of this chapter it must be concluded that persistent reactions of output and inflation to money growth shocks can only be explained in

<sup>20</sup>The nominal interest is however less volatile than a short term rate. Maußner (1994), p. 23, finds that a long term measure shows a reduced relative standard deviation of 5.69.

a MIU-model with GHH preferences and a high labor supply elasticity. All other economies considered fall short of reaching persistence.

An interesting future direction of research is to look at models that are generalized to include capital accumulation considerations. Results of Chari, Kehoe and McGrattan (2000) are very discouraging. They find almost no persistence in models with capital. The next chapter investigates such a model in depth. Of special interest is whether their results change in a CIA-model.

Another promising line of research is to analyze open economy models. Recently Ghironi (2002) has shown that once openness is taken into account a sticky price model can generate endogenous output persistence.<sup>21</sup> This depends crucially on incomplete asset markets. It would be interesting to generalize the model at hand to such a framework. This task is left for future research beyond this book.

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<sup>21</sup>See also Cavallo and Ghironi (2002).

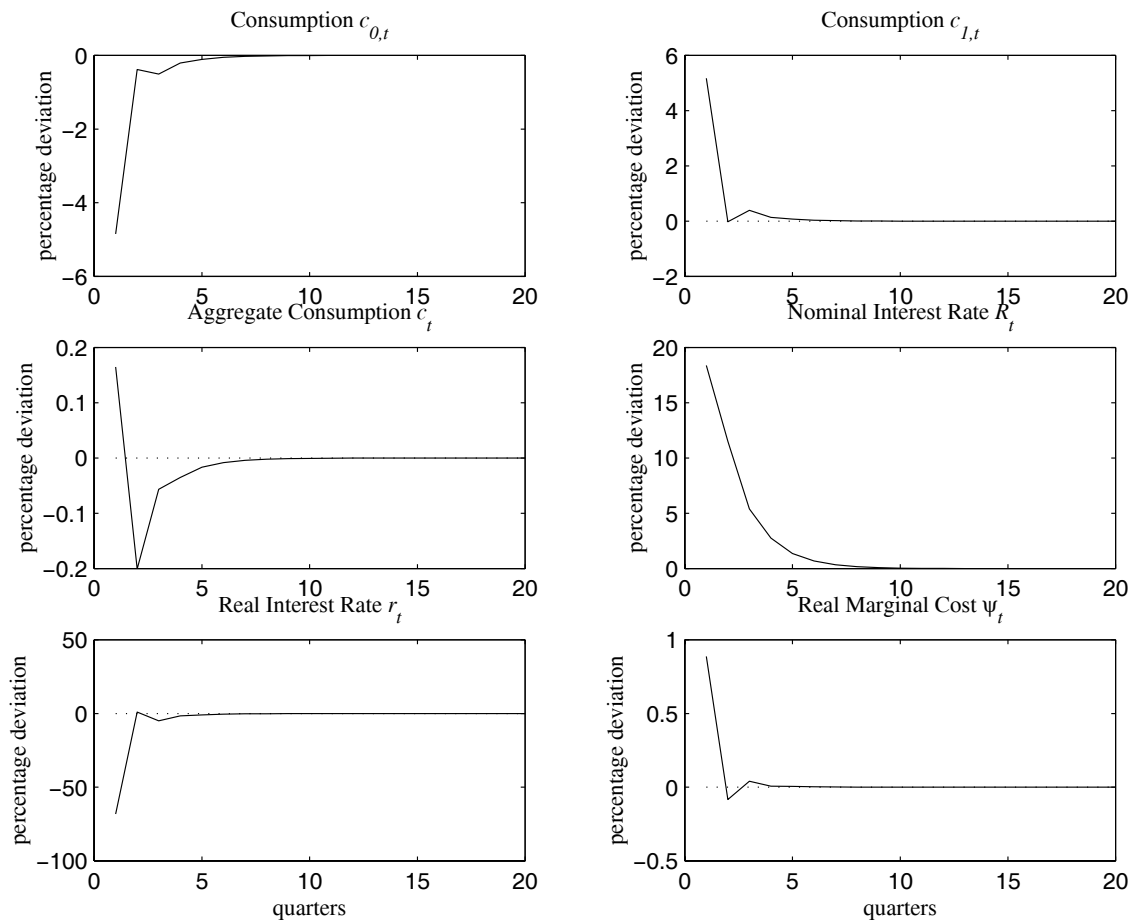


Figure 2.1: Impulse Response Functions for  $\widehat{c}_{0,t}, \widehat{c}_{1,t}, \widehat{c}_t, \widehat{R}_t, \widehat{r}_t, \widehat{\psi}_t$ , CIA-Model, GHH Preferences

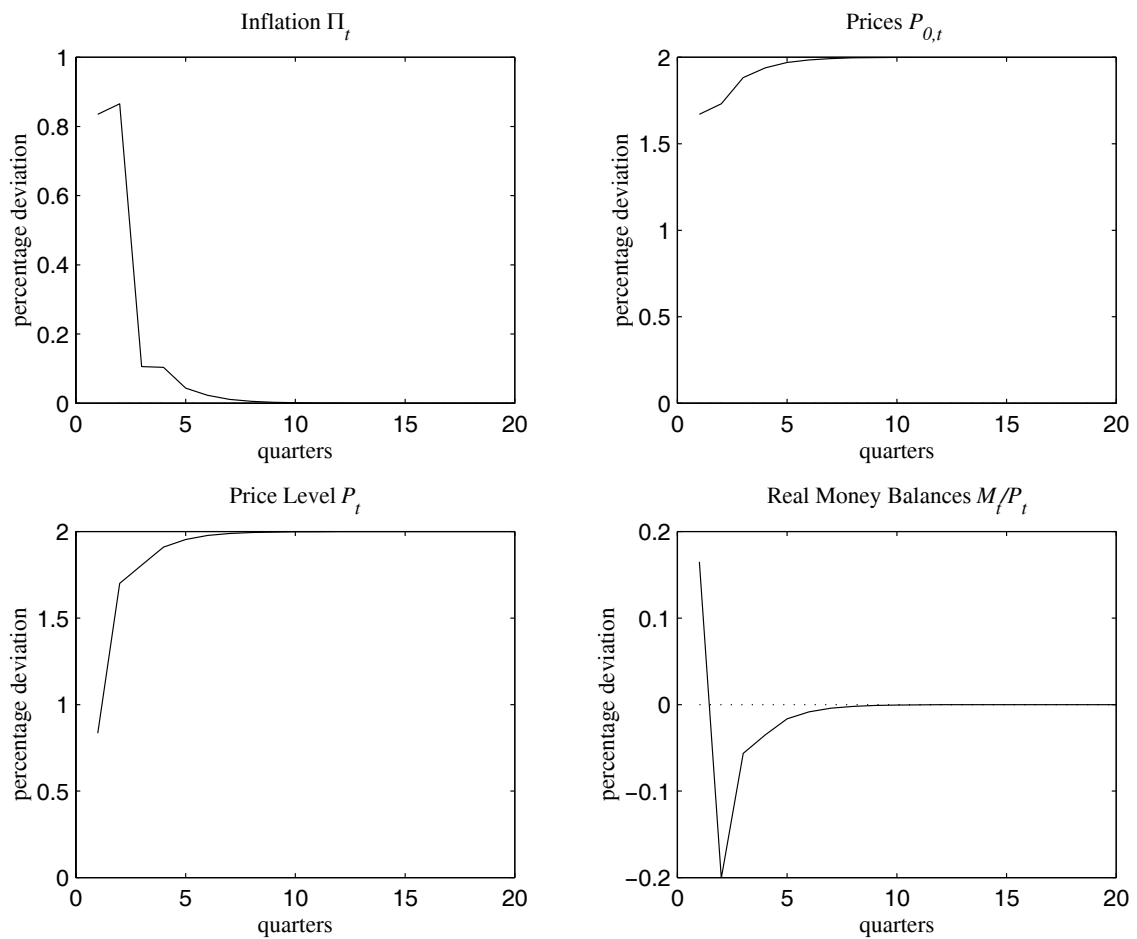


Figure 2.2: Impulse Response Functions for  $\widehat{\Pi}_t, \widehat{P}_{0,t}, \widehat{P}_t, \widehat{M}_t - \widehat{P}_t$ , CIA-Model, GHH Preferences

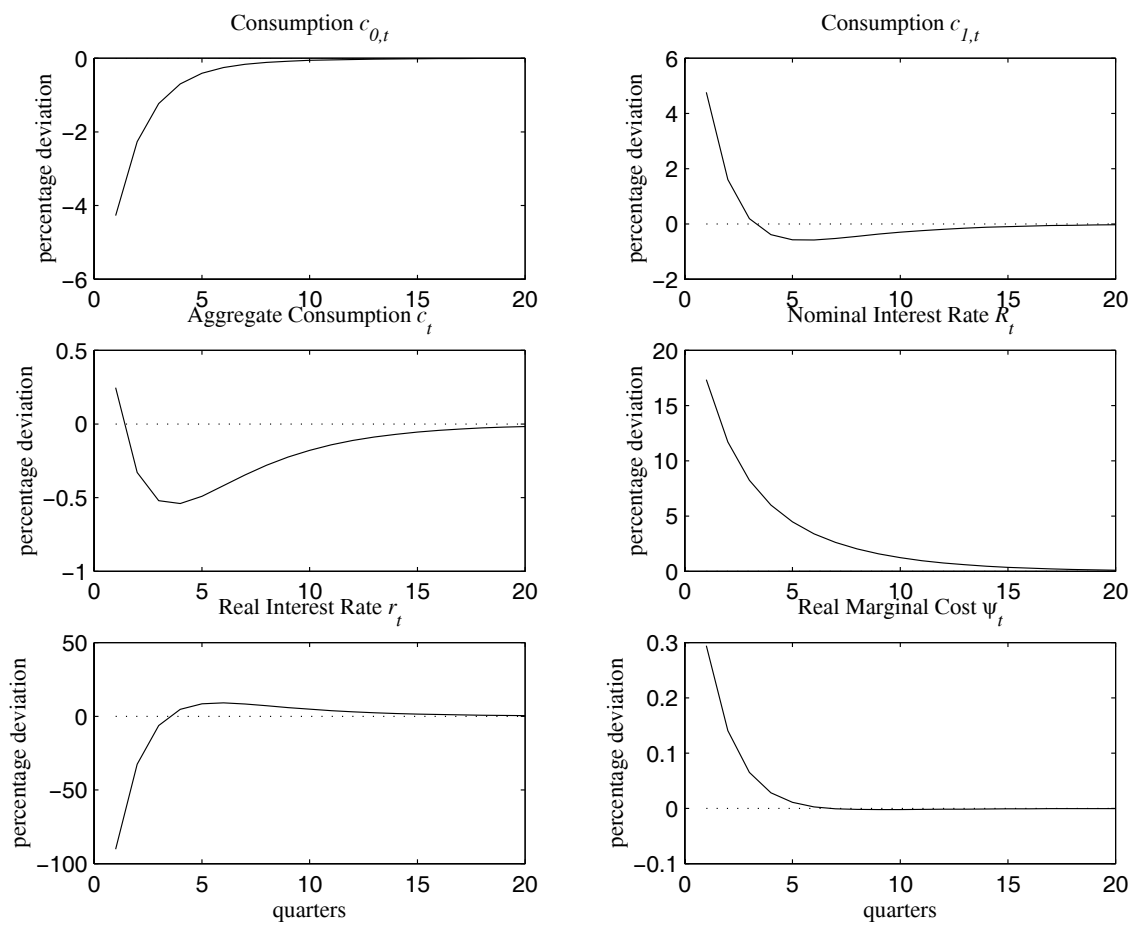


Figure 2.3: Impulse Response Functions for  $\widehat{c}_{0,t}, \widehat{c}_{1,t}, \widehat{c}_t, \widehat{R}_t, \widehat{r}_t, \widehat{\psi}_t$ , CIA-Model, GHH Preferences, high labor supply elasticity

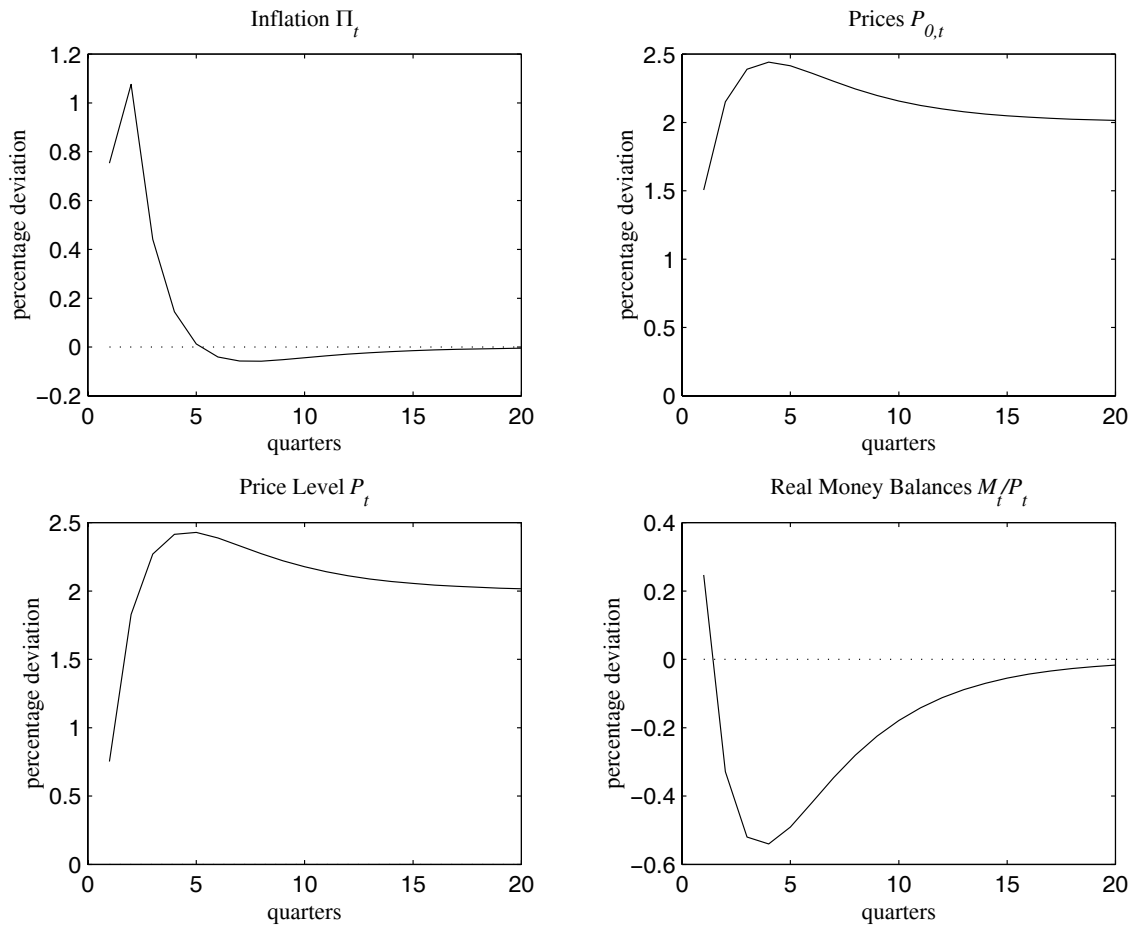


Figure 2.4: Impulse Response Functions for  $\widehat{\Pi}_t, \widehat{P}_{0,t}, \widehat{P}_t, \widehat{M}_t - \widehat{P}_t$ , CIA-Model, GHH Preferences, high labor supply elasticity

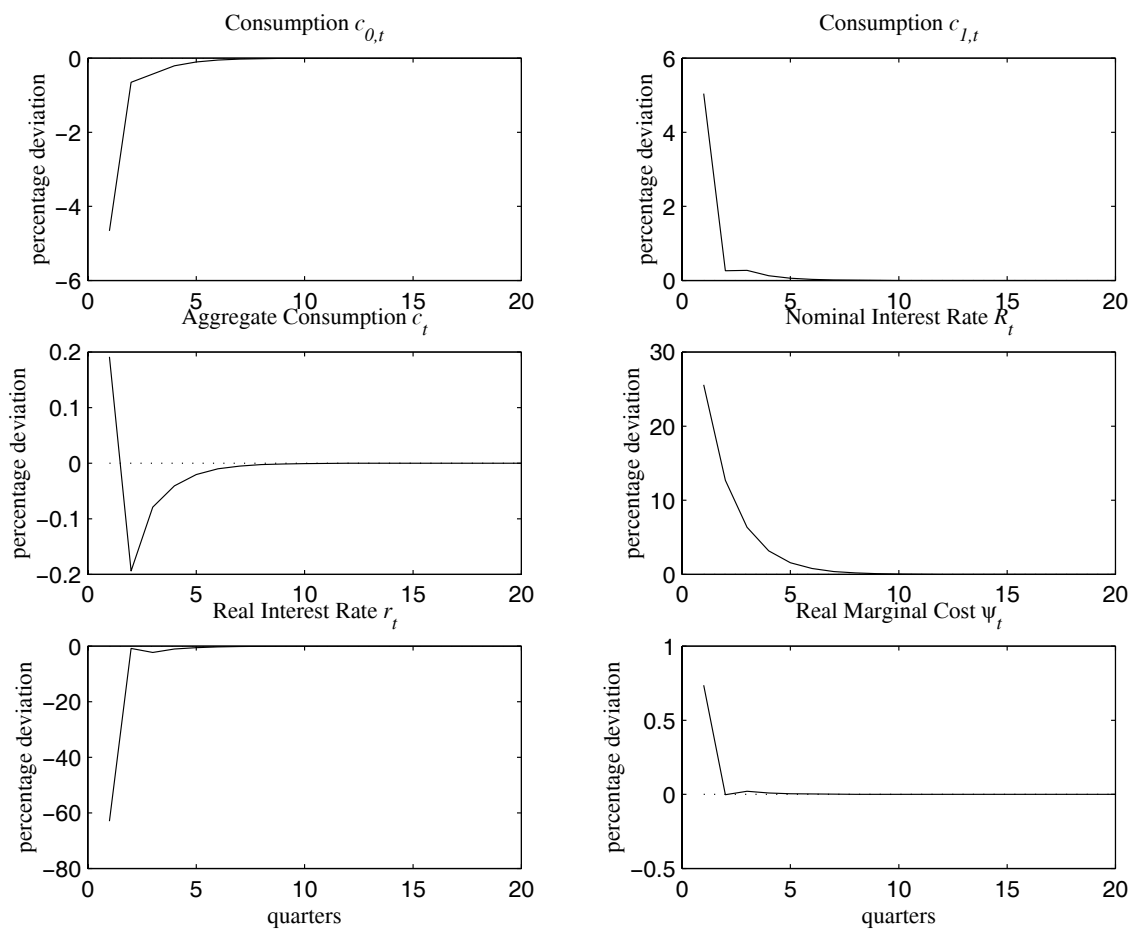


Figure 2.5: Impulse Response Functions for  $\widehat{c}_{0,t}, \widehat{c}_{1,t}, \widehat{c}_t, \widehat{R}_t, \widehat{r}_t, \widehat{\psi}_t$ , CIA-Model, CRRA Preferences

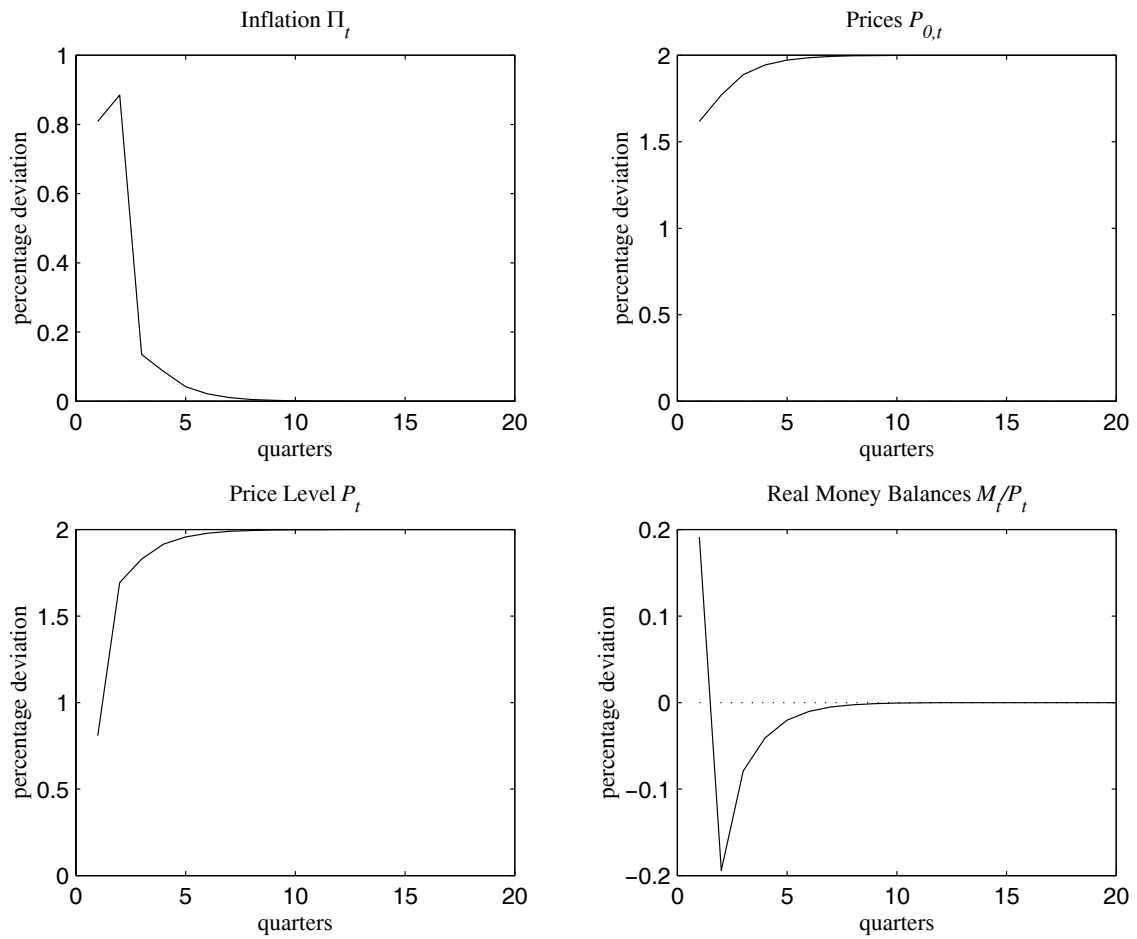


Figure 2.6: Impulse Response Functions for  $\widehat{\Pi}_t, \widehat{P}_{0,t}, \widehat{P}_t, \widehat{M}_t - \widehat{P}_t$ , CIA-Model, CRRA Preferences



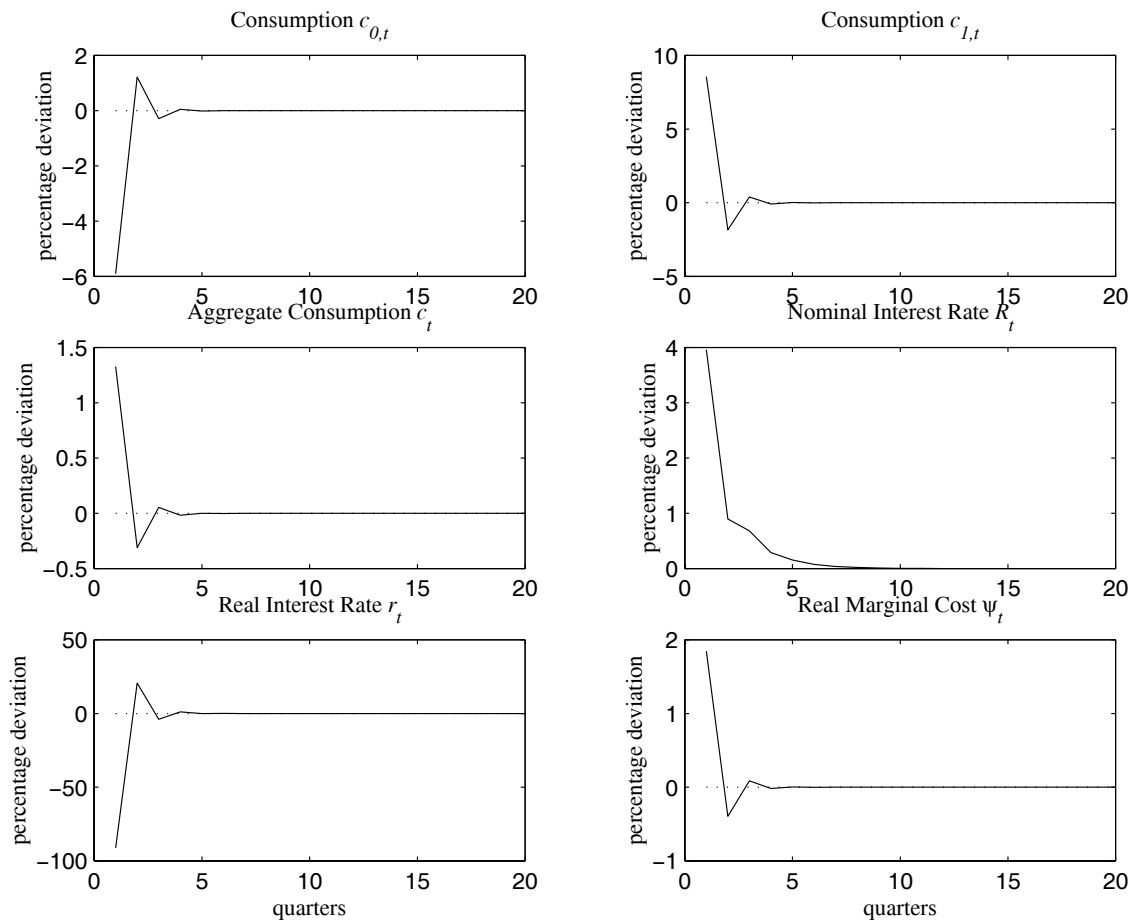


Figure 2.7: Impulse Response Functions for  $\widehat{c}_{0,t}, \widehat{c}_{1,t}, \widehat{c}_t, \widehat{R}_t, \widehat{r}_t, \widehat{\psi}_t$ , MIU-Model, GHH Preferences

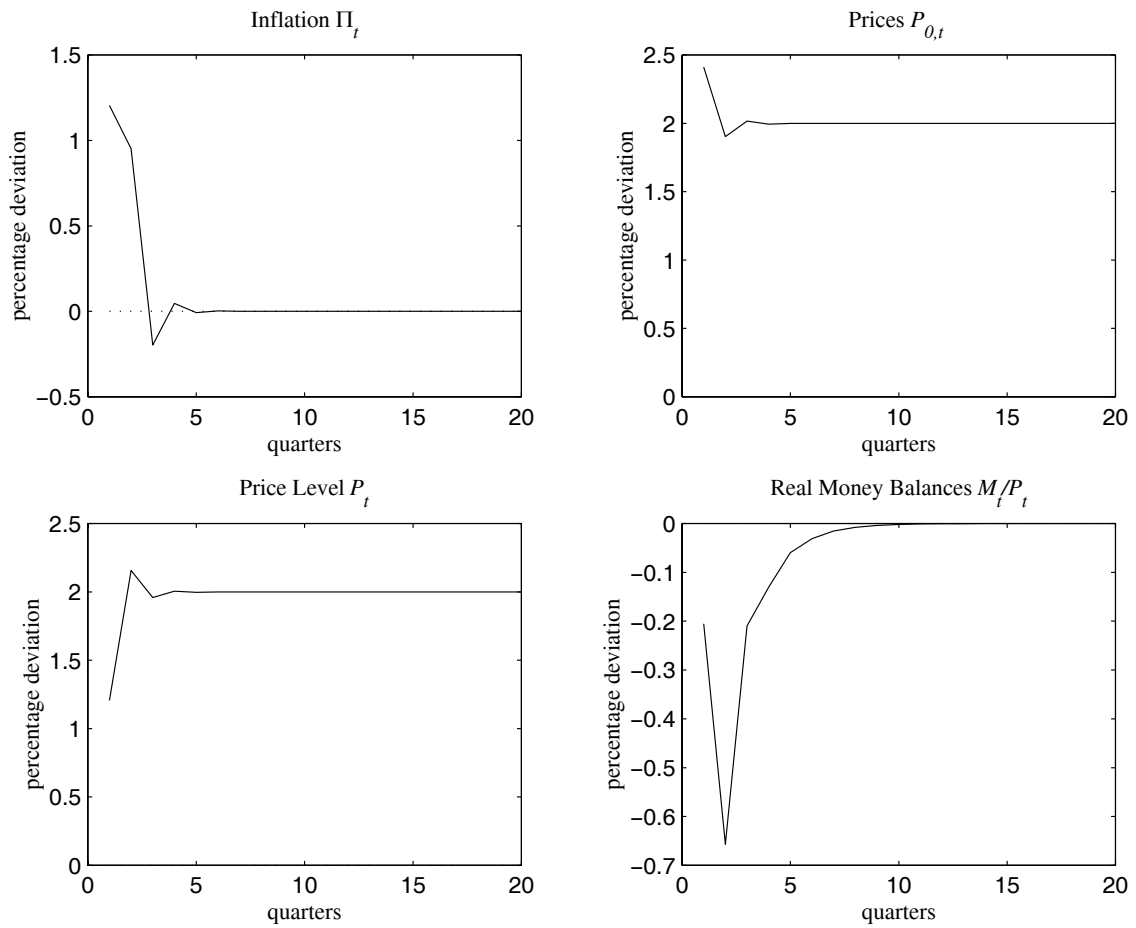


Figure 2.8: Impulse Response Functions for  $\widehat{\Pi}_t, \widehat{P}_{0,t}, \widehat{P}_t, \widehat{M}_t - \widehat{P}_t$ , MIU-Model, GHH Preferences

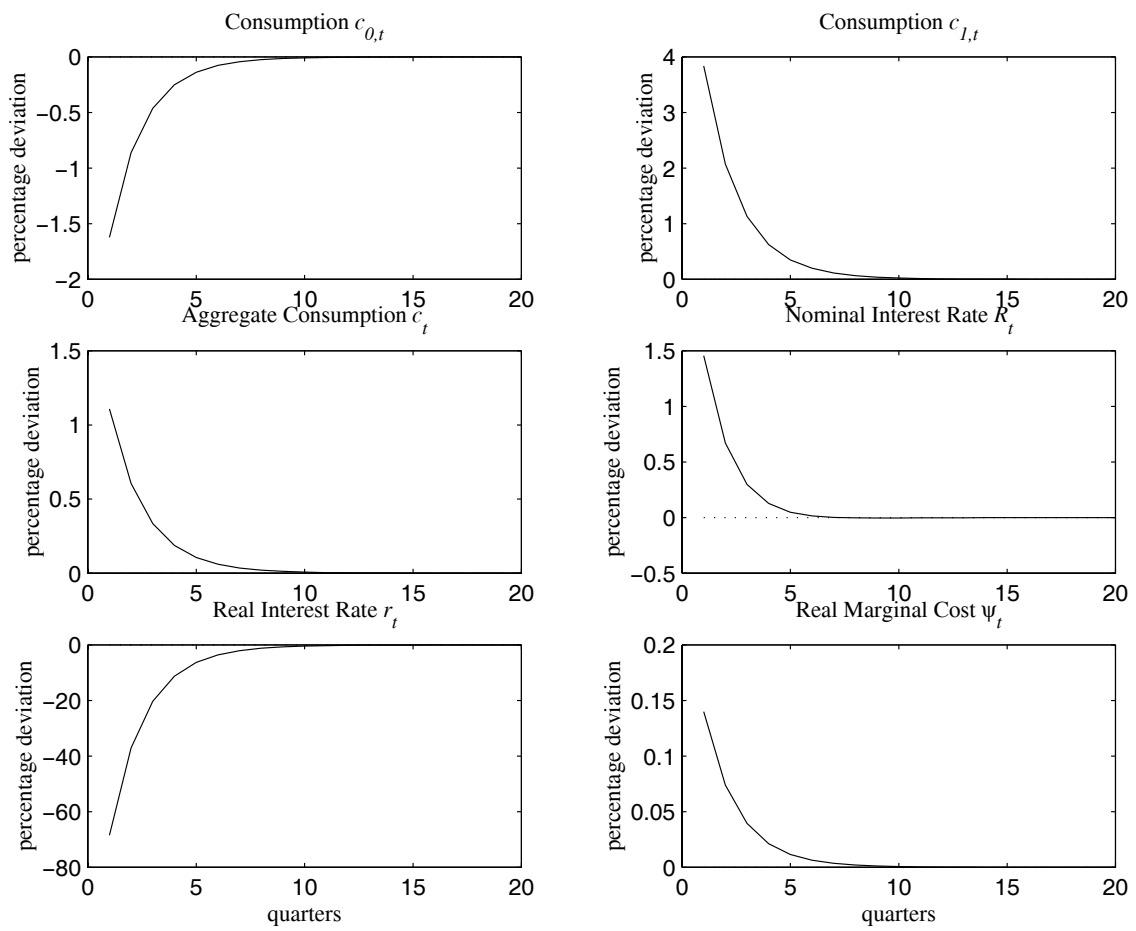


Figure 2.9: Impulse Response Functions for  $\widehat{c}_{0,t}, \widehat{c}_{1,t}, \widehat{c}_t, \widehat{R}_t, \widehat{r}_t, \widehat{\psi}_t$ , MIU-Model, GHH Preferences, high labor supply elasticity

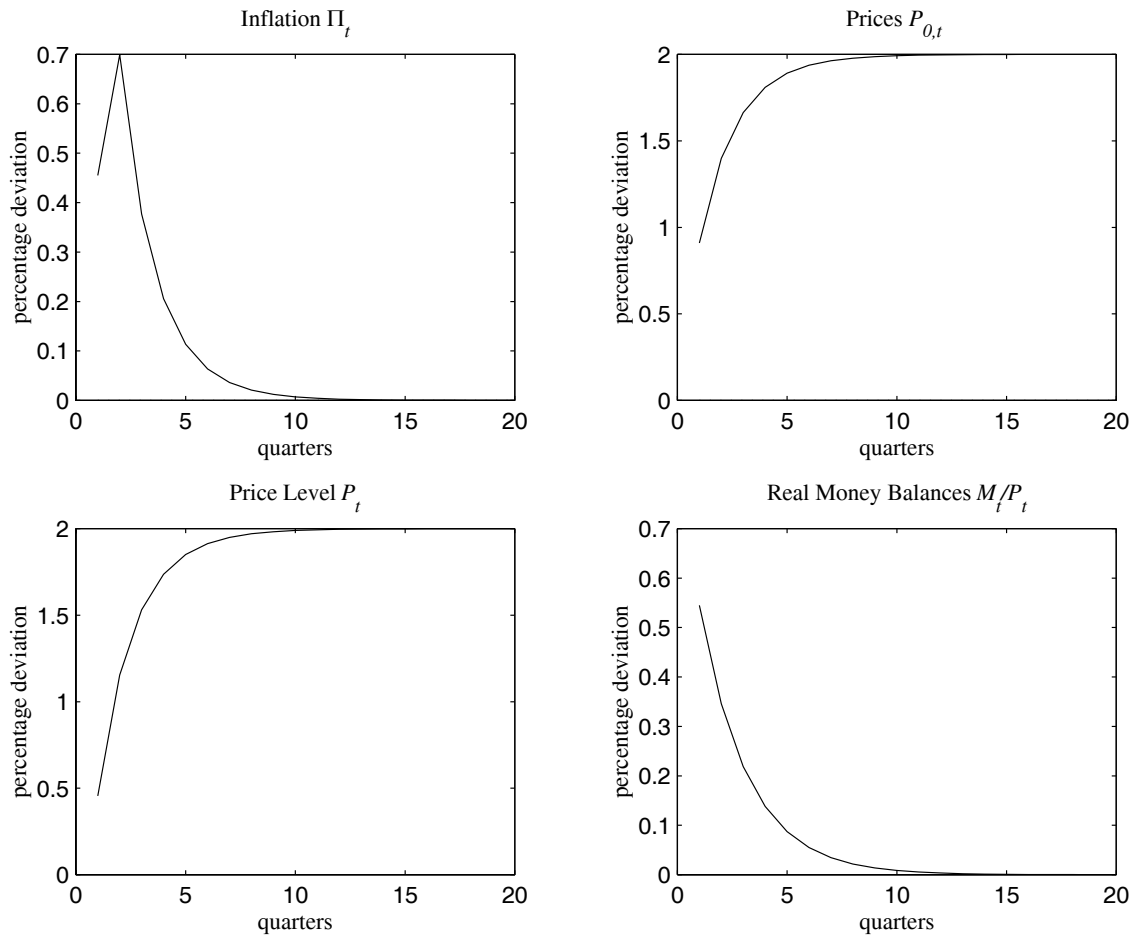


Figure 2.10: Impulse Response Functions for  $\widehat{\Pi}_t, \widehat{P}_{0,t}, \widehat{P}_t, \widehat{M}_t - \widehat{P}_t$ , MIU-Model, GHH Preferences, high labor supply elasticity

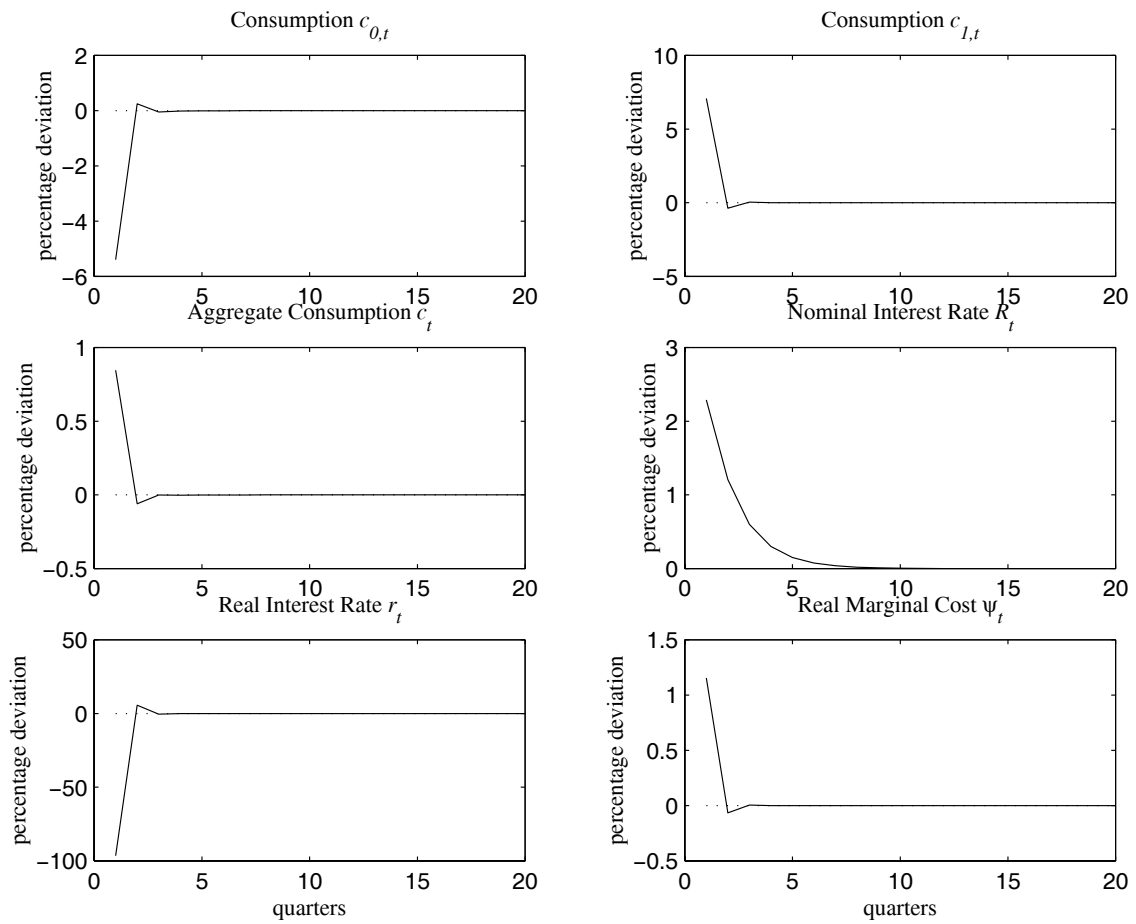


Figure 2.11: Impulse Response Functions for  $\hat{c}_{0,t}, \hat{c}_{1,t}, \hat{c}_t, \hat{R}_t, \hat{r}_t, \hat{\psi}_t$ , MIU-Model, CRRA Preferences

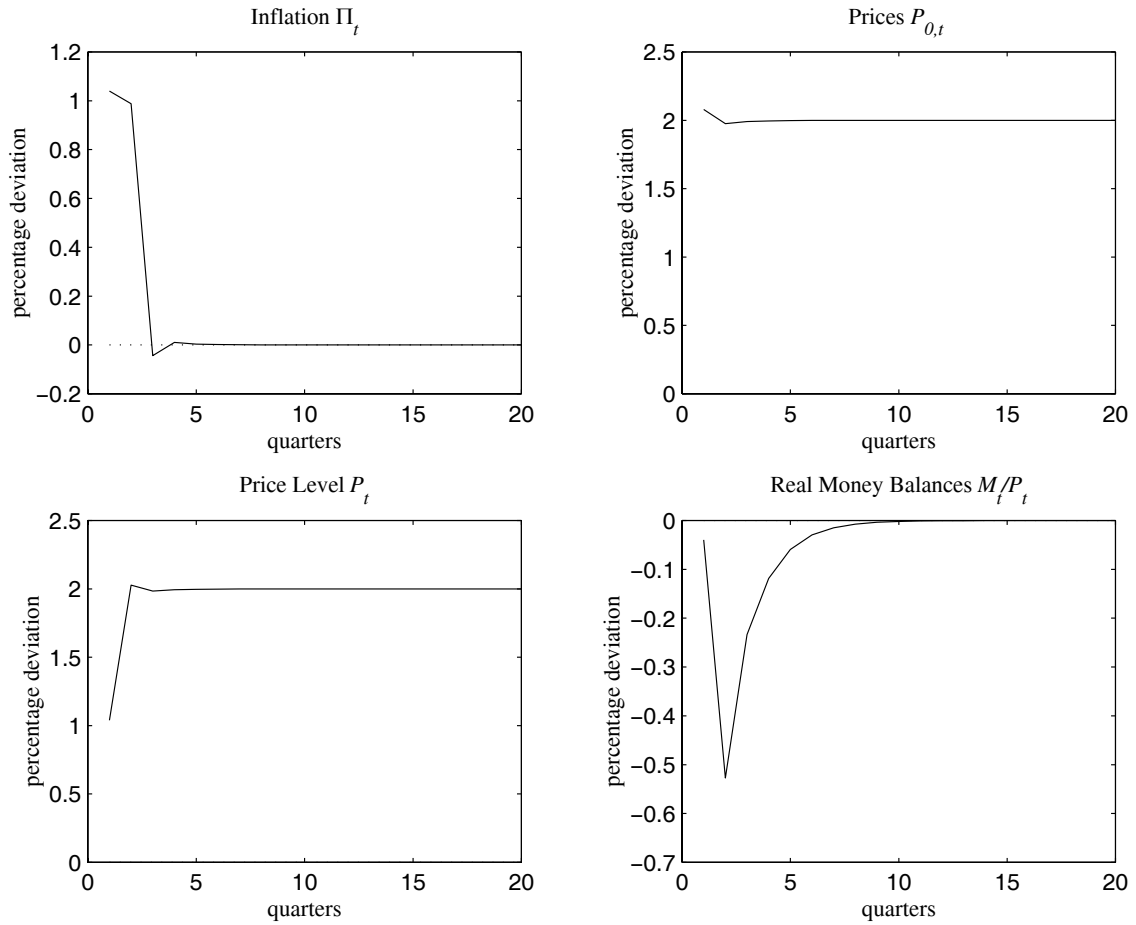


Figure 2.12: Impulse Response Functions for  $\widehat{\Pi}_t, \widehat{P}_{0,t}, \widehat{P}_t, \widehat{M}_t - \widehat{P}_t$ , MIU-Model, CRRA Preferences

# Chapter 3

## Price Staggering in a Monetary Stochastic Dynamic General Equilibrium Model with Labor and Capital

### 3.1 Introduction

In this chapter the model of Chapter 2 is augmented by capital accumulation considerations. Since the CIA-setup proved to be more successful in generating persistence the MIU-setup will not be considered again. Because persistence was found to be higher under CRRA preferences the GHH utility function will be also discarded. It can be shown that in contrast to models without capital the specific form of the utility function and the implied elasticity of labor supply with respect to the real wage play a minor role even in a MIU-setup.<sup>1</sup> Once intertemporal links are included persistent output reactions cannot be explained anymore. Real marginal costs react even stronger than in a labor only economy because there is additional upward pressure through the rental rate on capital. This confirms results of Chari, Kehoe and McGrattan (2000).

So instead of considering MIU-models a second way to introduce sticky prices will be analyzed, namely Calvo pricing. In the Calvo (1983) approach firms face a specific probability of being able to adjust their price while under Taylor staggering half of the firms can reoptimize their prices and the other half cannot. It turns out that under Calvo pricing persistent reactions of output and inflation can be explained

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<sup>1</sup>See Gail (2002*b*) on this point.

while in the Taylor version nearly all the variables show an enhanced cyclicity compared to the labor economy. This result confirms the conclusions in Kim (2003) who studies an economy with price and wage staggering and also compares Calvo and Taylor contracts. Kiley (2002) shows the same in simplified model versions. The reason for the failure of the models in Chari, Kehoe and McGrattan (2000) is thus to be found in the way staggered prices are modeled. With Calvo pricing a sticky price model is very well able to account for persistence in output.

Unfortunately it is not possible to study disaggregated variables because this leads to theoretical difficulties with respect to the aggregation of the production functions: It is impossible to consistently aggregate the capital stocks and labor inputs (the macroeconomic aggregation problem). But there would also be computational problems concerning the uniqueness of the model solution: The possibility of sunspots and multiple equilibria would be high. So a symmetric equilibrium will be considered.

The chapter is organized as follows: Section 3.2 describes in detail the different model versions, the steady state and the calibration. In Section 3.3 impulse responses are discussed for the Taylor staggering and Calvo pricing model. Business cycle implications are presented in Section 3.4 whereas Section 3.5 concludes and gives some suggestions for future research.

## 3.2 The Models

### 3.2.1 The Household

The representative household is assumed to have preferences over consumption ( $c_t$ ) and leisure ( $1 - n_t$ ). The utility function analyzed under this CIA-setup is the standard CRRA function.

$$u(c_t, n_t, a_t) = \frac{\left[ a_t c_t^\zeta (1 - n_t)^{1-\zeta} \right]^{1-\sigma} - 1}{1 - \sigma} \quad (3.1)$$

$a_t$  is the preference shock which can also act as a productivity shock.  $\sigma$  governs the degree of risk aversion and  $\zeta$  measures the relative weight of consumption for the representative agent.

The household's budget has to be modified in comparison to the pure labor economy of Chapter 2 since it can now invest  $i_t$  units of the final good to augment the capital stock  $k_t$ . It also receives factor payments  $z_t k_{t-1}$  for supplying capital to intermediate



goods producing firms. The new constraint is therefore given by

$$\begin{aligned} & P_t c_t + P_t i_t + M_t + B_t \\ = & P_t w_t n_t + P_t z_t k_{t-1} + M_{t-1} + (1 + R_{t-1}) B_{t-1} + \Xi_t + M_t^s \end{aligned} \quad (3.2)$$

where  $z_t$  denotes the real return on capital and where

$$\Xi_t = \int_0^1 \Xi_{j,t} dj \quad (3.3)$$

are the nominal profits of the intermediate goods producing firms. The uses of wealth are – in addition to real investment  $i_t$  – real consumption  $c_t$ , holdings of real money balances  $M_t/P_t$  and real bonds  $B_t/P_t$ . The household earns money working in the market at the real wage rate  $w_t$  and can spend its real money balances carried over from the previous period ( $M_{t-1}/P_t$ ). It receives income from previous period bond holdings including the interest on them  $(1 + R_{t-1})(B_{t-1}/P_t)$ . Finally, the household receives a monetary transfer  $M_t^s$  from the monetary authority and the profits from the intermediate goods firms  $\Xi_t$ , respectively. This transfer is equal to the change in money balances, i.e.

$$M_t^s = M_t - M_{t-1} \quad (3.4)$$

The household faces a cash-in-advance constraint. It can consume only out of cash balances it has received before. This condition is therefore given by

$$P_t c_t \leq M_{t-1} + M_t^s \quad (3.5)$$

The capital stock increases according to the following law of motion:

$$k_t = (1 - \delta) k_{t-1} + \phi \left( \frac{i_t}{k_{t-1}} \right) k_{t-1} \quad (3.6)$$

There are costs of adjusting the capital stock which are captured by the  $\phi$  function.  $\delta$  is the rate of depreciation. The detailed properties will be discussed in the calibration subsection. Because this equation cannot be explicitly solved for  $i_t$  a third Lagrange multiplier ( $\theta_t$ ) has to be introduced into the optimization problem of the household.

The Lagrangian is then given by:

$$\begin{aligned}
L = & E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c_t, n_t, a_t) \right. \\
& + \sum_{t=0}^{\infty} \beta^t \lambda_t \left( z_t k_{t-1} + w_t n_t + m_{t-1} \frac{P_{t-1}}{P_t} + \frac{\Xi_t}{P_t} + m_t^s \right. \\
& \quad \left. \left. + (1 + R_{t-1}) b_{t-1} \frac{P_{t-1}}{P_t} - c_t - i_t - m_t - b_t \right) \right. \\
& + \sum_{t=0}^{\infty} \beta^t \Omega_t \left( m_{t-1} \frac{P_{t-1}}{P_t} + m_t^s - c_t \right) \\
& \left. + \sum_{t=0}^{\infty} \beta^t \theta_t \left( (1 - \delta) k_{t-1} + \phi \left( \frac{i_t}{k_{t-1}} \right) k_{t-1} - k_t \right) \right] \quad (3.7)
\end{aligned}$$

Again small variables indicate real quantities, i.e. for example  $m_t = M_t/P_t$ . Households now optimize over  $c_t, n_t, i_t, k_t, m_t$  and  $b_t$  taking prices and the initial values of the price level  $P_0$  and the capital stock  $k_0$  as well as the outstanding stocks of money  $M_0$  and bonds  $B_0$  as given. The first order conditions for an interior solution are reported below.

$$\frac{\partial L}{\partial c_t} = \beta^t \frac{\partial u(c_t, n_t, a_t)}{\partial c_t} - \beta^t \lambda_t - \beta^t \Omega_t = 0 \quad (3.8)$$

$$\frac{\partial L}{\partial n_t} = \beta^t \frac{\partial u(c_t, n_t, a_t)}{\partial n_t} + \beta^t \lambda_t w_t = 0 \quad (3.9)$$

$$\frac{\partial L}{\partial i_t} = -\beta^t \lambda_t + \beta^t \theta_t \phi' \left( \frac{i_t}{k_{t-1}} \right) \left( \frac{1}{k_{t-1}} \right) k_{t-1} = 0 \quad (3.10)$$

$$\begin{aligned}
\frac{\partial L}{\partial k_t} = & E_t \beta^{t+1} \lambda_{t+1} z_{t+1} - \beta^t \theta_t + E_t \beta^{t+1} \theta_{t+1} \left[ (1 - \delta) \right. \\
& \left. + \phi \left( \frac{i_{t+1}}{k_t} \right) + \phi' \left( \frac{i_{t+1}}{k_t} \right) \left( -\frac{i_{t+1}}{k_t^2} \right) k_t \right] = 0 \quad (3.11)
\end{aligned}$$

$$\frac{\partial L}{\partial m_t} = -\beta^t \lambda_t + E_t \beta^{t+1} \lambda_{t+1} \frac{P_t}{P_{t+1}} + E_t \beta^{t+1} \Omega_{t+1} \frac{P_t}{P_{t+1}} = 0 \quad (3.12)$$

$$\frac{\partial L}{\partial b_t} = -\beta^t \lambda_t + E_t \beta^{t+1} \lambda_{t+1} (1 + R_t) \frac{P_t}{P_{t+1}} = 0 \quad (3.13)$$

The derivatives with respect to  $\lambda_t$  and  $\Omega_t$  are omitted since they are equal to the intertemporal budget constraint and the cash-in-advance constraint, respectively. The derivative with respect to  $\theta_t$  is not reported again since it is given by the capital accumulation condition stated above.  $\phi'$  denotes the derivative of the  $\phi$ -function with respect to the investment to capital ratio which is regarded as one

argument. Note again that these conditions result from the more general Kuhn-Tucker conditions assuming that all variables and multipliers are strictly positive. This implies especially that - given  $\Omega_t > 0$  - the CIA-constraint is always binding and that the nominal interest rate  $R_t$  is positive. Otherwise (3.12) and (3.13) will not be compatible. In addition the household's optimal choices must also satisfy the transversality conditions:

$$\lim_{t \rightarrow \infty} \beta^t \lambda_t x_t = 0 \quad \text{for } x = m, b, k \quad (3.14)$$

The familiar result that the first two efficiency conditions imply the equality of the marginal rate of substitution between consumption and labor and the real wage is distorted here by the cash-in-advance constraint. The real wage is now given by

$$w_t = -\frac{1}{\beta} \frac{\frac{\partial u(c_t, n_t, a_t)}{\partial n_t}}{\frac{\partial u(c_{t+1}, n_{t+1}, a_{t+1})}{\partial c_{t+1}}} \frac{P_{t+1}}{P_t} \quad (3.15)$$

As before this equation can be derived by eliminating  $\Omega_t$  in the efficiency condition for consumption using the efficiency condition for money. Note again the different timing of the marginal utility of consumption and labor which alters the dynamic evolution of  $w_t$ . There is also a direct influence of inflation on the real wage.

The efficiency condition for bond holdings establishes a relation between the nominal interest rate and the price level. Rearranging terms yields

$$(1 + R_t) = \frac{\lambda_t}{\lambda_{t+1}} \frac{1}{\beta} \frac{P_{t+1}}{P_t} \quad (3.16)$$

Supposed the Fisher equation is valid the real interest rate  $r_t$  is implicitly defined as

$$(1 + r_t) = \frac{\lambda_t}{\lambda_{t+1}} \frac{1}{\beta} \quad (3.17)$$

because  $P_{t+1}/P_t$  equals one plus the rate of expected inflation which is approximated by the ex-post-inflation rate.

### 3.2.2 The Finished Goods Producing Firm

The firm producing the final good  $y_t$  in the economy uses  $y_{j,t}$  units of each intermediate good  $j \in [0, 1]$  purchased at price  $P_{j,t}$  to produce  $y_t$  units of the finished good. Note that due to the inclusion of capital accumulation output no longer equals consumption. The production function is again assumed to be a CES aggregator as in Dixit and Stiglitz (1977) with  $\epsilon > 1$ .

$$y_t = \left( \int_0^1 y_{j,t}^{(\epsilon-1)/\epsilon} dj \right)^{\epsilon/(\epsilon-1)} \quad (3.18)$$

The firm maximizes its profits over  $y_{j,t}$  given the above production function and given the price  $P_t$ . So the problem can be written as

$$\max_{y_{j,t}} \left[ P_t y_t - \int_0^1 P_{j,t} y_{j,t} dj \right] \text{ s.t. } y_t = \left( \int_0^1 y_{j,t}^{(\epsilon-1)/\epsilon} dj \right)^{\epsilon/(\epsilon-1)} \quad (3.19)$$

The first order conditions for each good  $j$  imply

$$y_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{-\epsilon} y_t \quad (3.20)$$

where  $-\epsilon$  measures the constant price elasticity of demand for each good  $j$ . Because the firm operates under perfect competition profits are zero. Inserting the demand function into the profit function and imposing the zero profit condition reveals that the only price  $P_t$  that is consistent with this requirement is given by

$$P_t = \left( \int_0^1 P_{j,t}^{(1-\epsilon)} dj \right)^{1/(1-\epsilon)} \quad (3.21)$$

### 3.2.3 The Intermediate Goods Producing Firm

Intermediate goods firms can be considered to consist of a producing and a pricing unit. The producing unit is the same for both contract schemes and it will be presented in the next subsection. The pricing unit operates differently for Taylor and Calvo staggering and will thus be discussed separately in the following subsections.

#### 3.2.3.1 The Producing Unit

The producing unit operates under a Cobb-Douglas-technology which is subject to an aggregate random productivity shock  $a_t$ .

$$y_{j,t} = a_t n_{j,t}^\alpha k_{j,t-1}^{1-\alpha} \quad (3.22)$$

Here  $n_{j,t}$  is the labor input employed in period  $t$  by a firm  $j$ , similarly  $k_{j,t-1}$  is the capital stock, and  $0 < \alpha < 1$  is labor's share. The producing unit determines the optimal labor and capital inputs by minimizing costs. In models with capital the problem is given by

$$\begin{aligned} \min_{n_{j,t}, k_{j,t-1}} & [P_{j,t} w_{j,t} n_{j,t} + P_{j,t} z_{j,t} k_{j,t-1}] \\ \text{s.t. } & y_{j,t} = a_t n_{j,t}^\alpha k_{j,t-1}^{1-\alpha} \end{aligned} \quad (3.23)$$

It is useful for further calculations to define nominal marginal cost as  $\Psi_{j,t}$  which is equal to the Lagrange multiplier in the cost minimization problem stated above. The efficiency conditions are the following:

$$P_{j,t}w_{j,t} = \Psi_{j,t}\alpha a_t n_{j,t}^{\alpha-1} k_{j,t-1}^{1-\alpha} \quad (3.24)$$

$$P_{j,t}z_{j,t} = \Psi_{j,t}(1-\alpha) a_t n_{j,t}^{\alpha} k_{j,t-1}^{-\alpha} \quad (3.25)$$

In a symmetric equilibrium all choices of the producing unit of the firms are the same so that

$$P_{j,t} = P_t, w_{j,t} = w_t, z_{j,t} = z_t, \Psi_{j,t} = \Psi_t, n_{j,t} = n_t, k_{j,t-1} = k_{t-1} \text{ for all } t \quad (3.26)$$

So (3.24) and (3.25) hold with all  $j$ 's eliminated.

### 3.2.3.2 The Pricing Unit under Taylor Staggering

The pricing unit sets prices to maximize the present discounted value of profits. Those firms who do not adjust their prices in a given period can be interpreted as passive while those who do adjust do so optimally. Define the relative price by  $p_{j,t} = P_{j,t}/P_t$ . Because the production functions are homogenous of degree one real profit  $\xi_{j,t} = \Xi_{j,t}/P_t$  for a firm of type  $j$  is equal to

$$\xi_{j,t} = \xi(p_{j,t}, y_t, \psi_t) = p_{j,t}y_{j,t} - \psi_t y_{j,t} \quad (3.27)$$

where  $\psi_t = \Psi_t/P_t$  is real marginal cost. Using the demand function for the intermediate goods ( $y_{j,t} = p_{j,t}^{-\epsilon} y_t$ ) the profit function can be rewritten as

$$\xi_{j,t} = \xi(p_{j,t}, y_t, \psi_t) = y_{j,t}(p_{j,t} - \psi_t) = p_{j,t}^{-\epsilon} y_t (p_{j,t} - \psi_t) \quad (3.28)$$

When prices are fixed for two periods the firm has to take into account the effect of the price chosen in period  $t$  on current and future profits. The price in period  $t+1$  will be affected by the gross inflation rate  $\Pi_{t+1}$  between  $t$  and  $t+1$  ( $\Pi_{t+1} = P_{t+1}/P_t$ ).

$$p_{1,t+1} = \frac{p_{0,t}}{\Pi_{t+1}} \quad (3.29)$$

The optimal relative price has to balance the effects due to inflation between profits today and tomorrow. This intertemporal maximization problem is formally given by

$$\begin{aligned} \max_{p_{0,t}} E_t & \left[ \xi(p_{0,t}, y_t, \psi_t) + \beta \frac{\lambda_{t+1}}{\lambda_t} \xi(p_{1,t+1}, y_{t+1}, \psi_{t+1}) \right] \\ \text{s.t. } & p_{1,t+1} = \frac{p_{0,t}}{\Pi_{t+1}} \end{aligned} \quad (3.30)$$

The term  $\beta\lambda_{t+1}/\lambda_t$  is again the appropriate discount factor for real profits. Solving the efficiency condition for the optimal price to be set in period  $t$  using (3.28) yields a forward-looking form of the price equation and is in that respect similar to the one in Taylor (1980).

$$p_{0,t} = \frac{\epsilon}{\epsilon - 1} \frac{\lambda_t y_t \psi_t + \beta E_t \lambda_{t+1} (P_{t+1}/P_t)^\epsilon y_{t+1} \psi_{t+1}}{\lambda_t y_t + \beta E_t \lambda_{t+1} (P_{t+1}/P_t)^{\epsilon-1} y_{t+1}} \quad (3.31)$$

The optimal relative price  $p_{0,t}$  depends upon the current and future real marginal costs, the gross inflation rate, current and future output as well as today's and tomorrow's interest rates captured by the  $\lambda$ 's. To derive the Taylor approximation in Appendix B it is useful to write (3.31) as

$$P_{0,t} = \frac{\epsilon}{\epsilon - 1} \frac{\lambda_t P_t^\epsilon y_t \psi_t + \beta E_t \lambda_{t+1} P_{t+1}^\epsilon y_{t+1} \psi_{t+1}}{\lambda_t P_t^{\epsilon-1} y_t + \beta E_t \lambda_{t+1} P_{t+1}^{\epsilon-1} y_{t+1}} \quad (3.32)$$

With prices set for two periods half of the firms adjust their price in period  $t$  and half do not. Moreover all adjusting firms choose the same price. Then  $P_{j,t}$  is the nominal price at time  $t$  of any good whose price was set  $j$  periods ago and  $P_t$  is the price index (3.21) at time  $t$  and is given by

$$P_t = \left( \frac{1}{2} P_{0,t}^{1-\epsilon} + \frac{1}{2} P_{1,t}^{1-\epsilon} \right)^{1/(1-\epsilon)} \quad (3.33)$$

### 3.2.3.3 The Pricing Unit under Calvo Staggering

Under Calvo pricing there exists a constant probability  $\varphi$  that firms are not able to change their price so that  $P_{j,t} = P_{j,t-1}$ .<sup>2</sup> With a probability of  $1 - \varphi$  firms may reset their price independent of the time foregone since the last change in prices. Real profits can again be written as in (3.28) but it is useful to use the nominal prices as profits have to be evaluated  $s$  periods in the future.

$$\xi_{j,t+s} = \xi(p_{j,t+s}, y_{t+s}, \psi_{t+s}) = y_{j,t+s} \left( \frac{P_{j,t+s}}{P_{t+s}} - \psi_{t+s} \right) \quad (3.34)$$

The demand functions for the intermediate goods in period  $t + s$  are given by

$$y_{j,t+s} = P_{j,t+s}^{-\epsilon} P_{t+s}^\epsilon y_{t+s} = P_{0,t}^{-\epsilon} P_{t+s}^\epsilon y_{t+s} \quad (3.35)$$

<sup>2</sup>Some authors assume an indexation rule for these firms so that  $P_{j,t} = \bar{\Pi} P_{j,t-1}$  where  $\bar{\Pi}$  is the inflation factor, see e.g. Kim (2003). Others like Christiano, Eichenbaum and Evans (2003) propose an indexation rule that allows for a variable gross inflation rate  $\Pi_{t-1}$  to account for inertia in inflation. Since in the model here inflation is zero at the steady state these extensions are not considered.

The last equality holds because the price  $P_{j,t} = P_{0,t}$  has not been changed for  $s$  periods. Inserting these demand functions into the profit function yields

$$\xi_{j,t+s} = P_{0,t}^{1-\epsilon} P_{t+s}^{\epsilon-1} y_{t+s} - \psi_{t+s} P_{0,t}^{-\epsilon} P_{t+s}^{\epsilon} y_{t+s} \quad (3.36)$$

Now firms can reset their prices with a probability of  $1 - \varphi$ . With probability  $\varphi$  they could not change their price so with a probability of  $\varphi^s$  their old price is still valid in a period  $s$ . But differently  $P_{0,t}$  influences a firm  $j$ 's profits as long as it cannot reoptimize its price. The probability that this occurs for  $s$  periods is given by  $\varphi^s$ . Accordingly the intertemporal profit maximization problem can be written as follows:

$$\max_{P_{0,t}} E_t \left[ \sum_{s=0}^{\infty} (\beta\varphi)^s \frac{\lambda_{t+s}}{\lambda_t} \xi_{j,t+s} \right] \quad (3.37)$$

Intermediate goods firms maximize the present value of their profits as under Taylor staggering but now for an infinite horizon. In analogy to Taylor pricing  $\beta^s \lambda_{t+s}/\lambda_t$  is the appropriate discount factor. Using (3.36) and rearranging the optimal reset price is given by

$$P_{0,t} = \frac{\epsilon}{\epsilon - 1} \frac{\sum_{s=0}^{\infty} (\beta\varphi)^s E_t \lambda_{t+s} P_{t+s}^{\epsilon} y_{t+s} \psi_{t+s}}{\sum_{s=0}^{\infty} (\beta\varphi)^s E_t \lambda_{t+s} P_{t+s}^{\epsilon-1} y_{t+s}} \quad (3.38)$$

Using the pricing rule for non-adjusting firms  $P_{j,t} = P_{j,t-1}$  the price level (3.21) can be written as follows<sup>3</sup>

$$P_t = [\varphi P_{t-1}^{1-\epsilon} + (1 - \varphi) P_{0,t}^{1-\epsilon}]^{\frac{1}{1-\epsilon}} \quad (3.39)$$

One can now combine the optimum condition and the price level equation to derive the so called New Keynesian Phillips curve **as a Taylor approximation**.<sup>4</sup>

$$\hat{\pi}_t = (1 - \varphi) (1 - \beta\varphi) \varphi^{-1} \hat{\psi}_t + \beta E_t \hat{\pi}_{t+1} \quad (3.40)$$

This result is very important. Note that output, the optimal price and the Lagrange multiplier  $\lambda$  do not show up in this equation. It is the typical forward-looking Phillips curve where inflation  $\hat{\pi}_t$  depends on the expected inflation rate and on real marginal costs.

<sup>3</sup>This requires very tedious algebra. See Calvo (1983).

<sup>4</sup>A formal derivation of this equation can be found in the appendix of Schabert (2001) and also in Walsh (2003), p. 263-266. The same formula is obtained in Christiano, Eichenbaum and Evans (2003) while Kim (2003) uses a different way to solve the dynamic system.

### 3.2.4 Market Clearing Conditions and Other Equations

The aggregate resource constraint is derived using the resource constraint of households, firms, the government and the monetary authority. Since there are neither government expenditures nor taxes in this model, this condition is given by

$$y_t = c_t + i_t \quad (3.41)$$

As explained in Chapter 2 models like the one at hand imply multiple equilibria and sunspots because bonds are not determined. To escape this problem the household budget constraint is dropped and bonds are set to zero:  $b_t = 0$  for all  $t$ .

Substituting  $M_t^s$  in the CIA-constraint - holding with equality - allows to derive the implicit money demand function in the CIA-model.

$$M_t = P_t c_t \quad (3.42)$$

It is again a quantity theoretic type of money demand.

The markup  $\mu_t$  is just the reciprocal of real marginal cost so that

$$\mu_t = \frac{1}{\psi_t} \quad (3.43)$$

### 3.2.5 The Monetary Authority

The model is closed by adding a monetary policy rule. Therefore an exogenous process for the money growth rate is considered. To achieve persistent but non permanent effects the level of money again follows an AR(2)-process. Assume that money grows at a factor  $g_t$ :

$$M_t = g_t M_{t-1} \quad (3.44)$$

If  $\hat{g}_t$  follows an AR(1)-process  $\hat{g}_t = \rho_g \hat{g}_{t-1} + \epsilon_{g_t}$  then money will follow an AR(2)-process. As before  $\rho_g$  lies between 0 and 1 and  $\epsilon_{g_t}$  is white noise. Remember that inflation is zero at the steady state so also money growth is zero there ( $g = 1$ ).

The productivity and taste shock  $a_t$  follows an AR(1)-process

$$\hat{a}_t = \rho_a \hat{a}_{t-1} + \epsilon_{a_t} \quad (3.45)$$

with  $\epsilon_{a_t}$  white noise and  $0 < \rho_a < 1$ .

### 3.2.6 The Steady State

Imposing the condition of constancy of the price level in the steady state ( $P_t = P_{t-1} = P$ ) on the nominal interest rate equation reveals the familiar condition



from RBC models that  $\beta = 1/(1 + R)$ . In addition, as there is no steady state inflation,  $R = r$ . For Taylor staggering the two period price setting of the firms implies  $P_0 = P_1$ . Using this in the price index reveals that  $P_0 = P_1 = P$ . Under Calvo pricing  $P_0 = P$  holds in the symmetric equilibrium. The capital accumulation equation tells us that  $\phi(i/k) = \delta$  at the steady state. It is assumed that  $\phi' = 1$  in the steady state to ensure that Tobin's  $q$  is equal to one while Tobin's  $q$  is given by  $q = 1/\phi'$ . As a consequence of the requirement that the model with adjustment costs of capital should display the same steady state as the model without them  $i/k$  is equal to  $\phi(i/k)$ . Using this in the efficiency condition for capital it can be shown that the rental rate on capital is  $z = r + \delta$  as in a standard RBC model. With the help of (3.24) and the steady state for  $z$  it is possible to pin down  $k/n$  which amounts to

$$\frac{k}{n} = \left( \frac{r + \delta}{a} \frac{1}{1 - \alpha} \frac{1}{\psi} \right)^{-1/\alpha} \quad (3.46)$$

For the markup  $\mu$  it follows  $\mu = 1/\psi$  while  $\psi$  is determined by the steady state of the efficiency condition for maximizing profits, (3.32). This amounts to  $\psi = (\epsilon - 1)/\epsilon$ . This can be used to calculate  $w$  as well:

$$w = \psi a \alpha \left( \frac{k}{n} \right)^{1-\alpha} \quad (3.47)$$

The calculation of the steady state value of consumption is tedious because it takes quite a lot of steps. From the production function one knows that labor productivity is given by

$$\frac{y}{n} = a \left( \frac{k}{n} \right)^{1-\alpha} \quad (3.48)$$

This productivity can be combined with the investment to capital ratio to calculate the investment share:

$$\frac{i}{y} = \left( \frac{i}{k} \frac{k}{n} \right) / \left( \frac{y}{n} \right) \quad (3.49)$$

Now one can derive the consumption share using the aggregate resource constraint.

$$\frac{c}{y} = -\frac{i}{y} + 1 \quad (3.50)$$

To get the level of  $c$  the level of  $y$  and  $i$  have to be determined:  $y = n \cdot y/n$ ,  $i = y \cdot i/y$ . Finally  $c = y - i$  is the consumption steady state value.

(3.15) is used to pin down the preference parameter  $\zeta$  which is given by  $\zeta = c / [\beta(w - wn) + c]$ .

### 3.2.7 Calibration

In order to compute impulse responses the parameters of the model have to be calibrated. It is possible to either specify  $\beta$  or  $r$  exogenously. Here  $\beta$  will be set to 0.99 implying a value of  $r$  of about 0.0101 per quarter which is in line with other values used for the real interest rate in the literature.  $\psi$  and  $\mu$  can be determined by fixing a value for the elasticity of the demand functions for the differentiated products. This elasticity being equal to 4 causes the static markup  $\mu = \epsilon/(\epsilon - 1)$  to be 1.33 which is in line with the study of Linnemann (1999) about average markups.<sup>5</sup> In order to determine the steady state real wage  $w$  the productivity shock  $a$  has to be specified, along with calculating  $k/n$ , see below. As there is no information available about that parameter it is arbitrarily set at 10. Note that this is of no consequence for the Taylor approximations since the value has only an influence on levels that are of no interest here.  $n$  is specified to be equal to 0.25 implying that agents work 25 % of their non-sleeping time.

In the benchmark case,  $\sigma$ , the parameter governing the degree of risk aversion, is set to 2. The implied value of  $\zeta$  is given by  $\zeta = 0.3617$  which is in line with other studies.

As this model considers the role of capital accumulation several other technological parameters have to be calibrated. The most common one is the depreciation rate  $\delta$  which is set to 0.025 implying 10% depreciation per year. Labor's share  $\alpha$  is 0.64 whereas the elasticity of Tobin's  $q$  with respect to  $i/k$  is set to -0.5.<sup>6</sup> This value is also used in King and Wolman (1996). Adjustment costs of capital are introduced to dampen the volatility of investment. They are a common feature in equilibrium business cycle models but there are various ways to formalize them. Using  $r, \delta, a, \alpha$  and  $\psi$  the ratio  $k/n$  can be determined.

For the Calvo model the probability that firms can reset their price is given by  $1 - \varphi = 1/3$ . The probability that a price is still in effect in a period  $s$  is given by  $(1 - \varphi)\varphi^s$  because with  $1 - \varphi$  the price was once set optimally. So the average duration is given by  $(1 - \varphi) \sum_{s=0}^{\infty} s\varphi^s = \varphi/(1 - \varphi)$ .<sup>7</sup> This implies an average duration of price contracts of 2. Thus prices are on average fixed the same period of time as in the Taylor pricing version of the model. For the exogenous money growth process  $\rho_g = 0.5$  is used. As the focus here is on the persistence effects of money growth

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<sup>5</sup>This value is different from the one used in Chapter 2. But it does not have a significant influence on the model outcome.

<sup>6</sup>It can be shown that this elasticity is given by  $-\phi''/\phi' \cdot (i/k)$ .

<sup>7</sup>See Bénassy (2003b), p.12.

shocks productivity shocks will not be considered. But they can be used to check whether the model displays reasonable impulse responses to technology shocks.

### 3.3 Impulse Response Functions

As before the solution is conducted using an extended version of the algorithm of King, Plosser and Rebelo (2002) which allows for singularities in the system matrix of the reduced model. The results will be again discussed using impulse responses. They are presented in the next two subsections.

#### 3.3.1 Taylor Staggering

Here the impulse responses of the model variables to a 1% shock to the money growth rate will be discussed. Figures 3.1 – 3.4 display the reaction of selected variables to this shock. Compared to the results in the labor economy of the previous chapter all variables show even less persistence. Most aggregates are cyclical again. An exception is investment which stays above steady state but which is not persistent either. Consumption reacts a bit weaker than in a labor economy whereas output shows a stronger reaction. Price adjusting firms raise their price even more than before. Note the kink in the impulse response indicating the strength of the initial price adjustment. Inflation no longer displays a hump. The model cannot explain the liquidity effect, the nominal interest rate rises. The capital stock also rises and is even hump-shaped. But note the small percentage deviation of 0.03% from steady state. Overall these figures confirm the findings of Chari, Kehoe and McGrattan (2000) that once intertemporal links like capital accumulation are included in a Taylor price staggering model there is even less persistence than before.

What is the reason for the even worse performance of a model augmented by capital accumulation? The answer to this question can again be found in the reaction of real marginal costs.  $\hat{\psi}_t$ 's initial reaction is more than 1% while it was 0.7% in the labor only economy of Chapter 2. This is because the inclusion of the capital stock changes the marginal cost function. It now depends also on the rental of capital  $z_t$  so the dynamics are not only determined by the real wage rate  $w_t$  as discussed in Chapter 2. Formally  $\psi_t$  can be written as follows:

$$\psi_t = \left(\frac{w_t}{\alpha}\right)^\alpha \left(\frac{z_t}{1-\alpha}\right)^{1-\alpha} \frac{1}{a_t} \quad (3.51)$$

The money growth shock leads to an increase in the demand for labor  $n$  but also to higher demand for capital  $k$  because the demand for goods rises. Thus not only

the wage rate but also the rental rate on capital  $z_t$  rises. This results in additional upward pressure on marginal costs. Moreover  $w_t$  will also be determined from the production side of the economy. This means that in addition to the transmission channel discussed in (2.56) there is a direct influence on the wage rate through higher demand for labor and capital, see (3.24). Figure 3.2 reveals that  $\widehat{w}_t$  reacts stronger than in a labor economy (0.85% deviation).<sup>8</sup> The rental rate  $\widehat{z}_t$  has a 1.37 percentage deviation in the initial period resulting in the 1% reaction of  $\widehat{\psi}_t$  (because of the weighting parameters  $\alpha$  and  $1 - \alpha$  in (3.51)). In turn price adjusting firms will increase their optimal price stronger than in a world with only labor as a productive input factor. The fact that consumption no longer equals output also contributes to the stronger price increase. Note that it is output and not consumption that shows up in (3.32). As  $\widehat{y}_t$  reacts stronger than  $\widehat{c}_t$  firms raise their prices stronger.

### 3.3.2 Calvo Staggering

Figures 3.5 – 3.8 show the results for Calvo staggering. Note that the average length of price stickiness is the same as in the previous section under Taylor staggering. The results are very astonishing. Output, labor and investment show considerable persistence after a money growth shock. The contract multiplier for output is 0.86. This is even higher than in the MIU-model with GHH preferences and a high labor supply elasticity. There the multiplier was only 0.55. Consumption even shows a hump. The same holds for real money balances. The capital stock increase is higher than under Taylor pricing and the reaction is very smooth and long lasting. Unfortunately the model is again unable to account for the liquidity effect.

Why are the dynamics here completely different? This question is of special interest because real marginal costs rise *stronger* than in a Taylor staggering model.  $\widehat{\psi}_t$  deviates 1.4% from steady state in the initial period which is 40% higher here. Prices  $\widehat{P}_{0,t}$  even overshoot, see Figure 3.8. But the price level shows a remarkable persistence, too, as this figure reveals. Since both models are exactly equal with the exception of the price setting rule the answer to the question must be found there. As stated above the New Keynesian Phillips curve is valid in the Calvo model only as a Taylor approximation. It is derived by approximating (3.38) at the steady state. In (3.38) there are several sums over an infinite horizon. It can be shown that during the approximation all Lagrange multipliers  $\lambda_{t+s}$ , all outputs  $y_{t+s}$  and all  $\psi_{t+s}$  except  $\psi_t$  cancel. This eliminates an enormous amount of dynamic interaction resulting in

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<sup>8</sup>Remember that  $\widehat{w}_t = \widehat{\psi}_t$  in the labor economy.

an equation in which solely the expected inflation rate and current real marginal costs show up. Comparing (3.40) with (3.32) immediately confirms this intuition. Taken together this leads to the results presented in the figures.

Kim (2003) tries to get more intuition by simplifying his models so that he can obtain analytical results. In these stripped down versions he can show that the autoregressive coefficient in the pricing equation (which is actually a first order difference equation) is negative leading to the oscillatory behavior in the Taylor staggering model. In contrast the respective coefficient in the Calvo model can be shown to be positive. In the Taylor version this parameter depends on  $(1 - n)/n$  and on the price elasticity of the demand for intermediate goods  $\epsilon$  while in the Calvo setup it depends also on  $(1 - n)/n$  and on the probability that firms cannot adjust prices  $\varphi$ . He can demonstrate that the autoregressive coefficient in the Taylor model is *always* negative irrespective of the specific value of  $\epsilon$  while in the Calvo staggering model it is *always* positive. This result is confirmed in the model at hand. Using a very low value of  $\varphi$  lowers the contract multiplier considerably but does not lead to cyclical reactions of output. Figures 3.9 – 3.12 display the results for  $\varphi = 0.50$  implying that prices are on average fixed for only one period (quarter). Changing  $\epsilon$  in the Taylor model does not have any sizeable impact on the impulse responses. In particular the cyclical nature does not disappear.<sup>9</sup>

### 3.4 Business Cycle Properties

In order to explore the implications for the business cycle properties one has to specify the standard deviation of the AR(1)-process for money growth. Here the value estimated in Cooley and Hansen (1995), p.201, is used.<sup>10</sup> It implies a value of 0.0000792 for the variance  $\sigma_y^2$ . Table 3.1 shows the results for the Taylor staggering model after HP-filtering with  $\lambda = 1600$ .<sup>11</sup>

$\sigma_{\hat{x}}$  again denotes the percentage standard deviation of  $\hat{x}$  whereas  $\sigma_{\hat{x}}/\sigma_{\hat{y}}$  measures the respective standard deviation relative to that of output  $\hat{y}$ . The next two columns report the autocorrelations for one and two lags of the respective aggregate. The remaining columns display the cross correlations with output. A variable  $\hat{x}$  is leading

<sup>9</sup>Results are not shown in the figures.

<sup>10</sup>It is not intended to take the model explicitly to the data because of its overwhelming simplicity. This justifies the use of Cooley and Hansen's parameter values.

<sup>11</sup>Remember that all values in the tables have been rounded using the computer output. So it is possible that the relative standard deviations deliver a different value when using the values in the table.

Table 3.1: Moments in the Taylor Staggering Model

$\hat{x}_t$			autocorrelation		cross correlation of $\hat{x}_t$ with $\hat{y}$ in				
	$\sigma_{\hat{x}}$	$\sigma_{\hat{x}}/\sigma_{\hat{y}}$	1	2	$t-2$	$t-1$	$t$	$t+1$	$t+2$
$\hat{y}_t$	0.33	1.00	-0.42	-0.01	-0.01	-0.42	1.00	-0.42	-0.01
$\hat{i}_t$	0.98	2.97	-0.09	0.02	0.05	-0.08	0.88	-0.53	-0.06
$\hat{c}_t$	0.23	0.70	-0.30	-0.00	-0.07	-0.65	0.88	-0.20	0.03
$\hat{n}_t$	0.52	1.59	-0.40	-0.01	-0.02	-0.41	1.00	-0.40	-0.01
$\hat{w}_t$	0.73	2.21	-0.16	0.01	0.04	-0.14	0.92	-0.52	-0.05
$\hat{\mu}_t$	0.91	2.76	-0.23	0.01	-0.03	0.20	-0.95	0.50	0.04
$\hat{R}_t$	23.90	72.61	0.39	0.08	0.10	0.26	0.57	-0.52	-0.09
$\hat{\psi}_t$	0.91	2.76	-0.23	0.01	0.03	-0.20	0.95	-0.50	-0.04
$\hat{\Pi}_t$	0.98	2.99	0.45	-0.07	-0.04	0.60	0.28	-0.43	-0.09
$\hat{P}_t$	1.96	5.95	0.87	0.63	0.10	0.12	-0.18	-0.32	-0.10

$\hat{y}$  if the absolute value of the correlation  $\rho(\hat{x}_t, \hat{y}_{t+i})$  is highest for  $i > 0$ . Accordingly a variable  $\hat{x}$  is lagging  $\hat{y}$  if the absolute value of the correlation  $\rho(\hat{x}_t, \hat{y}_{t+i})$  has a maximum for  $i < 0$ . In case that this correlation is positive one speaks of a procyclical variable while it is called anticyclical if it is negative. If the maximum correlation occurs at lag 0 ( $i = 0$ ) the variable is moving with the cycle. Including capital accumulation improves the model outcome because it allows for a richer set of different volatilities and correlations. This applies especially to the relative standard deviations and the contemporaneous cross correlations. The model can account for investment's variability which is about three times the volatility of output. Consumption is about 70% as volatile as output. The cross correlations of these aggregates with output are 0.88, well below 1.00. Empirically, I found a relative volatility of consumption of 0.92 while investment is 2.73 times as volatile as output, see Gail (1998), p. 52. Hours worked are only about half as variable as output (0.57) so that the value of 1.59 is too high here. Also the real wage and real marginal costs show a reduced correlation with output of 0.92 and 0.95, respectively. As is already clear from the impulse responses the model cannot improve upon the autocorrelations of the aggregates. Again most variables are negatively autocorrelated. Only prices, inflation and the nominal rate display positive autocorrelation coefficients. The absolute standard deviation of output is slightly higher than in a labor only economy but still too low. Prices, inflation and especially the nominal rate are by far too volatile again.

Table 3.2 shows the results for the Calvo staggering model after HP-filtering with

$\lambda = 1600$ . Now all aggregates are positively autocorrelated. Output's volatility rises

Table 3.2: Moments in the Calvo Staggering Model

$\hat{x}_t$	$\sigma_{\hat{x}}$	$\sigma_{\hat{x}}/\sigma_{\hat{y}}$	autocorrelation		cross correlation of $\hat{x}_t$ with $\hat{y}$ in				
			1	2	$t-2$	$t-1$	$t$	$t+1$	$t+2$
$\hat{y}_t$	0.59	1.00	0.64	0.29	0.29	0.64	1.00	0.64	0.29
$\hat{i}_t$	1.57	2.68	0.54	0.21	0.16	0.50	0.99	0.66	0.34
$\hat{c}_t$	0.36	0.62	0.72	0.37	0.42	0.76	0.98	0.59	0.24
$\hat{n}_t$	0.92	1.57	0.63	0.29	0.26	0.61	1.00	0.65	0.32
$\hat{w}_t$	1.13	1.92	0.56	0.23	0.20	0.54	0.99	0.65	0.32
$\hat{\mu}_t$	1.46	2.49	0.58	0.24	-0.20	-0.54	-0.99	-0.66	-0.33
$\hat{R}_t$	25.04	42.76	0.38	0.08	-0.02	0.29	0.92	0.65	0.36
$\hat{\psi}_t$	1.46	2.49	0.58	0.24	0.20	0.54	0.99	0.66	0.33
$\hat{\Pi}_t$	0.71	1.22	0.52	0.20	0.14	0.48	0.98	0.66	0.34
$\hat{P}_t$	1.71	2.91	0.91	0.73	0.60	0.54	0.34	-0.07	-0.34

to 0.59 still being smaller than empirical estimates but considerably higher than in the Taylor staggering model. Investment's and consumption's relative variability is slightly reduced while labor fluctuates about as strong as before. The most striking difference concerns the cross correlations which rise very strongly: correlations at leads and lags are positive and the contemporaneous correlations are near 1.00. Such high values are not observed empirically. In German data, consumption has a contemporaneous correlation of 0.62 and investment a correlation of 0.78. The relative volatility of the price level, inflation and the nominal interest rate fall approaching their empirical counterparts. Prices are now clearly lagging procyclically (0.60) – which is counterfactual – while the inflation rate is procyclical. Interestingly the standard deviation of real marginal costs rises by more than 50% compared to the Taylor model. Overall this model performs much better in explaining actual business cycles without assuming an implausibly high labor supply elasticity as the MIU-model with GHH preference in the previous chapter.

### 3.5 Conclusions

This chapter has made clear that the results of Chari, Kehoe and McGrattan (2000) depend to a large extent on Taylor price staggering. It has been shown that in a model with Calvo pricing where firms face a specific probability of being able to

adjust their price persistent output and inflation responses to a money growth shock can be explained without having to rely on implausibly high price stickiness.

So it is thus also crucial how to implement sticky prices in a monetary dynamic general equilibrium model. Different assumptions on the staggering mechanism can lead to very different model outcomes. So besides the way in which money is introduced researchers have to be careful when implementing sticky prices. The conclusion of Chari, Kehoe and McGrattan (2000) that ‘mechanisms to solve the persistence problem must be found elsewhere’ has to be seriously questioned. Calvo price staggering models are very well able to account for persistence in output.

The paper of Christiano, Eichenbaum and Evans (2003) claims that habit persistence in consumption is an important real rigidity that can enhance the propagation of a monetary policy shock. This question is taken up in the next chapter in a model with Taylor staggering in order to explore the specific role of habits in consumption.



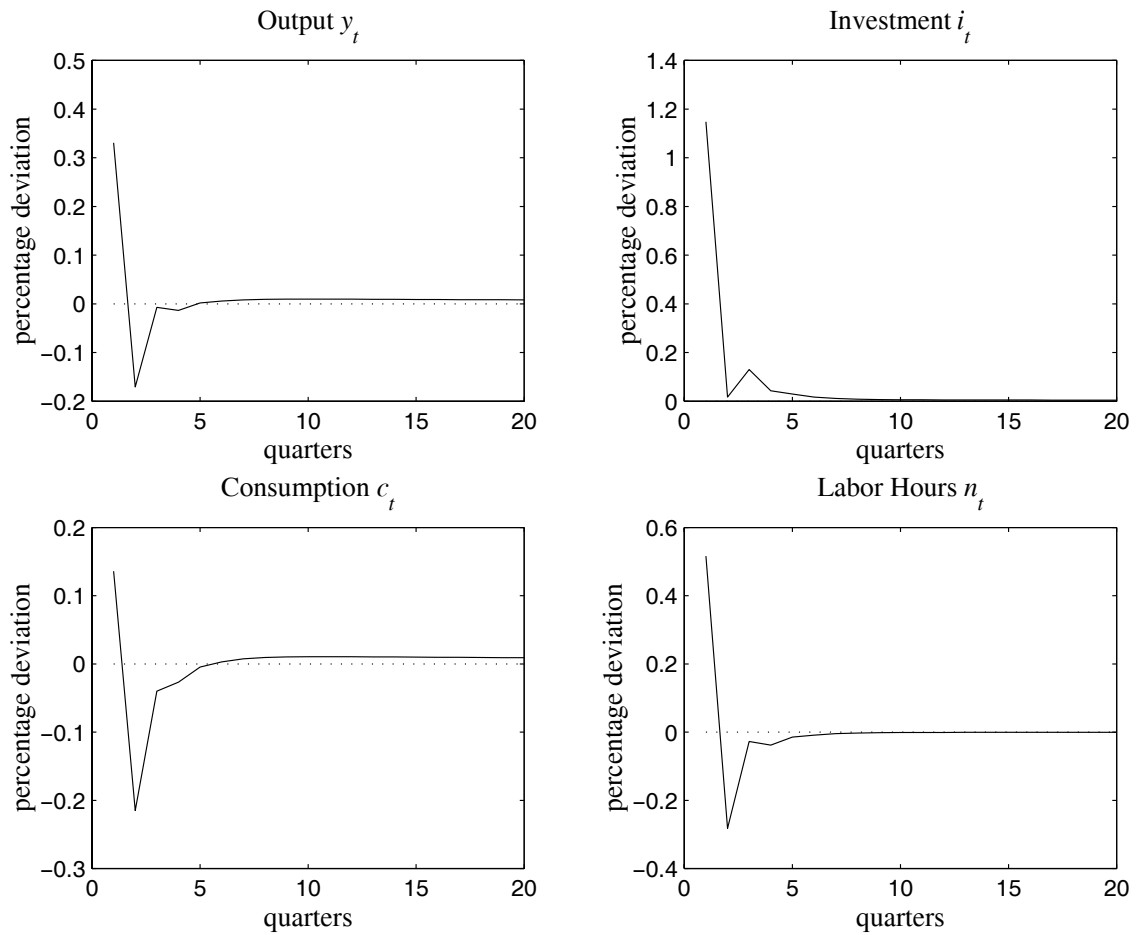


Figure 3.1: Impulse Response Functions for  $\hat{y}_t, \hat{i}_t, \hat{c}_t, \hat{n}_t$ , CIA-Model, CRRA Preferences, Taylor Staggering

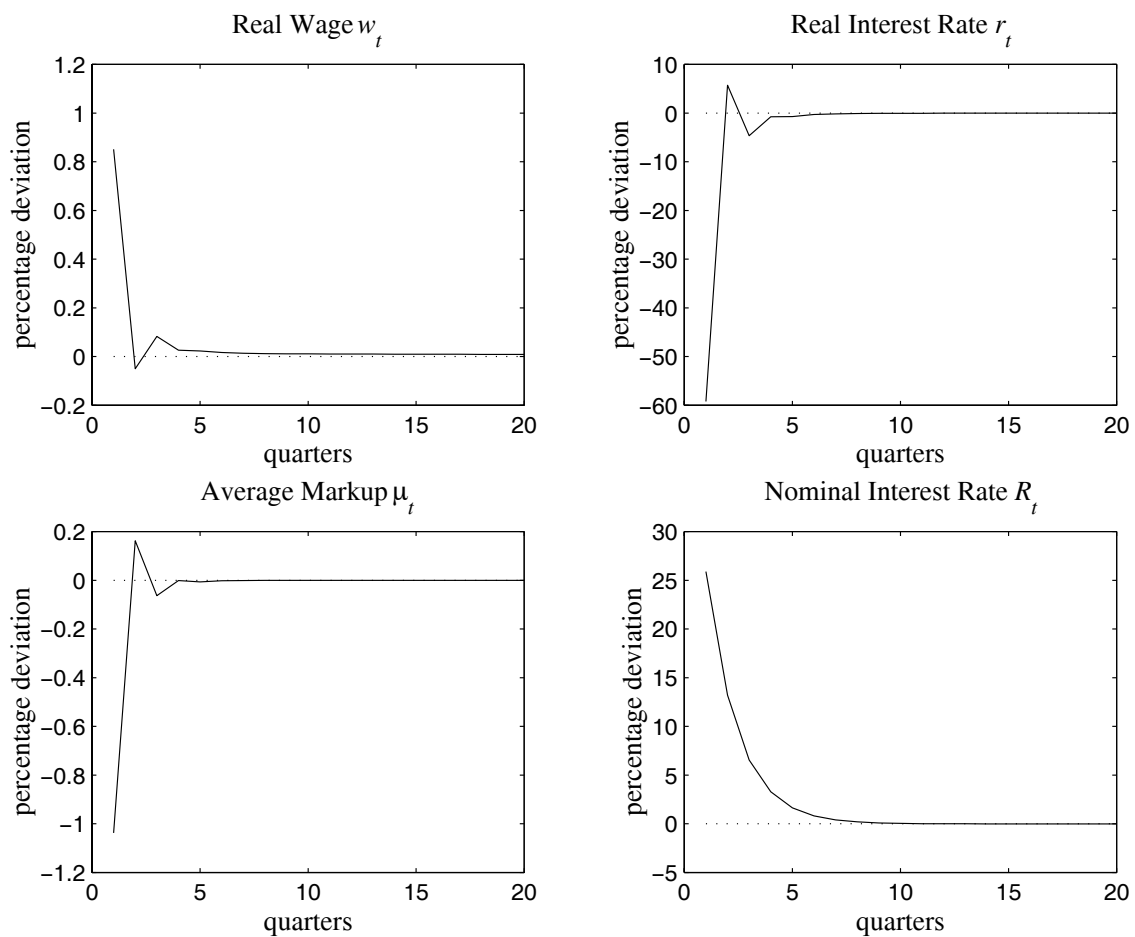


Figure 3.2: Impulse Response Functions for  $\hat{w}_t, \hat{r}_t, \hat{\mu}_t, \hat{R}_t$ , CIA-Model, CRRA Preferences, Taylor Staggering

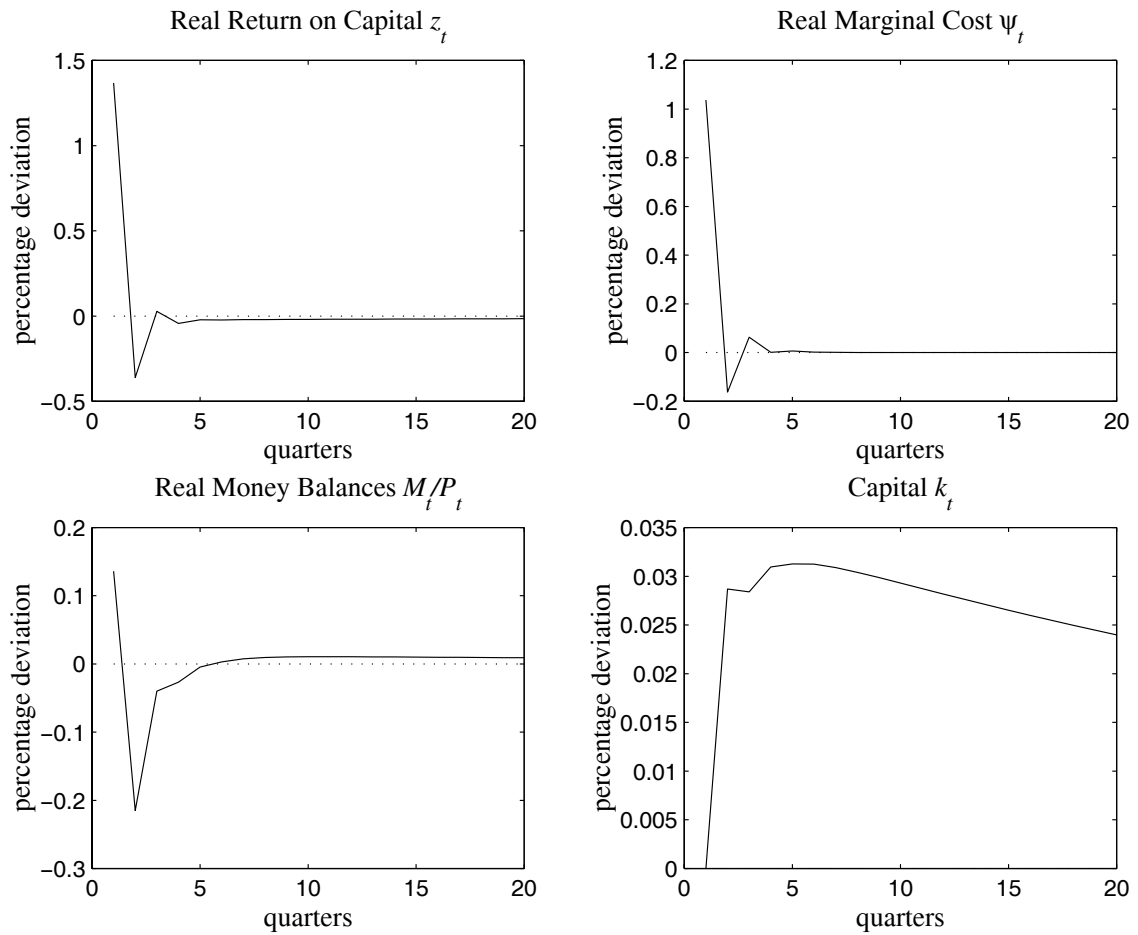


Figure 3.3: Impulse Response Functions for  $\hat{z}_t, \hat{\psi}_t, \hat{M}_t - \hat{P}_t, \hat{k}_t$ , CIA-Model, CRRA Preferences, Taylor Staggering

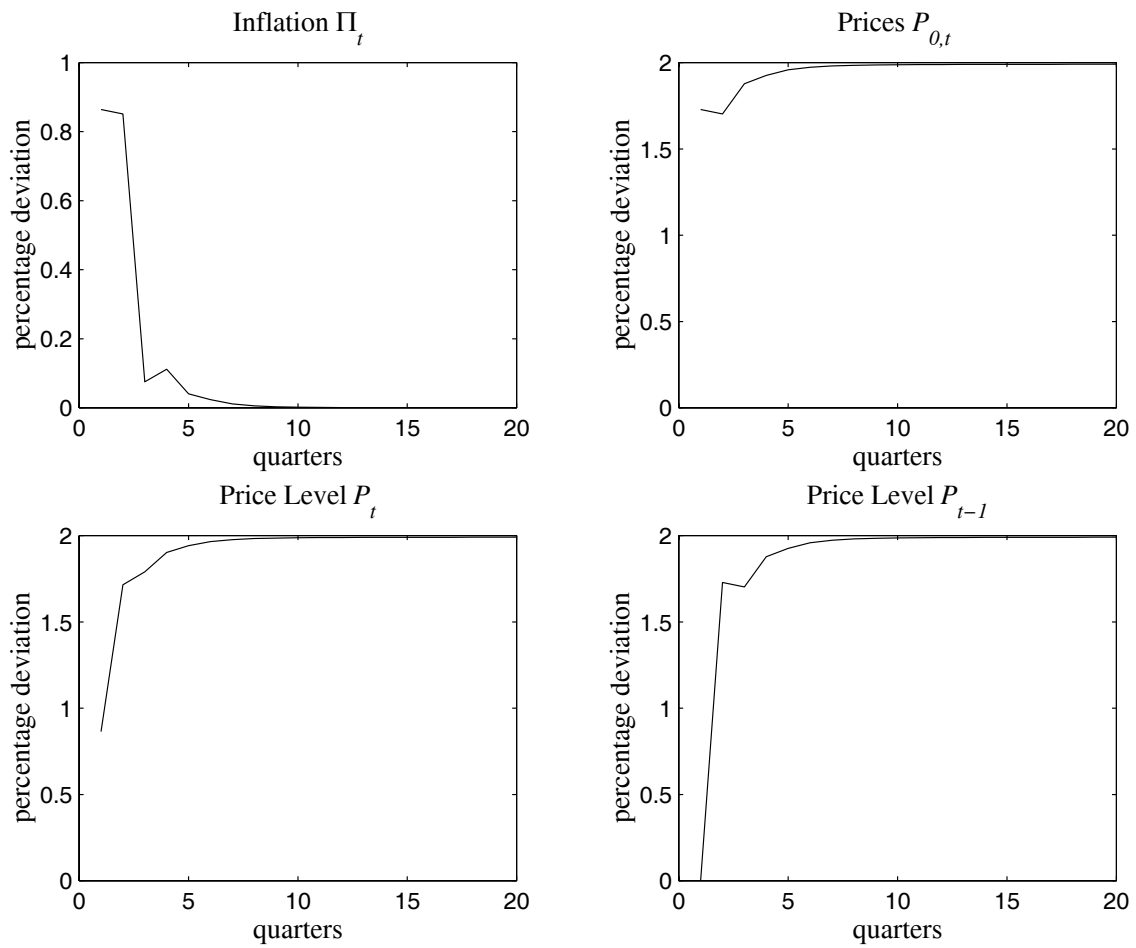


Figure 3.4: Impulse Response Functions for  $\hat{\Pi}_t, \hat{P}_{0,t}, \hat{P}_t, \hat{P}_{t-1}$ , CIA-Model, CRRA Preferences, Taylor Staggering

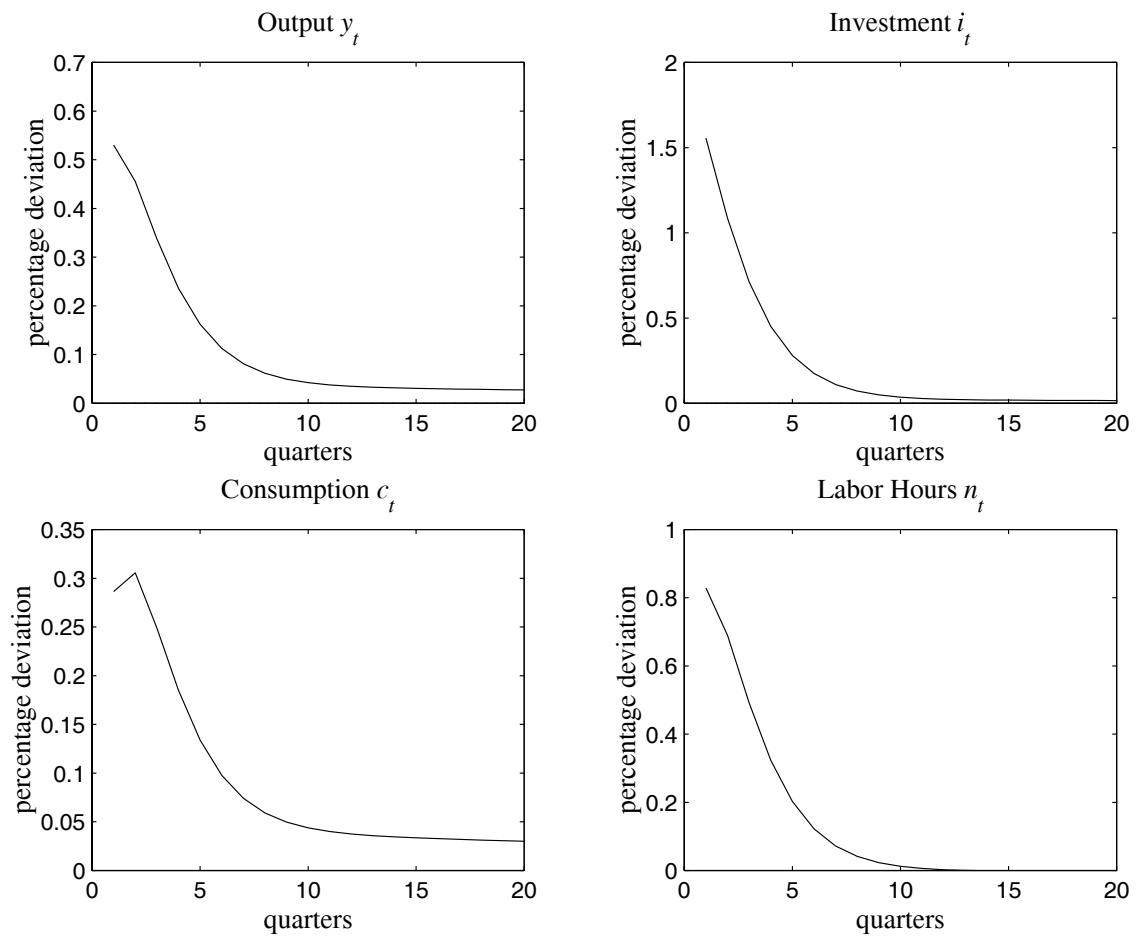


Figure 3.5: Impulse Response Functions for  $\hat{y}_t, \hat{i}_t, \hat{c}_t, \hat{n}_t$ , CIA-Model, CRRA Preferences, Calvo Staggering

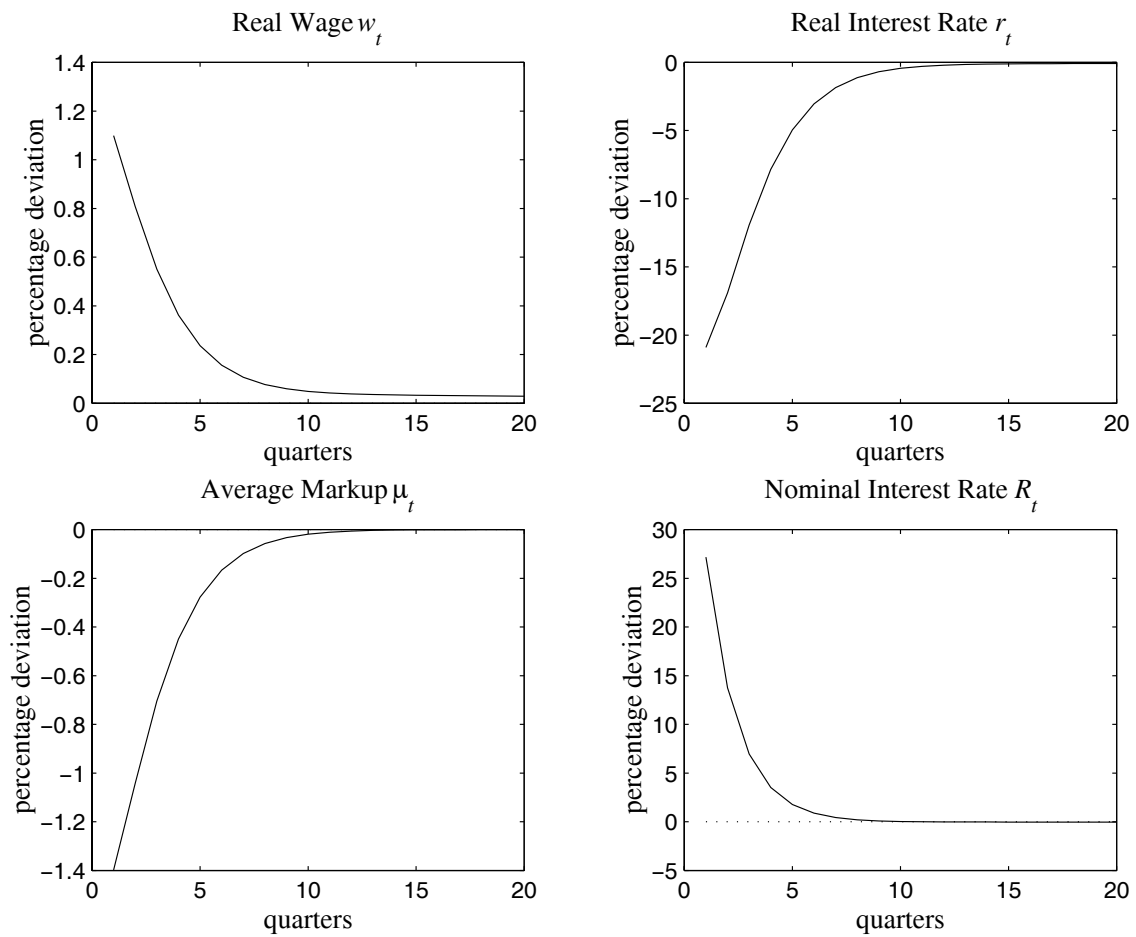


Figure 3.6: Impulse Response Functions for  $\hat{w}_t, \hat{r}_t, \hat{\mu}_t, \hat{R}_t$ , CIA-Model, CRRA Preferences, Calvo Staggering

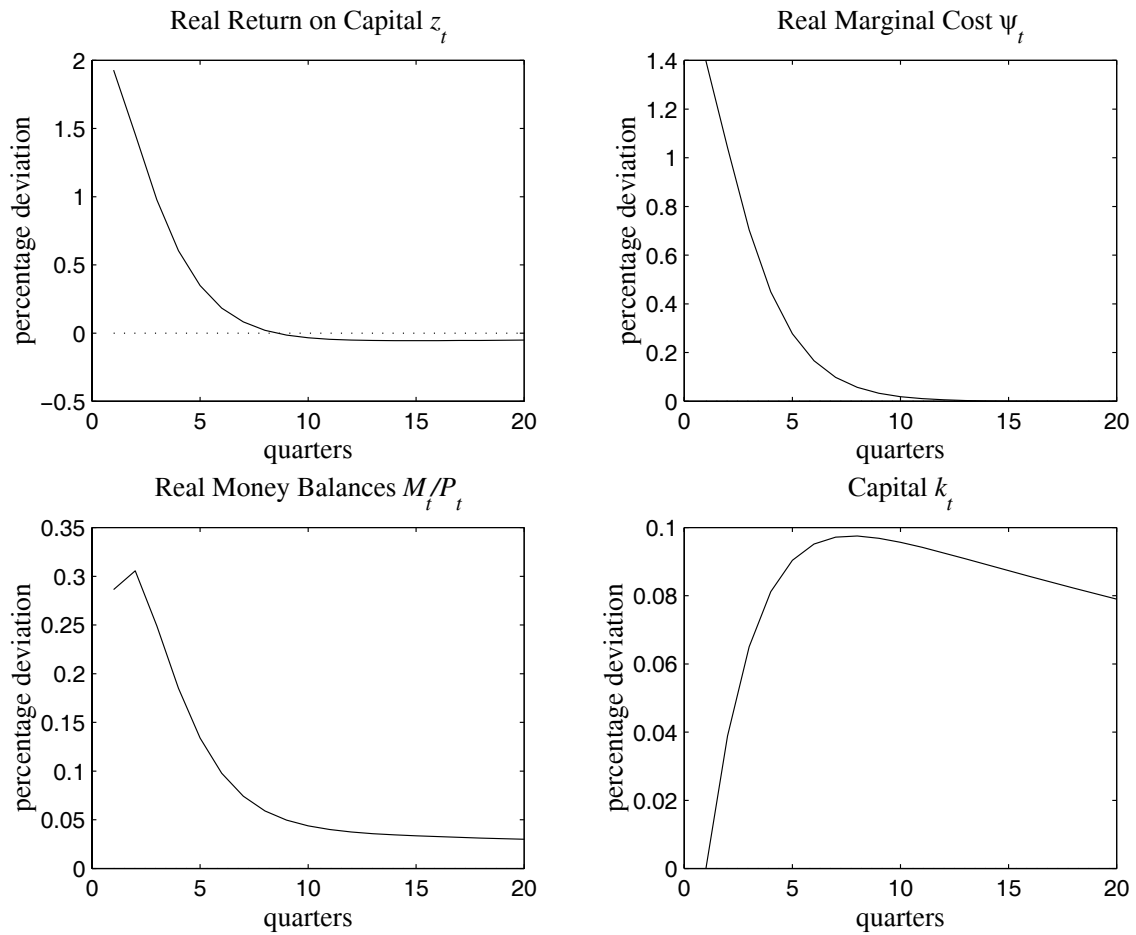


Figure 3.7: Impulse Response Functions for  $\hat{z}_t, \hat{\psi}_t, \hat{M}_t - \hat{P}_t, \hat{k}_t$ , CIA-Model, CRRA Preferences, Calvo Staggering

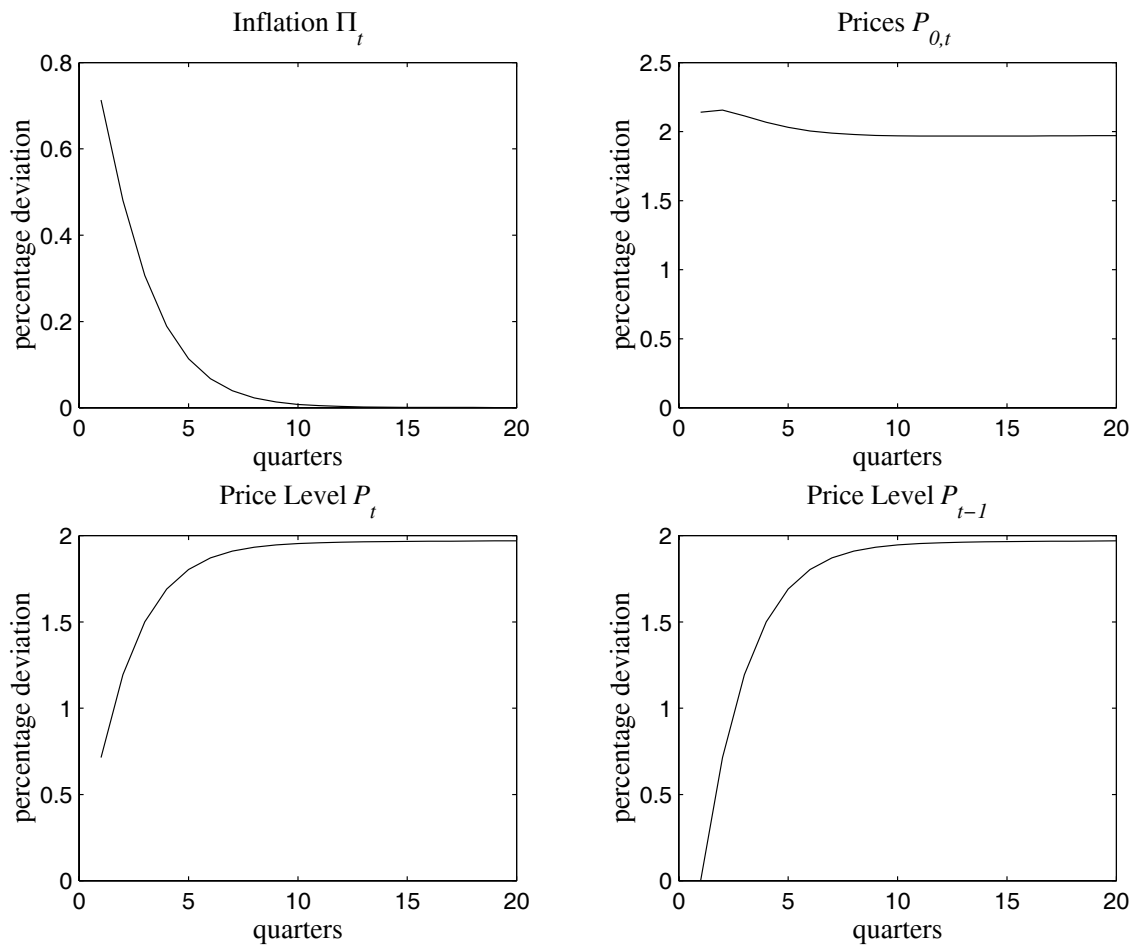


Figure 3.8: Impulse Response Functions for  $\hat{\Pi}_t, \hat{P}_{0,t}, \hat{P}_t, \hat{P}_{t-1}$ , CIA-Model, CRRA Preferences, Calvo Staggering



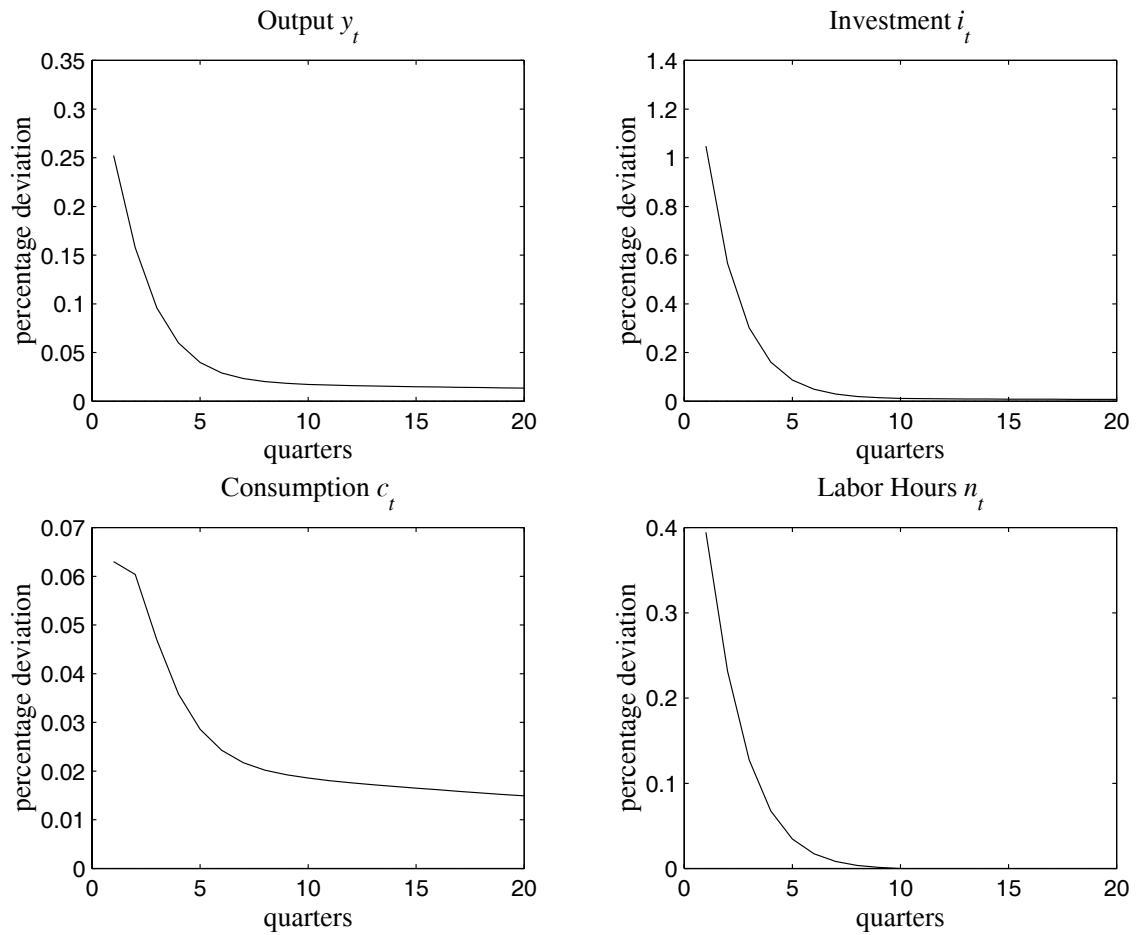


Figure 3.9: Impulse Response Functions for  $\hat{y}_t, \hat{i}_t, \hat{c}_t, \hat{n}_t$ , CIA-Model, CRRA Preferences, Calvo Staggering,  $\varphi = 0.5$

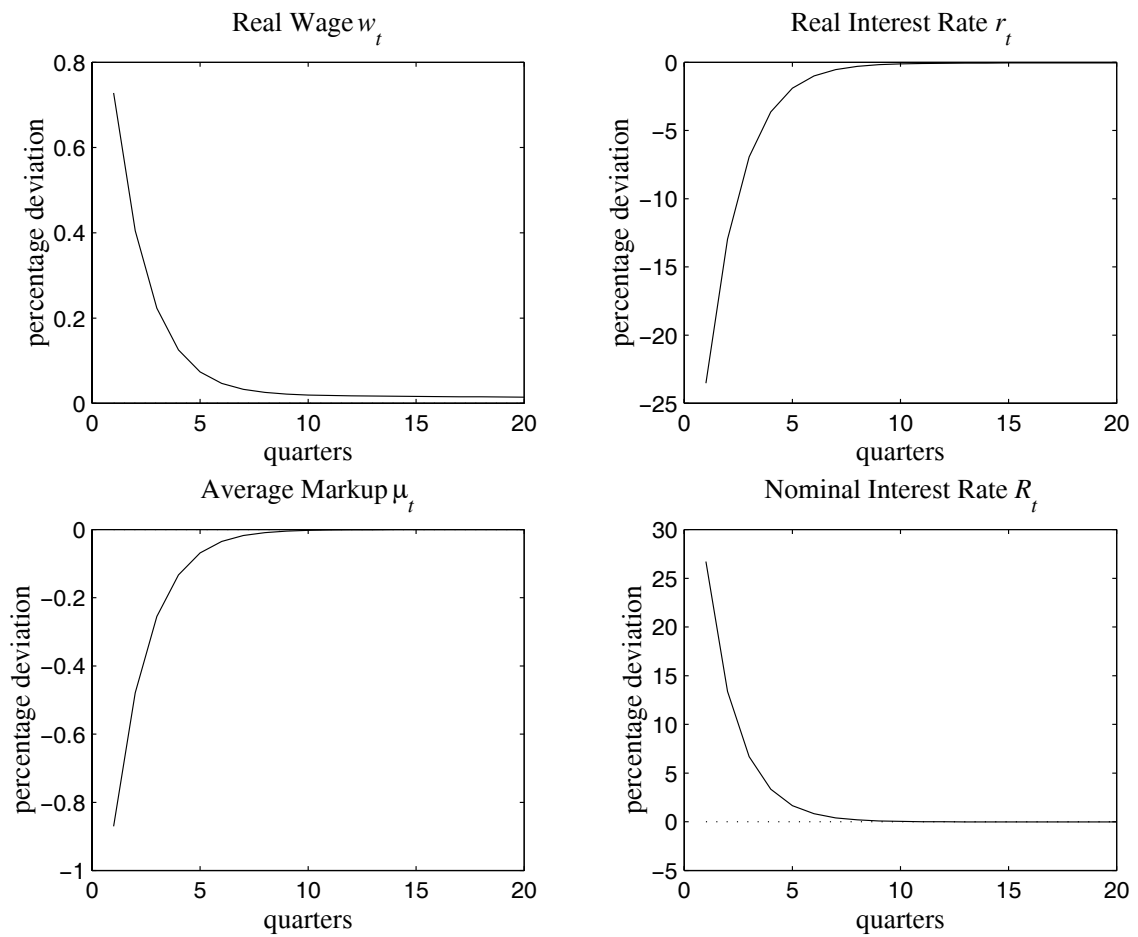


Figure 3.10: Impulse Response Functions for  $\hat{w}_t, \hat{r}_t, \hat{\mu}_t, \hat{R}_t$ , CIA-Model, CRRA Preferences, Calvo Staggering,  $\varphi = 0.5$

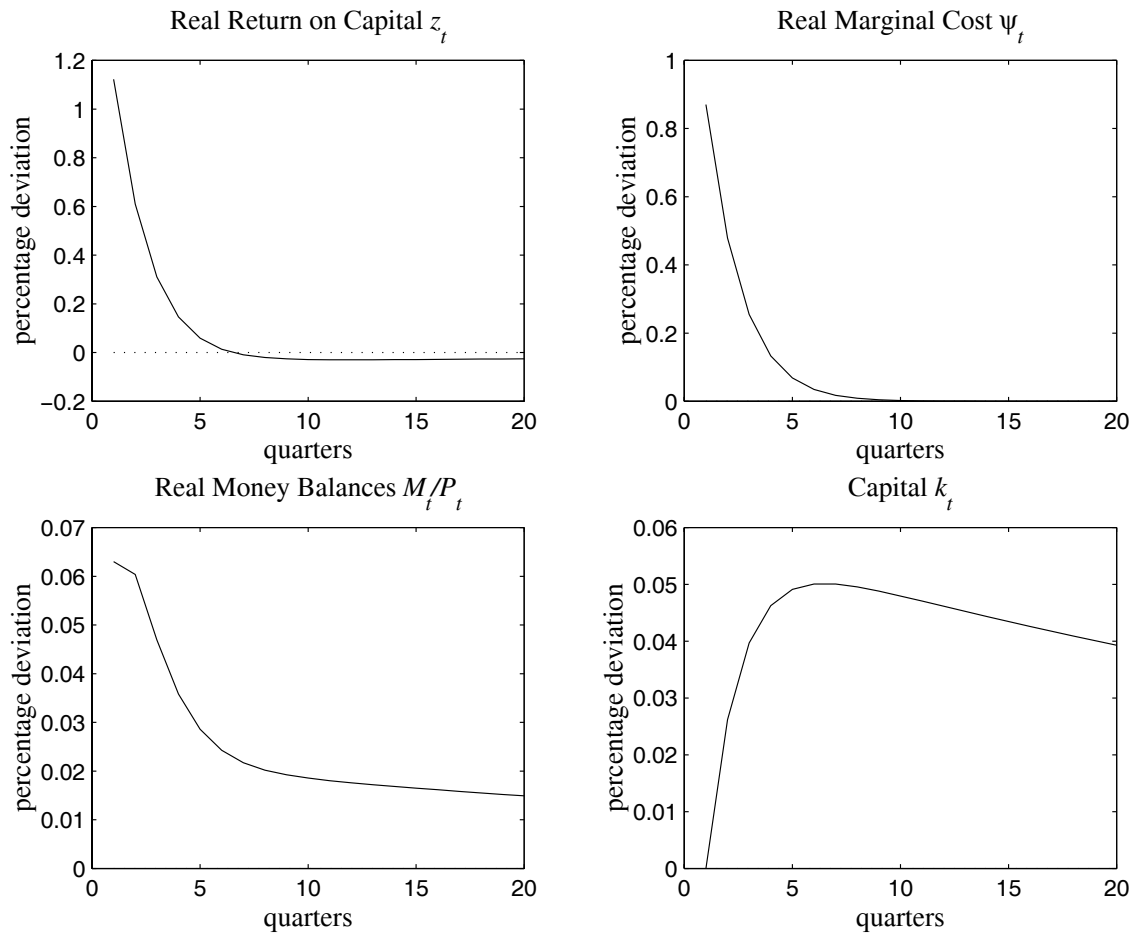


Figure 3.11: Impulse Response Functions for  $\hat{z}_t, \hat{\psi}_t, \widehat{M}_t - \widehat{P}_t, \hat{k}_t$ , CIA-Model, CRRA Preferences, Calvo Staggering,  $\varphi = 0.5$

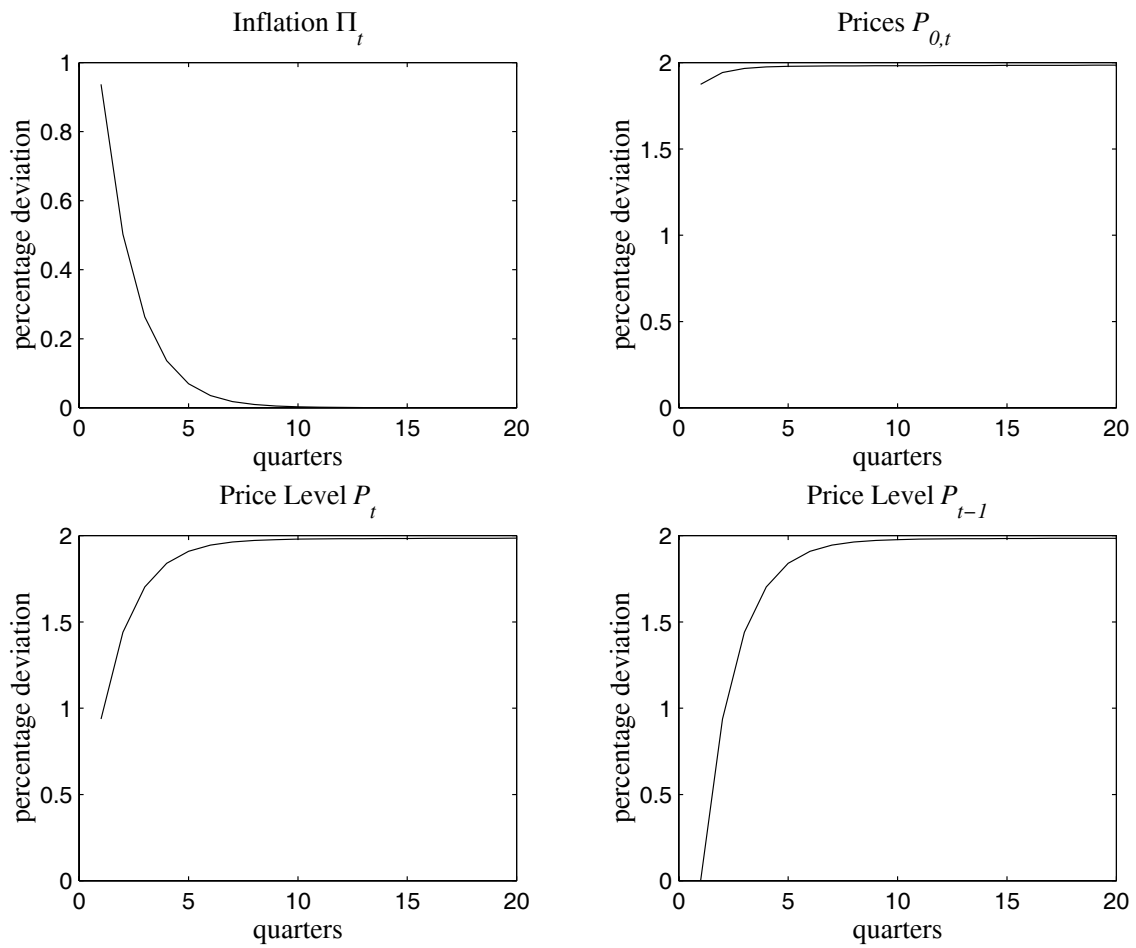


Figure 3.12: Impulse Response Functions for  $\widehat{\Pi}_t, \widehat{P}_{0,t}, \widehat{P}_t, \widehat{P}_{t-1}$ , CIA-Model, CRRA Preferences, Calvo Staggering,  $\varphi = 0.5$

# Chapter 4

## Habit Persistence and Price Staggering in a Monetary Stochastic Dynamic General Equilibrium Model with Labor and Capital

### 4.1 Introduction

In this chapter a MIU-model with Taylor price staggering is augmented by habit persistence in consumption. Several authors have shown that empirically there is an influence of last period's consumption on actual consumption. In addition habit persistence has already been successfully included in monetary stochastic dynamic general equilibrium models, for example in Christiano, Eichenbaum and Evans (2003). But their approach is too broad to allow for an analysis of the specific contribution of habits in consumption to create persistence in output after a money growth shock. This is the purpose of the present chapter. The main result is that only the behavior of consumption after a monetary policy shock can be improved upon. Output and inflation responses are very strong on impact and are cyclical thereafter. The business cycle properties do not match well empirical estimates.

Recently Bouakez, Cardia and Ruge-Murcia (2002) have estimated a similar model to the one presented here using US data. In addition to habit formation they consider the influence of adjustment costs to capital on the persistence of output after a money growth shock. They conclude that both features give rise to a hump-shaped response of output to a monetary shock. In light of the results obtained here the main reason for their success in matching empirical regularities must lie in the

different modeling of the price adjustment process: While these authors consider Calvo pricing I essentially assume Taylor price staggering. As shown in Chapter 3 it is of fundamental importance which price staggering scheme is used.

McCallum and Nelson (1999a) incorporate habit formation in an open economy model of nominal income targeting and find – contrary to the results obtained here – an important role for increasing the ability to match quarterly US data.

Auray, Collard and Fève (2002) consider habit formation in conjunction with a CIA-model to explain the liquidity effect. They show that high enough habit persistence can generate a falling nominal interest rate after a positive money growth shock but that it leads also to real indeterminacy. In the model at hand the nominal rate rises. The difference may be due to the fact that these authors do not incorporate sticky prices. In a related paper Auray, Collard and Fève (2004) show that in a MIU-model there is always determinacy of the equilibrium.

The chapter is organized as follows: Section 4.2 describes in detail the model, the steady state and the calibration. In Section 4.3 impulse responses are discussed while Section 4.4 gives results for the business cycle properties of the model. Section 4.5 concludes and gives some suggestions for future research.

## 4.2 The Model

### 4.2.1 The Household

The representative household is assumed to have preferences over consumption ( $c_t$ ), leisure ( $1 - n_t$ ) (where  $n_t$  is labor) and real money balances  $M_t/P_t$  since they facilitate transactions. Here I use the simplest specification in a separable form – an additively separable CRRA function – since the more complicated nonseparable variant does not enhance much - if at all - the persistency effects of money growth shocks in standard sticky price models.

$$u \left( c_t, c_{t-1}, \frac{M_t}{P_t}, n_t \right) = \frac{1}{1 - \sigma} \left[ \left( \frac{c_t}{c_{t-1}^b} \right)^{1-\sigma} + \gamma (1 - n_t)^{1-\sigma} + \left( \frac{M_t}{P_t} \right)^{1-\sigma} \right] \quad (4.1)$$

As usual  $\sigma$  governs the degree of risk aversion.  $\gamma$  is a positive parameter while  $b$  is a measure for the degree of habit persistence. Lagged consumption  $c_{t-1}$  is the habit reference level while  $b$  indexes the importance of this reference level relative to current consumption. With  $b = 0$  the standard model with actual consumption  $c_t$  only results, but with  $b = 1$  only consumption relative to previous consumption

matters. This can be seen more clearly when rewriting the consumption term as

$$\left(\frac{c_t}{c_{t-1}^b}\right) = \left(\frac{c_t}{c_{t-1}}c_{t-1}^{1-b}\right) \quad (4.2)$$

Now with  $b = 1$  the second term with lagged consumption has no influence any more so that the *level* of  $c_{t-1}$  does not matter.  $b$  cannot exceed 1 because otherwise steady state utility would be falling in consumption.<sup>1</sup>

This formulation of habit persistence neglects the possibility of memory in the habit reference level. Fuhrer (2000) considers the more general case introducing a new variable  $S_t$  for the reference level replacing  $c_{t-1}$  in (4.1). He assumes then that  $S_t$  evolves according to

$$S_t = \rho S_{t-1} + (1 - \rho) c_{t-1} \quad (4.3)$$

With  $\rho = 0$  only last period's consumption matters while for higher  $\rho$  past period's consumption levels become more and more important. Using this formulation leads to a very complex Euler equation which will not be used in this chapter (see e.g. Fuhrer (2000), p. 371).

Some authors (e.g. Christiano, Eichenbaum and Evans (2003)) consider the difference in consumption levels in the utility function, not the ratio. So the term corresponding to (4.2) looks like

$$c_t - h c_{t-1} \quad (4.4)$$

Deaton (1992) shows that this is a special case of the Fuhrer (2000) formulation where  $h$  captures both the influence of  $b$  and  $\rho$ . It is the result when setting  $\rho = 1$  so that there is no 'depreciation' of the habit reference level. In the model considered here persistence in habits does not have a great influence on the dynamics so it will not be used.<sup>2</sup>

The budget constraint is given by

$$\begin{aligned} & P_t c_t + P_t i_t + M_t + B_t \\ = & P_t w_t n_t + P_t z_t k_{t-1} + M_{t-1} + (1 + R_{t-1}) B_{t-1} + \Xi_t + M_t^s \end{aligned} \quad (4.5)$$

where

$$\Xi_t = \int_0^1 \Xi_{j,t} dj \quad (4.6)$$

are the nominal profits of the intermediate goods producing firms. The household can invest  $i_t$  units of the final good to augment the capital stock  $k_t$ . Further it can

<sup>1</sup>McCallum and Nelson (1999a) also use this formulation for modeling habit persistence.

<sup>2</sup>See Deaton (1992), p. 30.

decide how much to consume ( $c_t$ ) and how much real money balances  $M_t/P_t$  and real bonds  $B_t/P_t$  to hold. The household has a labor income  $w_t n_t$  working in the market at the real wage rate  $w_t$  and can spend its money holdings carried over from the previous period ( $M_{t-1}/P_t$ ). It also receives factor payments  $z_t k_{t-1}$  for supplying capital to intermediate goods producing firms where  $z_t$  denotes the real return on capital. There are also previous period bond holdings including the interest on them  $(1 + R_{t-1})(B_{t-1}/P_t)$ . Finally, the household receives a monetary transfer  $M_t^s$  from the monetary authority and the profits from the intermediate goods firms  $\Xi_t$ , respectively. This transfer is equal to the change in money balances, i.e.

$$M_t^s = M_t - M_{t-1} \quad (4.7)$$

The capital stock increases according to the following law of motion:

$$k_t = (1 - \delta) k_{t-1} + \phi \left( \frac{i_t}{k_{t-1}} \right) k_{t-1} \quad (4.8)$$

There are costs of adjusting the capital stock which are captured by the  $\phi$  function.  $\delta$  is the rate of depreciation. The detailed properties will be discussed in the calibration subsection.<sup>3</sup> Because this equation cannot be explicitly solved for  $i_t$  a second Lagrange multiplier ( $\theta_t$ ) has to be introduced into the optimization problem of the household. The Lagrangian is then given by:

$$\begin{aligned} L = & E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c_t, c_{t-1}, m_t, n_t) \right. \\ & + \sum_{t=0}^{\infty} \beta^t \lambda_t \left( z_t k_{t-1} + w_t n_t + m_{t-1} \frac{P_{t-1}}{P_t} + \frac{\Xi_t}{P_t} + m_t^s \right. \\ & \left. \left. + (1 + R_{t-1}) b_{t-1} \frac{P_{t-1}}{P_t} - c_t - i_t - m_t - b_t \right) \right. \\ & \left. + \sum_{t=0}^{\infty} \beta^t \theta_t \left( (1 - \delta) k_{t-1} + \phi \left( \frac{i_t}{k_{t-1}} \right) k_{t-1} - k_t \right) \right] \quad (4.9) \end{aligned}$$

Here small variables indicate real quantities, i.e. for example  $m_t = M_t/P_t$ . Households optimize over  $c_t, n_t, i_t, k_t, m_t$  and  $b_t$  taking prices and the initial values of the price level  $P_0$  and the capital stock  $k_0$  as well as the outstanding stocks of money  $M_0$  and bonds  $B_0$  as given. The first order conditions are reported below.

$$\frac{\partial L}{\partial c_t} = \beta^t \frac{\partial u(c_t, c_{t-1}, m_t, n_t)}{\partial c_t} + \beta^{t+1} \frac{\partial u(c_{t+1}, c_t, m_{t+1}, n_{t+1})}{\partial c_t} - \beta^t \lambda_t = 0 \quad (4.10)$$

---

<sup>3</sup>Bouakez, Cardia and Ruge-Murcia (2002), p.4, assume a quadratic and strictly convex adjustment cost function.



$$\frac{\partial L}{\partial n_t} = \beta^t \frac{\partial u(c_t, c_{t-1}, m_t, n_t)}{\partial n_t} + \beta^t \lambda_t w_t = 0 \quad (4.11)$$

$$\frac{\partial L}{\partial i_t} = -\beta^t \lambda_t + \beta^t \theta_t \phi' \left( \frac{i_t}{k_{t-1}} \right) \left( \frac{1}{k_{t-1}} \right) k_{t-1} = 0 \quad (4.12)$$

$$\begin{aligned} \frac{\partial L}{\partial k_t} = & E_t \beta^{t+1} \lambda_{t+1} z_{t+1} - \beta^t \theta_t + E_t \beta^{t+1} \theta_{t+1} \left[ (1 - \delta) \right. \\ & \left. + \phi \left( \frac{i_{t+1}}{k_t} \right) + \phi' \left( \frac{i_{t+1}}{k_t} \right) \left( -\frac{i_{t+1}}{k_t^2} \right) k_t \right] = 0 \end{aligned} \quad (4.13)$$

$$\frac{\partial L}{\partial m_t} = \beta^t \frac{\partial u(c_t, c_{t-1}, m_t, n_t)}{\partial m_t} - \beta^t \lambda_t + E_t \beta^{t+1} \lambda_{t+1} \frac{P_t}{P_{t+1}} = 0 \quad (4.14)$$

$$\frac{\partial L}{\partial b_t} = -\beta^t \lambda_t + E_t \beta^{t+1} \lambda_{t+1} (1 + R_t) \frac{P_t}{P_{t+1}} = 0 \quad (4.15)$$

The derivative with respect to  $\lambda_t$  is omitted since it is equal to the intertemporal budget constraint. The derivative with respect to  $\theta_t$  is not reported again since it is given by the capital accumulation condition stated above.  $\phi'$  denotes the derivative of the  $\phi$ -function with respect to the investment to capital ratio which is regarded as one argument. Note the different consumption Euler equation. Due to habit formation the marginal utility of consumption enters two times indicating the influence of last period's consumption on today's utility. In addition the household's optimal choices must also satisfy the transversality conditions:

$$\lim_{t \rightarrow \infty} \beta^t \lambda_t x_t = 0 \quad \text{for } x = m, b, k \quad (4.16)$$

The familiar result that the first two efficiency conditions imply the equality of the marginal rate of substitution between consumption and labor and the real wage is altered here through the influence of habit formation in consumption. The real wage is now given by

$$w_t = - \frac{\frac{\partial u(c_t, c_{t-1}, m_t, n_t)}{\partial n_t}}{\frac{\partial u(c_t, c_{t-1}, m_t, n_t)}{\partial c_t} + \beta \frac{\partial u(c_{t+1}, c_t, m_{t+1}, n_{t+1})}{\partial c_t}} \quad (4.17)$$

Note that the marginal utility of consumption enters twice in the denominator which alters the dynamic evolution of  $w_t$ .

The efficiency condition for bond holdings establishes a relation between the nominal interest rate and the price level. Rearranging terms yields

$$(1 + R_t) = \frac{\lambda_t}{\lambda_{t+1}} \frac{1}{\beta} \frac{P_{t+1}}{P_t} \quad (4.18)$$

Supposed the Fisher equation is valid the real interest rate  $r_t$  is implicitly defined as

$$(1 + r_t) = \frac{\lambda_t}{\lambda_{t+1}} \frac{1}{\beta} \quad (4.19)$$

because  $P_{t+1}/P_t$  equals one plus the rate of expected inflation which is approximated by the ex-post-inflation rate.

In the efficiency condition for money the marginal utility of real balances has to be considered. This derivative determines the endogenous money demand function. Combining the optimum conditions for consumption, bonds and money yields the following equation:

$$\frac{\partial u(c_t, c_{t-1}, m_t, n_t)}{\partial m_t} = \left[ \frac{\partial u(c_t, c_{t-1}, m_t, n_t)}{\partial c_t} + \beta \frac{\partial u(c_{t+1}, c_t, m_{t+1}, n_{t+1})}{\partial c_t} \right] \frac{R_t}{1 + R_t} \quad (4.20)$$

In principal this specification allows to estimate an empirical money demand function. But this approach will not be pursued here since the dynamic structure involves consumption at three different points in time, a specification normally not considered to be appropriate for the estimation of an empirical money demand function. In addition the utility function (4.1) has been chosen such that there is no need for an estimation of parameters such as  $\nu$  and  $\eta$  in (2.4).

### 4.2.2 The Finished Goods Producing Firm

The firm producing the final good  $y_t$  in the economy uses  $y_{j,t}$  units of each intermediate good  $j \in [0, 1]$  purchased at price  $P_{j,t}$  to produce  $y_t$  units of the finished good. The production function is assumed to be a CES aggregator as in Dixit and Stiglitz (1977) with  $\epsilon > 1$ .

$$y_t = \left( \int_0^1 y_{j,t}^{(\epsilon-1)/\epsilon} dj \right)^{\epsilon/(\epsilon-1)} \quad (4.21)$$

The firm maximizes its profits over  $y_{j,t}$  given the above production function and given the price  $P_t$ . So the problem can be written as

$$\max_{y_{j,t}} \left[ P_t y_t - \int_0^1 P_{j,t} y_{j,t} dj \right] \text{ s.t. } y_t = \left( \int_0^1 y_{j,t}^{(\epsilon-1)/\epsilon} dj \right)^{\epsilon/(\epsilon-1)} \quad (4.22)$$

The first order conditions for each good  $j$  imply

$$y_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{-\epsilon} y_t \quad (4.23)$$

where  $-\epsilon$  measures the constant price elasticity of demand for each good  $j$ . Since the firm operates under perfect competition it does not make any profits. Inserting

the demand function into the profit function and imposing the zero profit condition reveals that the only price  $P_t$  that is consistent with this requirement is given by

$$P_t = \left( \int_0^1 P_{j,t}^{(1-\epsilon)} dj \right)^{1/(1-\epsilon)} \quad (4.24)$$

### 4.2.3 The Intermediate Goods Producing Firm

Intermediate good firms can be considered to consist of a producing and a pricing unit. The producing unit operates under a Cobb-Douglas-technology which is subject to an aggregate random productivity shock  $a_t$ .

$$y_{j,t} = a_t n_{j,t}^\alpha k_{j,t-1}^{1-\alpha} \quad (4.25)$$

Here  $n_{j,t}$  is the labor input employed in period  $t$  by a firm who set the price in period  $t - j$ , similarly  $k_{j,t-1}$  is the capital stock, and  $0 < \alpha < 1$  is labor's share.

The producing unit chooses labor and capital to minimize costs. In models with capital the problem is given by

$$\begin{aligned} \min_{n_{j,t}, k_{j,t-1}} & [P_{j,t} w_{j,t} n_{j,t} + P_{j,t} z_{j,t} k_{j,t-1}] \\ \text{s.t.} & y_{j,t} = a_t n_{j,t}^\alpha k_{j,t-1}^{1-\alpha} \end{aligned} \quad (4.26)$$

It is useful for further calculations to define nominal marginal cost as  $\Psi_{j,t}$  which is equal to the Lagrange multiplier in the cost minimization problem stated above. The efficiency conditions are the following:

$$P_{j,t} w_{j,t} = \Psi_{j,t} \alpha a_t n_{j,t}^{\alpha-1} k_{j,t-1}^{1-\alpha} \quad (4.27)$$

$$P_{j,t} z_{j,t} = \Psi_{j,t} (1 - \alpha) a_t n_{j,t}^\alpha k_{j,t-1}^{-\alpha} \quad (4.28)$$

In a symmetric equilibrium all choices of the producing unit of the firms are the same so that

$$P_{j,t} = P_t, w_{j,t} = w_t, z_{j,t} = z_t, \Psi_{j,t} = \Psi_t, n_{j,t} = n_t, k_{j,t-1} = k_{t-1} \text{ for all } t \quad (4.29)$$

So (4.27) and (4.28) hold with all  $j$ 's eliminated.

The pricing unit sets prices to maximize the present discounted value of profits. Those firms who do not adjust their prices in a given period can be interpreted as passive while those who do adjust do so optimally. Define the relative price by  $p_{j,t} = P_{j,t}/P_t$ . Because the production functions are homogenous of degree one real profit  $\xi_{j,t} = \Xi_{j,t}/P_t$  for a firm of type  $j$  is equal to

$$\xi_{j,t} = \xi(p_{j,t}, y_t, \psi_t) = p_{j,t} y_{j,t} - \psi_t y_{j,t} \quad (4.30)$$

where  $\psi_t = \Psi_t/P_t$  is real marginal cost. Using the demand function for the intermediate goods ( $y_{j,t} = p_{j,t}^{-\epsilon} y_t$ ) the profit function can be rewritten as

$$\xi_{j,t} = \xi(p_{j,t}, y_t, \psi_t) = y_{j,t}(p_{j,t} - \psi_t) = p_{j,t}^{-\epsilon} y_t (p_{j,t} - \psi_t) \quad (4.31)$$

When prices are fixed for two periods the firm has to take into account the effect of the price chosen in period  $t$  on current and future profits. The price in period  $t+1$  will be affected by the gross inflation rate  $\Pi_{t+1}$  between  $t$  and  $t+1$  ( $\Pi_{t+1} = P_{t+1}/P_t$ ).

$$p_{1,t+1} = \frac{p_{0,t}}{\Pi_{t+1}} \quad (4.32)$$

The optimal relative price has to balance the effects due to inflation between profits today and tomorrow. This intertemporal maximization problem is formally given by

$$\begin{aligned} \max_{p_{0,t}} E_t \left[ \xi(p_{0,t}, y_t, \psi_t) + \beta \frac{\lambda_{t+1}}{\lambda_t} \xi(p_{1,t+1}, y_{t+1}, \psi_{t+1}) \right] \\ \text{s.t. } p_{1,t+1} = \frac{p_{0,t}}{\Pi_{t+1}} \end{aligned} \quad (4.33)$$

The term  $\beta\lambda_{t+1}/\lambda_t$  is again the appropriate discount factor for real profits. Solving the efficiency condition for the optimal price to be set in period  $t$  using (4.31) yields a forward-looking form of the price equation and is in that respect similar to the one in Taylor (1980).

$$P_{0,t} = \frac{\epsilon}{\epsilon - 1} \frac{\lambda_t P_t^\epsilon y_t \psi_t + \beta E_t \lambda_{t+1} P_{t+1}^\epsilon y_{t+1} \psi_{t+1}}{\lambda_t P_t^{\epsilon-1} y_t + \beta E_t \lambda_{t+1} P_{t+1}^{\epsilon-1} y_{t+1}} \quad (4.34)$$

The optimal price  $P_{0,t}$  depends upon the current and future real marginal costs, current and future price levels and output as well as today's and tomorrow's interest rates captured by the  $\lambda$ 's.

With prices set for two periods half of the firms adjust their price in period  $t$  and half do not. Moreover all adjusting firms choose the same price. Then  $P_{j,t}$  is the nominal price at time  $t$  of any good whose price was set  $j$  periods ago and  $P_t$  is the price index (4.24) at time  $t$  and is given by

$$P_t = \left( \frac{1}{2} P_{0,t}^{1-\epsilon} + \frac{1}{2} P_{1,t}^{1-\epsilon} \right)^{1/(1-\epsilon)} \quad (4.35)$$

Remember that Bouakez, Cardia and Ruge-Murcia (2002) essentially use a New Keynesian Phillips curve as a Taylor approximation since they consider Calvo pricing similar to the exercise in Chapter 3.

#### 4.2.4 Market Clearing Conditions and Other Equations

The aggregate resource constraint is derived using the resource constraint of households, firms, the government and the monetary authority. Since there are neither government expenditures nor taxes in this model, this condition is given by

$$y_t = c_t + i_t \quad (4.36)$$

It is well known that models like the one at hand imply multiple equilibria and sunspots because bonds are not determined. To escape this problem the household budget constraint is dropped and bonds are set to zero:  $b_t = 0$  for all  $t$ .

The markup  $\mu_t$  is just the reciprocal of real marginal cost so that

$$\mu_t = \frac{1}{\psi_t} \quad (4.37)$$

#### 4.2.5 The Monetary Authority

The model is closed by adding a monetary policy rule. Therefore an exogenous process for the money growth rate is considered. To achieve persistent but non permanent effects the level of money again follows an AR(2)-process. Assume that money grows at a factor  $g_t$ :

$$M_t = g_t M_{t-1} \quad (4.38)$$

If  $\hat{g}_t$  follows an AR(1)-process  $\hat{g}_t = \rho_g \hat{g}_{t-1} + \epsilon_{g_t}$  then money will follow an AR(2)-process. As before  $\rho_g$  lies between 0 and 1 and  $\epsilon_{g_t}$  is white noise. Remember that inflation is zero at the steady state so also money growth is zero there ( $g = 1$ ).

The productivity and taste shock  $a_t$  follows an AR(1)-process

$$\hat{a}_t = \rho_a \hat{a}_{t-1} + \epsilon_{a_t} \quad (4.39)$$

with  $\epsilon_{a_t}$  white noise and  $0 < \rho_a < 1$ .

#### 4.2.6 The Steady State

Imposing the condition of constancy of the price level in the steady state ( $P_t = P_{t-1} = P$ ) on the nominal interest rate equation reveals the familiar condition from RBC models that  $\beta = 1/(1 + R)$ . In addition, as there is no steady state inflation,  $R = r$ . The two period price setting of the firms implies  $P_0 = P_1$ . Using this in the price index reveals that  $P_0 = P_1 = P$ . The capital accumulation equation tells us that  $\phi(i/k) = \delta$  at the steady state. It is assumed that  $\phi' = 1$  in the steady state to ensure that Tobin's  $q$  is equal to one ( $q = 1/\phi'$ ). As a consequence of

the requirement that the model with adjustment costs of capital should display the same steady state as the model without them  $i/k$  is equal to  $\phi(i/k)$ . Using this in the efficiency condition for capital it can be shown that the rental rate on capital is  $z = r + \delta$  as in a standard RBC model. With the help of (4.27) and the steady state for  $z$  it is possible to pin down  $k/n$  which amounts to

$$\frac{k}{n} = \left( \frac{r + \delta}{a} \frac{1}{1 - \alpha \psi} \right)^{-1/\alpha} \quad (4.40)$$

For the markup  $\mu$  it follows  $\mu = 1/\psi$  while  $\psi$  is determined by the steady state of the efficiency condition for maximizing profits, (4.34). This amounts to  $\psi = (\epsilon - 1)/\epsilon$ . This can be used to calculate  $w$  as well:

$$w = \psi a \alpha \left( \frac{k}{n} \right)^{1-\alpha} \quad (4.41)$$

The calculation of the steady state value of consumption is tedious because it takes quite a lot of steps. From the production function one knows that labor productivity is given by

$$\frac{y}{n} = a \left( \frac{k}{n} \right)^{1-\alpha} \quad (4.42)$$

This productivity can be combined with the investment to capital ratio to calculate the investment share:

$$\frac{i}{y} = \left( \frac{i}{k} \frac{k}{n} \right) / \left( \frac{y}{n} \right) \quad (4.43)$$

Now one can derive the consumption share using the aggregate resource constraint.

$$\frac{c}{y} = -\frac{i}{y} + 1 \quad (4.44)$$

To get the level of  $c$  the level of  $y$  and  $i$  have to be determined:  $y = n \cdot y/n$ ,  $i = y \cdot i/y$ . Finally  $c = y - i$  is the consumption steady state value.

The marginal rate of substitution (4.17) between consumption and labor can also be used to calculate the preference parameter  $\gamma$ .

$$\gamma = (1 - \beta b) c^{\sigma b - \sigma - b} w (1 - n)^\sigma \quad (4.45)$$

Using the efficiency condition for money  $m$  depends only upon  $\beta, b, c$  and  $\sigma$  and can be written as

$$m = (1 - \beta)^{-\frac{1}{\sigma}} (1 - \beta b)^{-\frac{1}{\sigma}} c^{\frac{\sigma + b - \sigma b}{b}} \quad (4.46)$$

### 4.2.7 Calibration

To compute impulse responses the parameters of the model have to be calibrated.

It is possible to either specify  $\beta$  or  $r$  exogenously. Here  $\beta$  will be set to 0.99 implying a value of  $r$  of about 0.0101 per quarter which is in line with other values used for the real interest rate in the literature.  $\psi$  and  $\mu$  can be determined by fixing a value for the elasticity of the demand functions for the differentiated products. This elasticity being equal to 4 causes the static markup  $\mu = \epsilon/(\epsilon - 1)$  to be 1.33 which is in line with the study of Linnemann (1999) about average markups. In order to determine the steady state real wage  $w$  the productivity shock  $a$  has to be specified, along with calculating  $k/n$ , see below. As there is no information available about that parameter it is arbitrarily set at 10.  $n$  is specified to be equal to 0.25 implying that agents work 25 % of their non-sleeping time.

In the benchmark case,  $\sigma$ , the parameter governing the degree of risk aversion, is set to 2. The value of  $b$  which measures the degree of habit persistence is set to 0.8 as in McCallum and Nelson (1999a) in the benchmark case, implying a value for  $\gamma$  of 0.1483.

As this model considers the role of capital accumulation several other technological parameters have to be calibrated. The most common one is the depreciation rate  $\delta$  which is set to 0.025 implying 10% depreciation per year. Labor's share  $\alpha$  is 0.64 whereas the elasticity of Tobin's  $q$  with respect to  $i/k$  is set to -0.5.<sup>4</sup> This value is also used in King and Wolman (1996). The presence of adjustment costs of capital dampens the volatility of investment and is a common feature in equilibrium business cycle models. Using  $r, \delta, a, \alpha$  and  $\psi$  the ratio  $k/n$  can be determined.

For the exogenous money growth process  $\rho_g = 0.5$  is used. As the focus of the chapter is on the persistence effects of money growth shocks productivity shocks will not be considered. But they can be used to check whether the model displays reasonable impulse responses to technology shocks.

## 4.3 Impulse Response Functions

As before the solution is conducted using an extended version of the algorithm of King, Plosser and Rebelo (2002) which allows for singularities in the system matrix of the reduced model. The results will be again discussed using impulse responses.

Figure 4.1 shows the reaction of output, investment, consumption and labor

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<sup>4</sup>It can be shown that this elasticity is given by  $-\left[\frac{\phi''}{\phi'} \cdot (i/k)\right]$ .

hours to a one percent shock to the money growth rate. The immediate impression is the cyclical responses of  $\hat{y}_t$ ,  $\hat{n}_t$  and  $\hat{i}_t$ . They display almost no persistence at all. But consumption displays quite a persistent response although the magnitude is very small. Nevertheless the effects last for more than five periods. This is due to the habit formation in consumption. With the respective parameter  $b$  equal to 0.8 there is a sizeable influence of past period's consumption on today's utility so that households smooth their consumption expenditures. The contract multiplier for consumption is 0.42. Figure 4.2 mirrors the response of the real wage, the real interest rate, the markup and the nominal interest rate. Counterfactually the nominal rate rises so the model does not generate the liquidity effect. But this variable is quite persistent as opposed to the other three which are again cyclical. The strong rise in real marginal costs displayed in Figure 4.3 causes firms to raise prices very strongly. They overshoot their new equilibrium value considerably. This rise is stronger than the rise in money so real balances even fall and approach the steady state from below. The capital stock is hump-shaped but the magnitude of the increase is very small while nevertheless the effects are long lasting. Inflation does not show a hump but peaks in the first period, as shown in Figure 4.4.

Is there an intuition for this model result? Again it is helpful to examine the dynamics of the real wage. Using (4.17) and inserting the marginal utilities of labor and consumption allows to derive the following equation:<sup>5</sup>

$$w_t = \frac{\gamma (1 - n_t)^{-\sigma}}{c_t^{-\sigma} c_{t-1}^{b(\sigma-1)} - \beta b c_{t+1}^{1-\sigma} c_t^{b(\sigma-1)-1}} \quad (4.47)$$

Now a positive money growth shock again causes a rise in  $n$  since firms face higher demand and hire more workers. This leads to a rise in the numerator. The increase in consumption  $c_t$  leads to a decrease in the first term in the denominator whereas the second term decreases as long as  $b < 1$  for  $\sigma = 2$ . But this second term is subtracted so that the overall effect is not definite. In addition there is an influence of future consumption  $c_{t+1}$  which increases as can be seen in Figure 4.1. This leads to a further decrease of the second term.  $c_{t-1}$  enters the first term but is unchanged in period  $t$  thus having no effect here. Overall as long as  $b > 0$  the second term will dampen the decline of the numerator so that the rise of the real wage rate will be dampened as well. The impulse response of  $w_t$  reveals that the dampening effect is not very strong as the real wage deviates 1.25% from the steady state. As the money growth shock also leads to an increase in the demand for capital  $k$  the rental

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<sup>5</sup>It should be kept in mind that  $w_t$  is also influenced from the production side.



rate on capital  $z_t$  rises. This results in additional upward pressure on real marginal cost.  $\widehat{\psi}_t$ 's initial response is a 1.48% deviation from steady state.

There are two important special cases to be considered in (4.47). The first is  $\sigma = 1$  which implies log linear utility. This will eliminate  $c_{t+1}$  as well as  $c_{t-1}$  so that the real wage rate will be solely determined by current consumption.

$$w_t = \frac{\gamma(1 - n_t)^{-1}}{c_t^{-1} - \beta b c_t^{-1}} \quad (4.48)$$

The impulse responses are presented in Figures 4.5 – 4.8. The exercise has little effect on the wage rate (1.27% deviation) but a dramatic effect on consumption that is cyclical again as output, investment and labor. The reason for this is that with  $\sigma = 1$  consumption from the previous period  $c_{t-1}$  drops out of the dynamic system. This essentially eliminates consumption habits and thus persistence from the model. Because the rental rate  $\widehat{z}_t$  rises stronger now real marginal cost also show an increased reaction of 1.79% deviation. The second limiting case is  $b = 0$  which would eliminate habit persistence from the model.

$$w_t = \frac{\gamma(1 - n_t)^{-1}}{c_t^{-1}} = \frac{\gamma c_t}{(1 - n_t)} \quad (4.49)$$

This condition is familiar from Chapter 2, see (2.62). Figures 4.9 – 4.12 display the results. Now the real wage response is strongest (1.41%) because the rise in labor and in consumption can exert fully their influence as in the MIU-model of the labor only economy. In (4.48) the factor  $1 - \beta b = 0.208$  dampens consumption's rise on  $w_t$ . Comparing Figures 4.5 and 4.9 reveals a stronger reaction of output, labor and consumption without habit persistence than with log linear utility. So the characteristic of habit persistence to dampen consumption's reaction is overturned for log linear utility.

It can be concluded that habit persistence improves only the response of consumption to a money growth shock in a model with Taylor price staggering. Assigning  $b$  the highest possible value of 1 so that only the ratio of current to past consumption matters (see the discussion of the utility function above) allows consumption to be hump-shaped reaching a peak 12 periods after the shock (see Figure 4.13). The contract multiplier is now very high: 0.96. But note the very small value of the reaction:  $\widehat{c}_t$  deviates only about 0.01 percent from steady state due to a 1 percent shock to money growth. In the next section the model is evaluated along this dimension.

## 4.4 Business Cycle Properties

In order to explore the implications for the business cycle properties one has to specify the standard deviation of the AR(1)-process for money growth. Here the value estimated in Cooley and Hansen (1995), p.201, is used.<sup>6</sup> It implies a value of 0.0000792 for the variance  $\sigma_g^2$ . Table 4.1 shows the results for the benchmark model with  $b = 0.8$  after HP-filtering with  $\lambda = 1600$ .<sup>7</sup>

Table 4.1: Moments in the Benchmark Habit Persistence Model

$\hat{x}_t$	$\sigma_{\hat{x}}$	$\sigma_{\hat{x}}/\sigma_{\hat{y}}$	autocorrelation		cross correlation of $\hat{x}_t$ with $\hat{y}$ in				
			1	2	$t-2$	$t-1$	$t$	$t+1$	$t+2$
$\hat{y}_t$	0.34	1.00	-0.14	-0.02	-0.02	-0.14	1.00	-0.14	-0.02
$\hat{i}_t$	1.42	4.14	-0.23	-0.04	-0.06	-0.25	0.99	-0.12	-0.01
$\hat{c}_t$	0.10	0.29	0.28	0.06	0.10	0.26	0.90	-0.18	-0.08
$\hat{n}_t$	0.54	1.59	-0.15	-0.03	-0.04	-0.16	0.99	-0.13	-0.01
$\hat{w}_t$	1.09	3.19	-0.20	-0.04	-0.05	-0.22	0.99	-0.13	-0.01
$\hat{\mu}_t$	1.29	3.78	-0.20	-0.04	0.05	0.21	-0.99	0.12	0.01
$\hat{R}_t$	1.30	3.79	0.60	0.12	0.17	0.62	0.62	-0.13	-0.07
$\hat{\psi}_t$	1.29	3.78	-0.20	-0.04	-0.05	-0.21	0.99	-0.12	-0.01
$\hat{\Pi}_t$	1.26	3.67	0.35	-0.20	-0.21	0.61	0.69	-0.11	-0.04
$\hat{P}_t$	2.17	6.33	0.83	0.54	0.31	0.43	0.07	-0.32	-0.26

$\sigma_{\hat{x}}$  again denotes the percentage standard deviation of  $\hat{x}$  whereas  $\sigma_{\hat{x}}/\sigma_{\hat{y}}$  measures the respective standard deviation relative to that of output  $\hat{y}$ . The next two columns report the autocorrelations for one and two lags of the respective aggregate. The remaining columns display the cross correlations with output. A variable  $\hat{x}$  is leading  $\hat{y}$  if the absolute value of the correlation  $\rho(\hat{x}_t, \hat{y}_{t+i})$  is highest for  $i > 0$ . Accordingly a variable  $\hat{x}$  is lagging  $\hat{y}$  if the absolute value of the correlation  $\rho(\hat{x}_t, \hat{y}_{t+i})$  has a maximum for  $i < 0$ . In case that this correlation is positive one speaks of a procyclical variable while it is called anticyclical if it is negative. If the maximum correlation occurs at lag 0 ( $i = 0$ ) the variable is moving with the cycle. This table strenghtens the insights from the impulse response functions. First, the cyclical character of most variables is displayed in their negative autocorrelations, see e.g.

<sup>6</sup>It is not intended to take the model explicitly to the data because of its overwhelming simplicity. This justifies the use of Cooley and Hansen's parameter values.

<sup>7</sup>Remember that all values in the tables have been rounded using the computer output. So it is possible that the relative standard deviations deliver a different value when using the values in the table.

output and investment. Second, investment, labor, the real wage and real marginal cost are nearly perfectly correlated with output whereas the correlations at leads and lags are negative. Third, the relative variability of consumption (0.29) is very low while also most absolute volatilities of the real variables are too small compared to empirical estimates. This applies especially to output and investment giving support to the claim that money growth shocks cannot account for the observed volatilities of real aggregates. The opposite is true for nominal variables such as the inflation rate which is by far too volatile. The same result concerns the price level. Fourth, only consumption and the nominal interest rate show a small portion of persistence since their autocorrelations are positive and well above 0.25 at the first lag.

In the limiting case with  $b = 1$  the relative variability of consumption falls to 0.04 while the absolute value is only 0.01% (see also Figure 4.13). But the autocorrelations rise to 0.75 and 0.57 respectively. On the other hand investment is now 5.40 times as volatile as output which is by far too high. Labor's relative variability does not change. Finally considering  $b = 0$  worsens the performance of the model even more. Of course now consumption shows more variation, its relative volatility rises to 0.65. But the autocorrelations as well as the lead/lag correlations get negative while the contemporaneous correlation with output is perfect.

## 4.5 Conclusions

Adding habit persistence in consumption to a monetary stochastic dynamic general equilibrium model with Taylor price staggering does not enhance very much the ability to account for persistent effects of money growth shocks. It is only the behavior of consumption that can be improved.

This stands in contrast to results in Bouakez, Cardia and Ruge-Murcia (2002) who consider a similar model. But these authors use a different way to model adjustment costs of capital. The most important difference is that they assume Calvo pricing. As shown in Chapter 3 the latter feature is responsible for the difference.

The model presented here can also be extended to include wage staggering as another nominal rigidity. It would be particularly interesting to investigate the interaction with sticky prices to create inflation and output persistence. In addition the inclusion of variable capital utilization could further enhance persistence, as suggested by Christiano, Eichenbaum and Evans (2003).

The analysis of wage staggering is of particular interest since Woodford has

recently shown that ‘allowing for wage stickiness does not matter all that much, if the goal is simply to construct a positive model of the co-movement of inflation and output, and the way that both can be affected by monetary policy’.<sup>8</sup> This gives a justification to neglect wage staggering in positive stochastic dynamic general equilibrium models and casts some doubt on the role some authors give to sticky wages. The issue will be taken up in the next chapter.

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<sup>8</sup>See Woodford (2003*a*), p. 235.

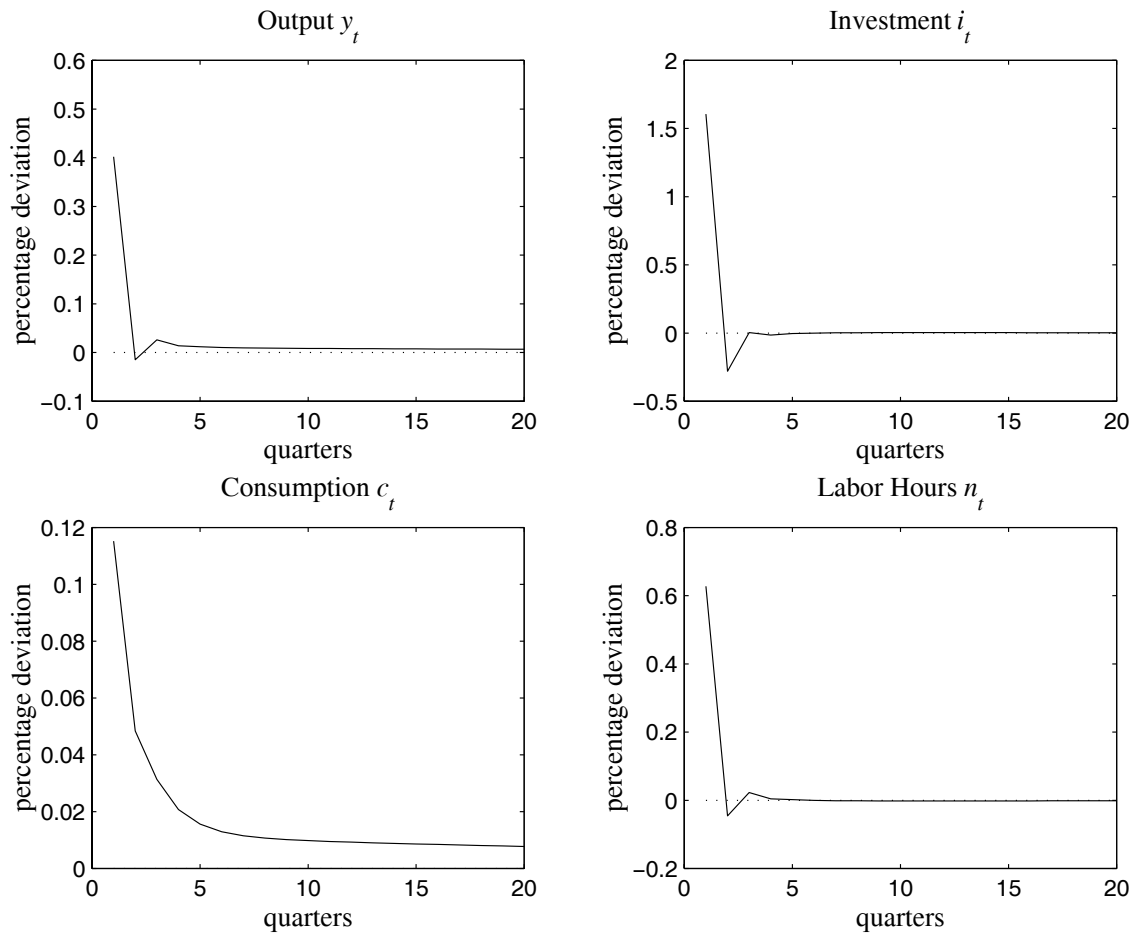


Figure 4.1: Impulse Response Functions for  $\hat{y}_t, \hat{i}_t, \hat{c}_t, \hat{n}_t$ ,  $b = 0.8$

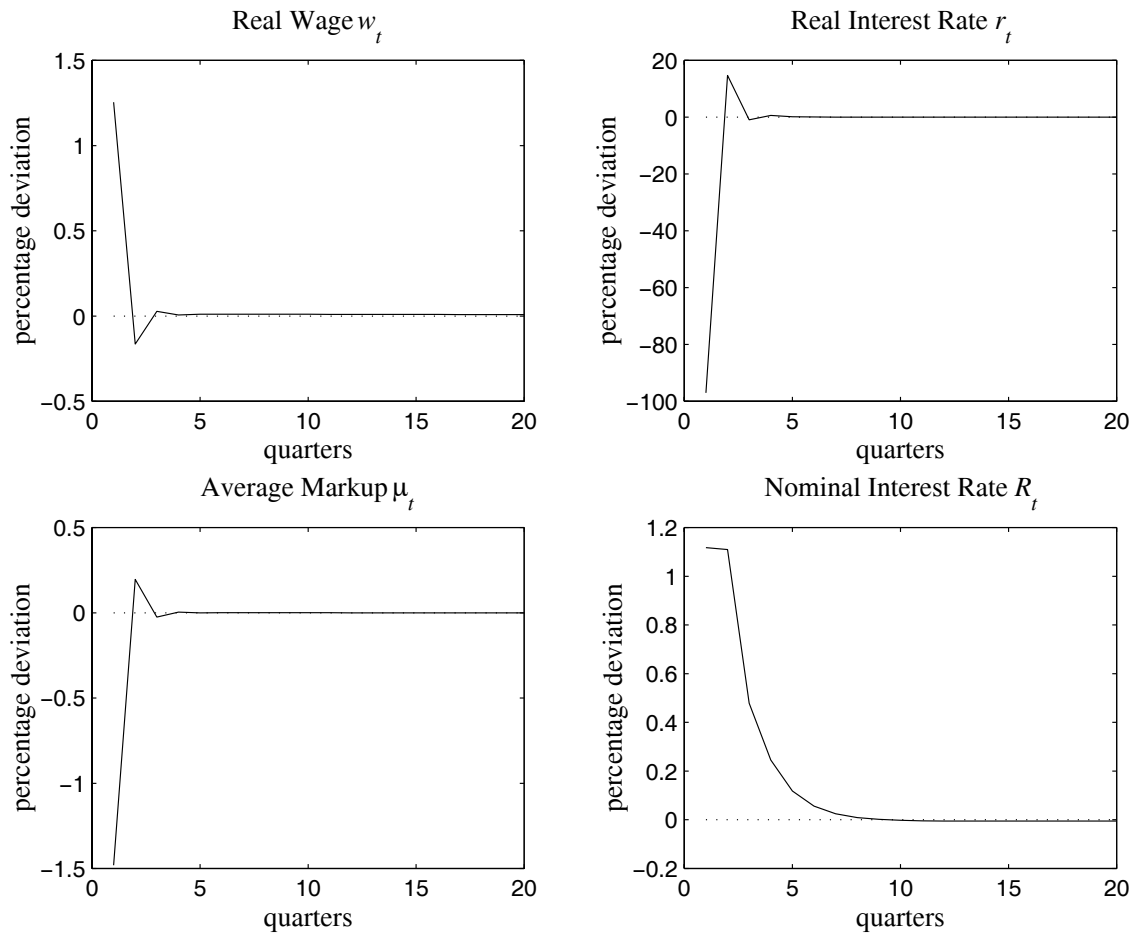


Figure 4.2: Impulse Response Functions for  $\hat{w}_t, \hat{r}_t, \hat{\mu}_t, \hat{R}_t$   $b = 0.8$

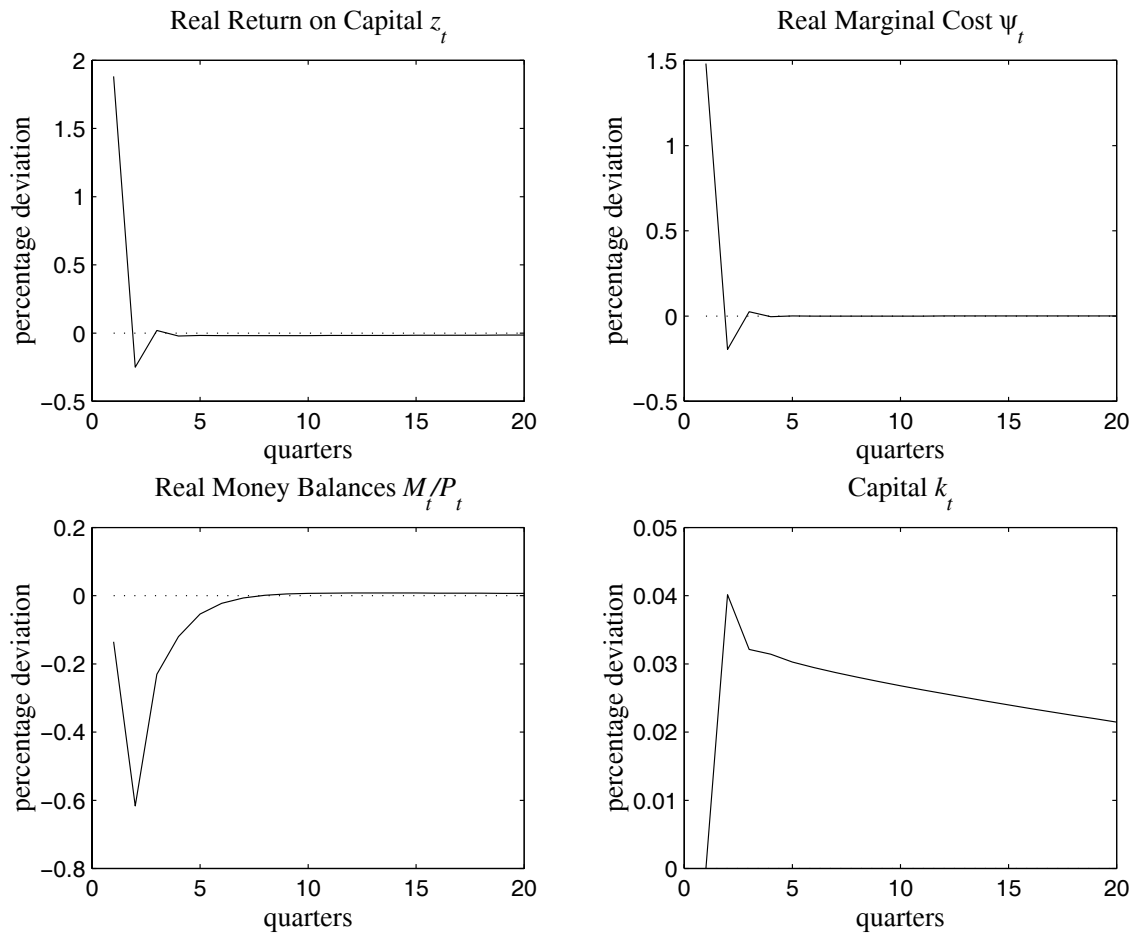


Figure 4.3: Impulse Response Functions for  $\hat{z}_t, \hat{\psi}_t, \hat{M}_t - \hat{P}_t, \hat{k}_t$   $b = 0.8$

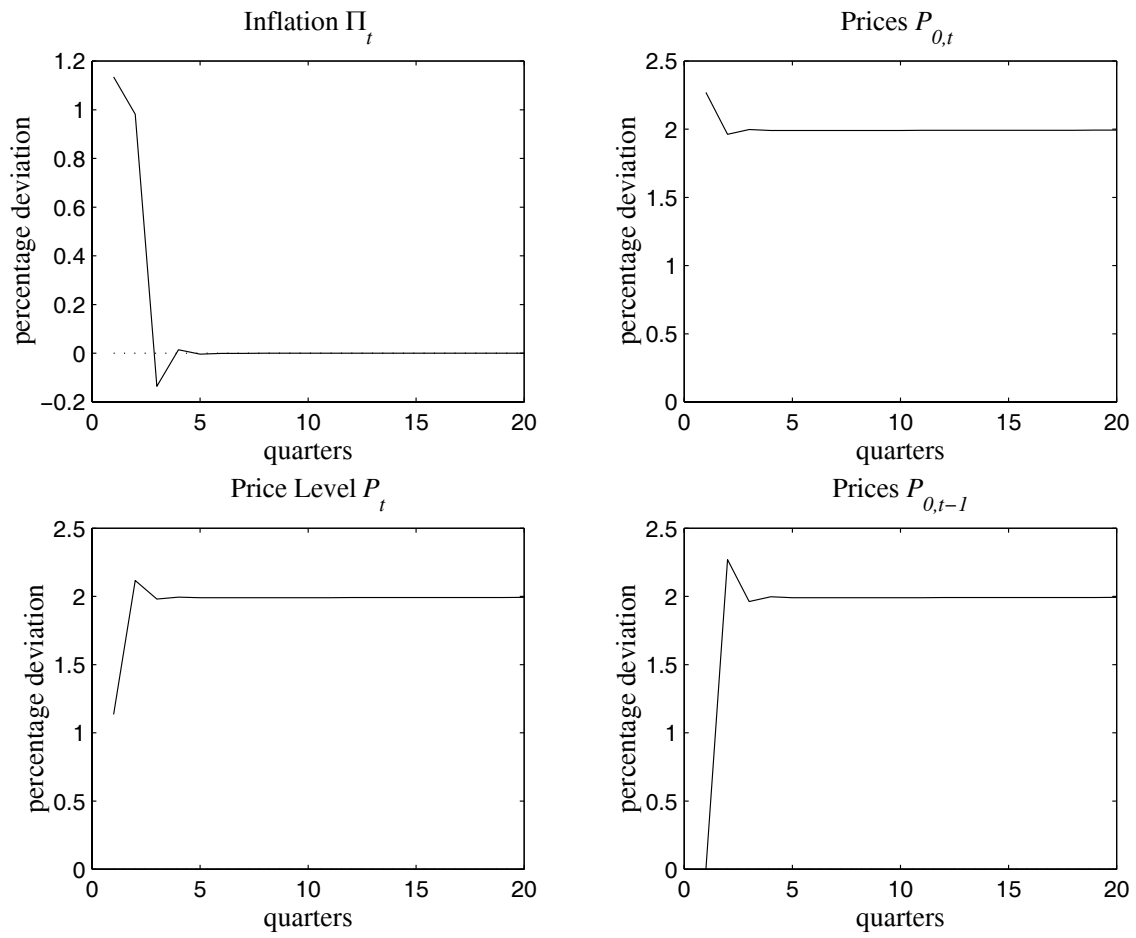


Figure 4.4: Impulse Response Functions for  $\hat{\Pi}_t, \hat{P}_{0,t}, \hat{P}_t, \hat{P}_{0,t-1}$   $b = 0.8$



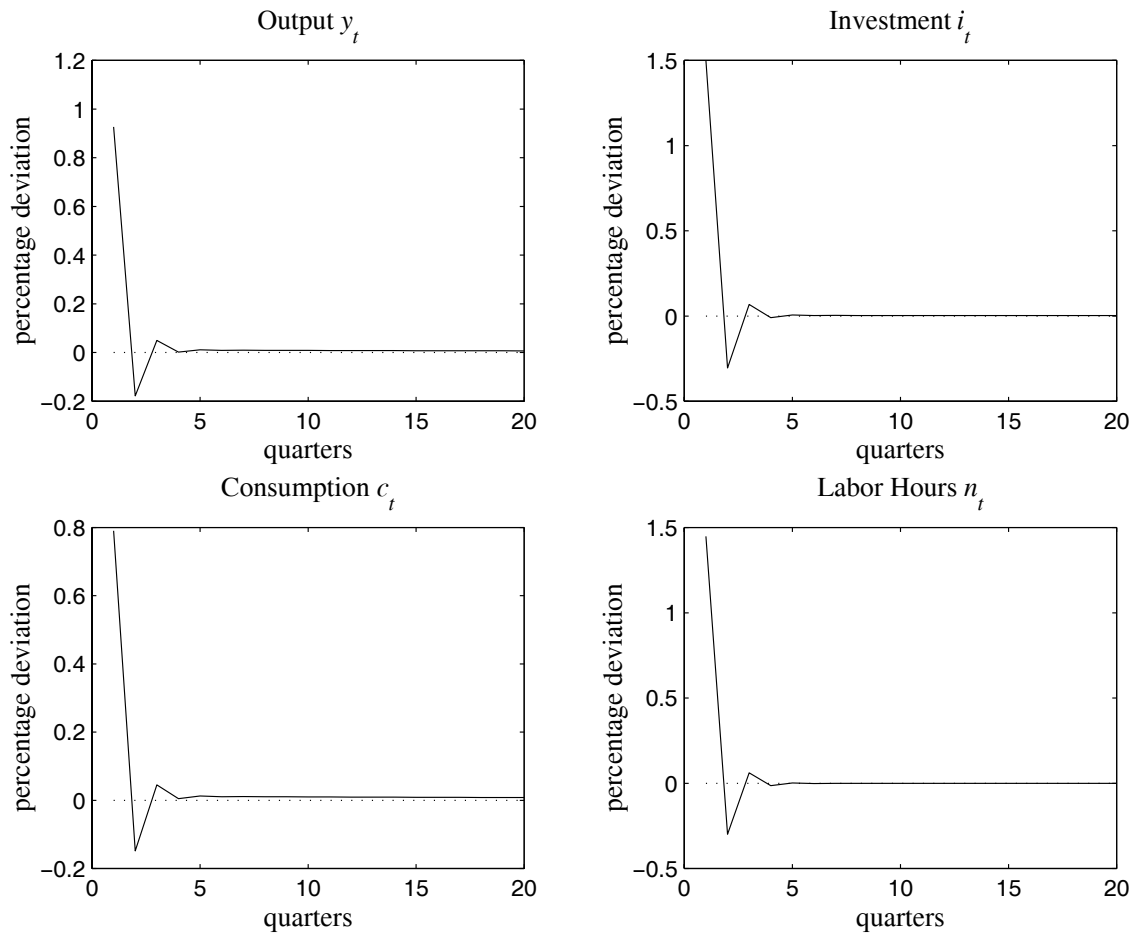


Figure 4.5: Impulse Response Functions for  $\hat{y}_t, \hat{i}_t, \hat{c}_t, \hat{n}_t, \sigma = 1$

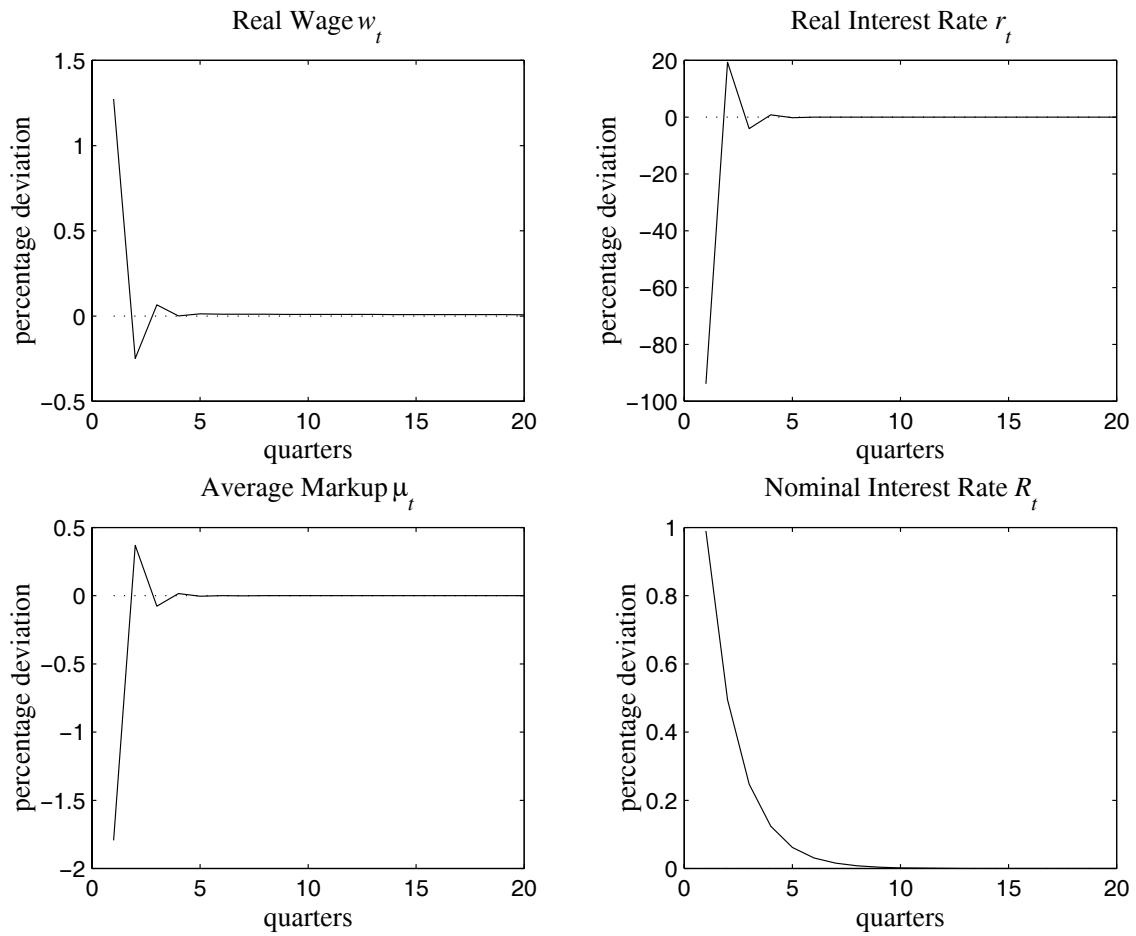


Figure 4.6: Impulse Response Functions for  $\hat{w}_t, \hat{r}_t, \hat{\mu}_t, \hat{R}_t, \sigma = 1$

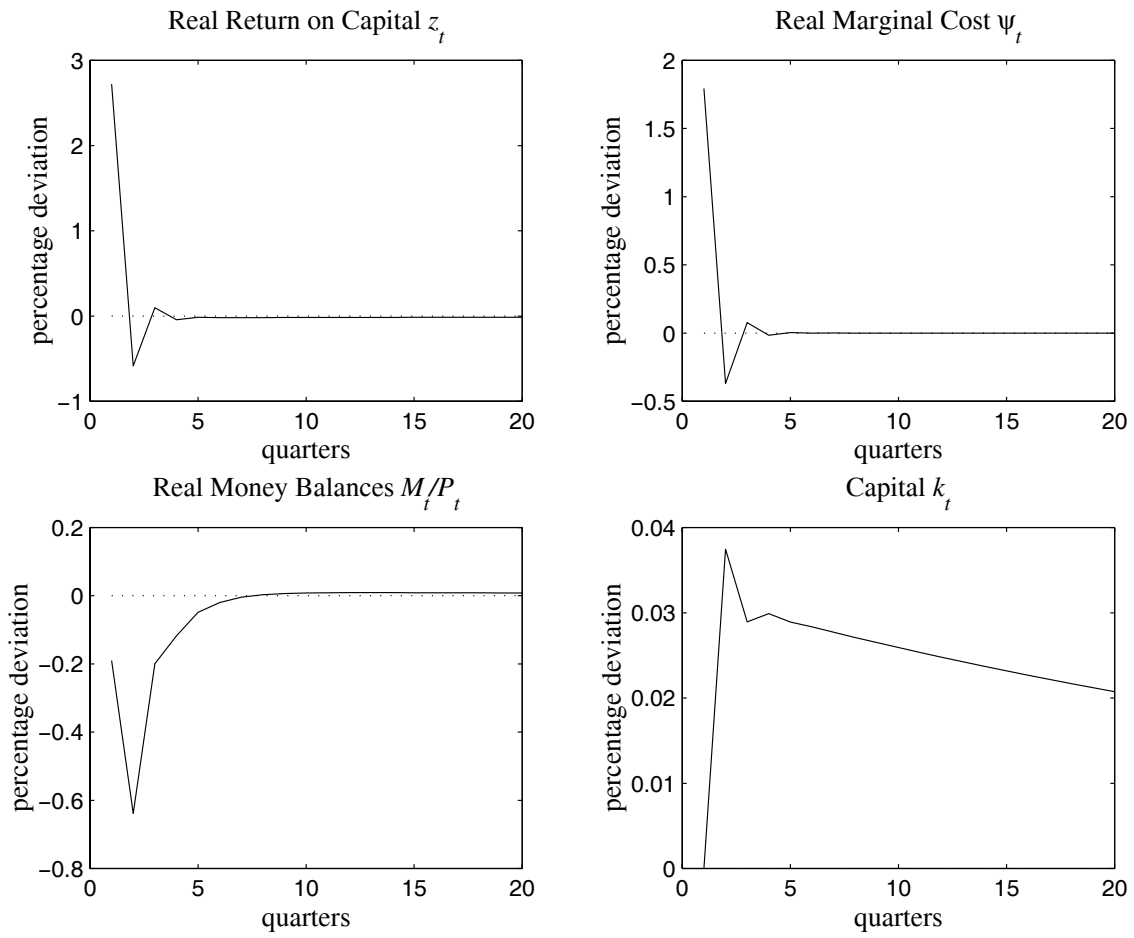


Figure 4.7: Impulse Response Functions for  $\widehat{z}_t, \widehat{\psi}_t, \widehat{M}_t - \widehat{P}_t, \widehat{k}_t, \sigma = 1$

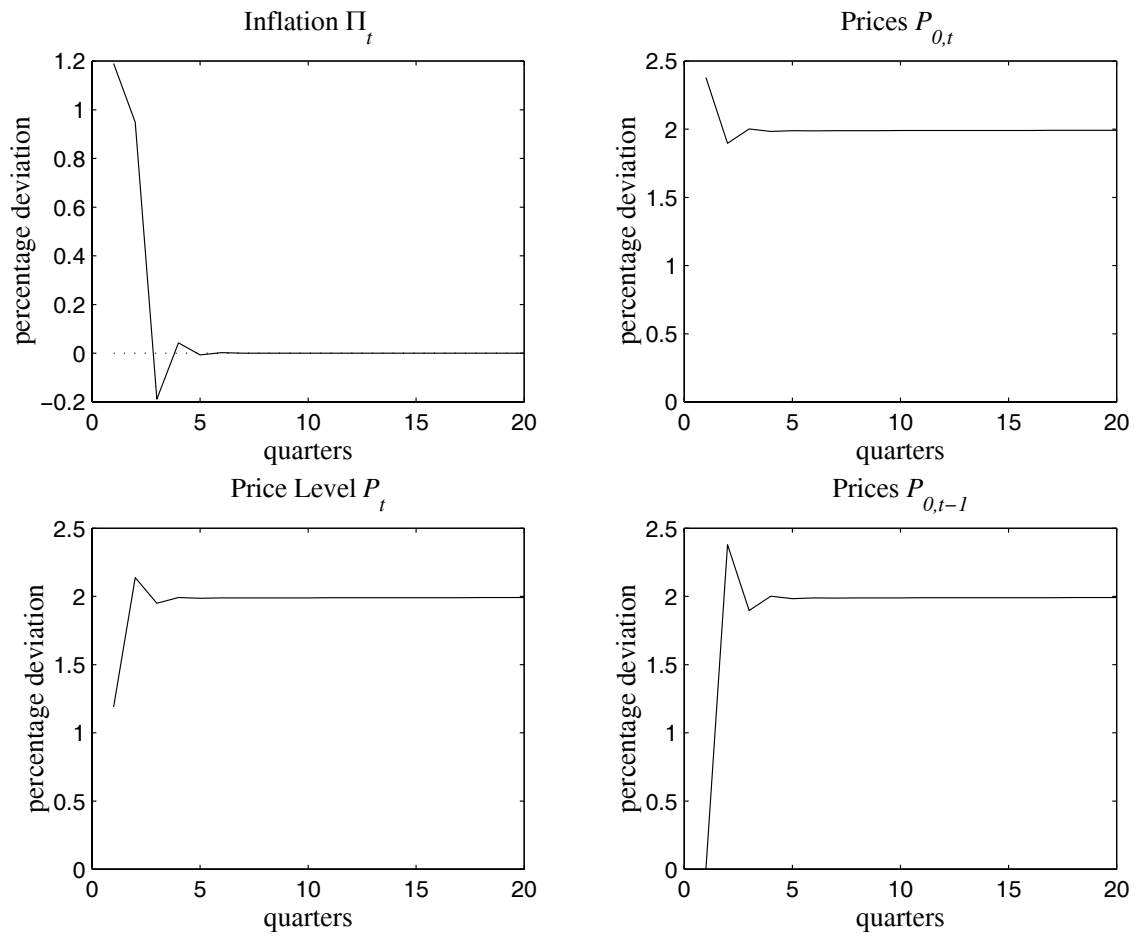


Figure 4.8: Impulse Response Functions for  $\hat{\Pi}_t, \hat{P}_{0,t}, \hat{P}_t, \hat{P}_{0,t-1}, \sigma = 1$

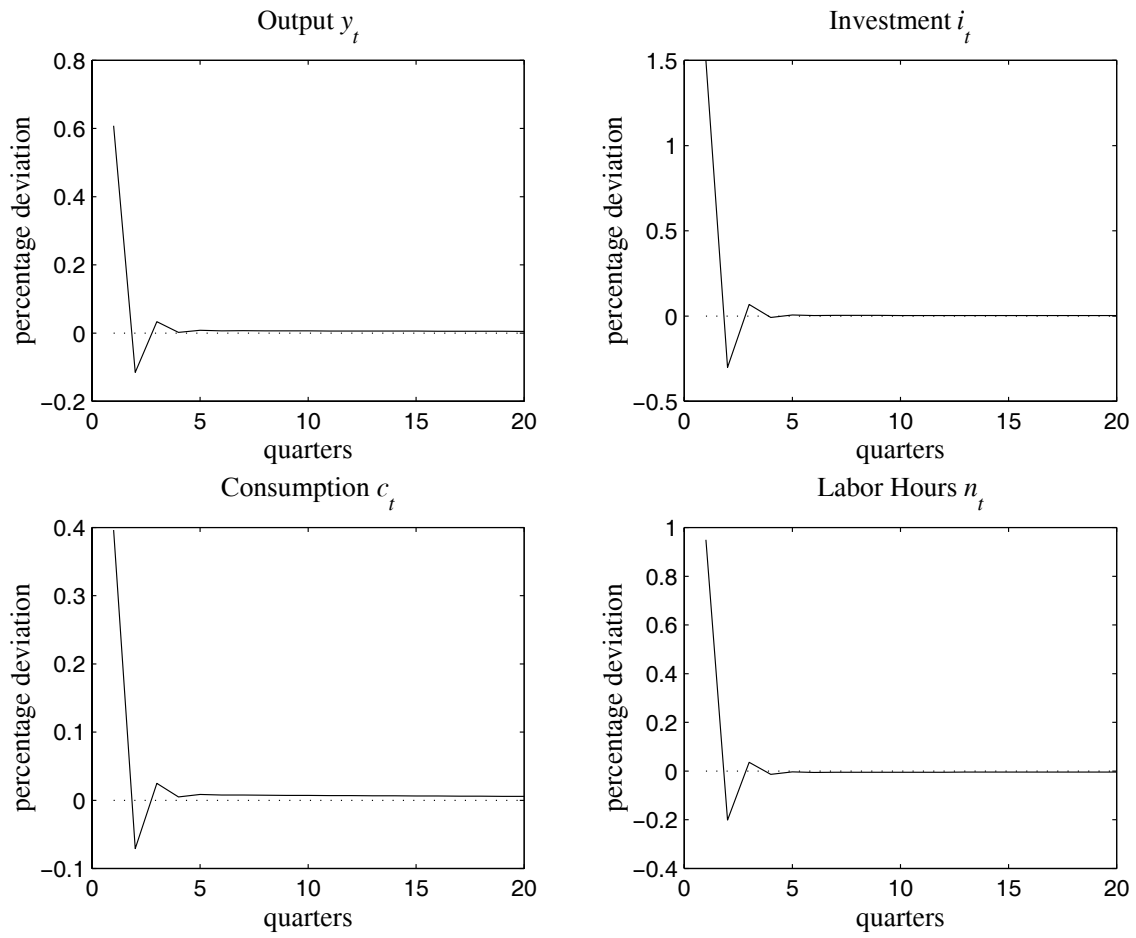


Figure 4.9: Impulse Response Functions for  $\hat{y}_t, \hat{i}_t, \hat{c}_t, \hat{n}_t, b = 0$

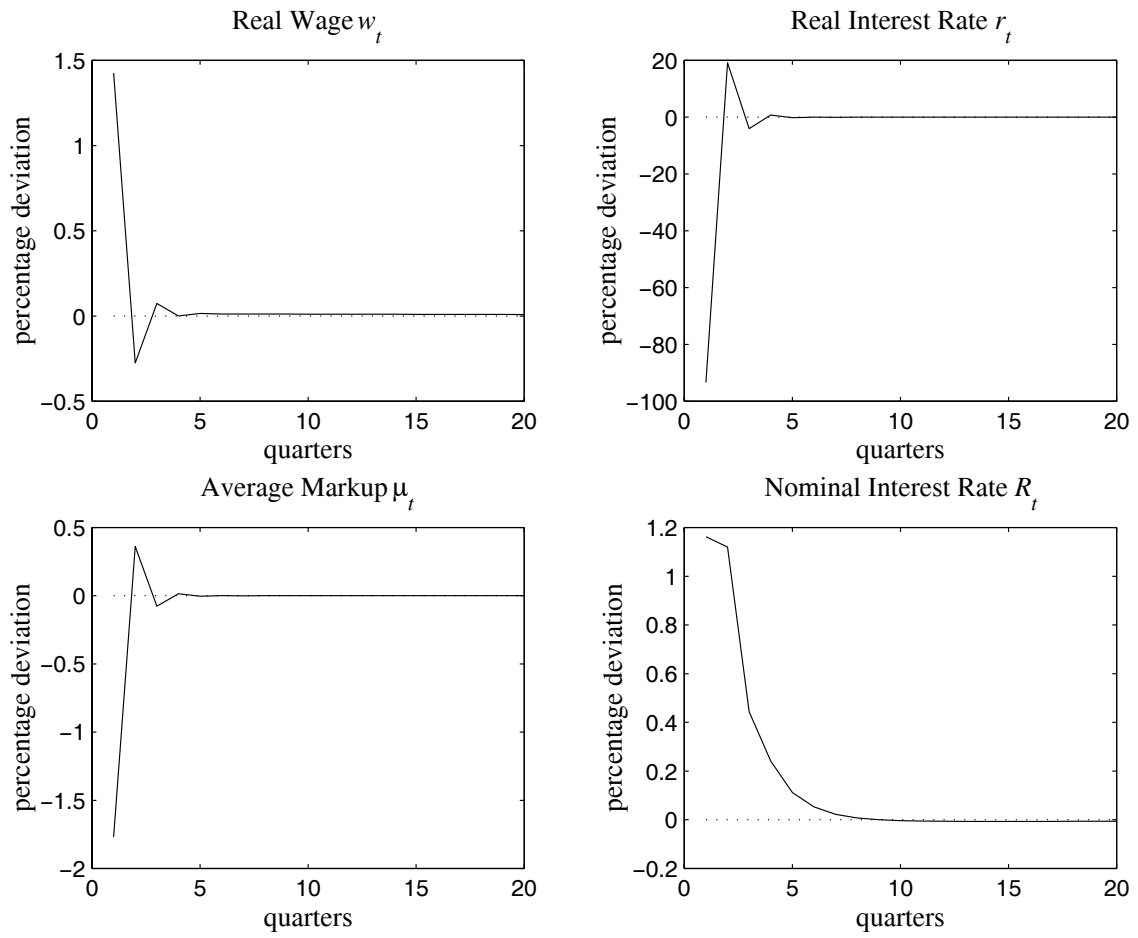


Figure 4.10: Impulse Response Functions for  $\hat{w}_t, \hat{r}_t, \hat{\mu}_t, \hat{R}_t, b = 0$

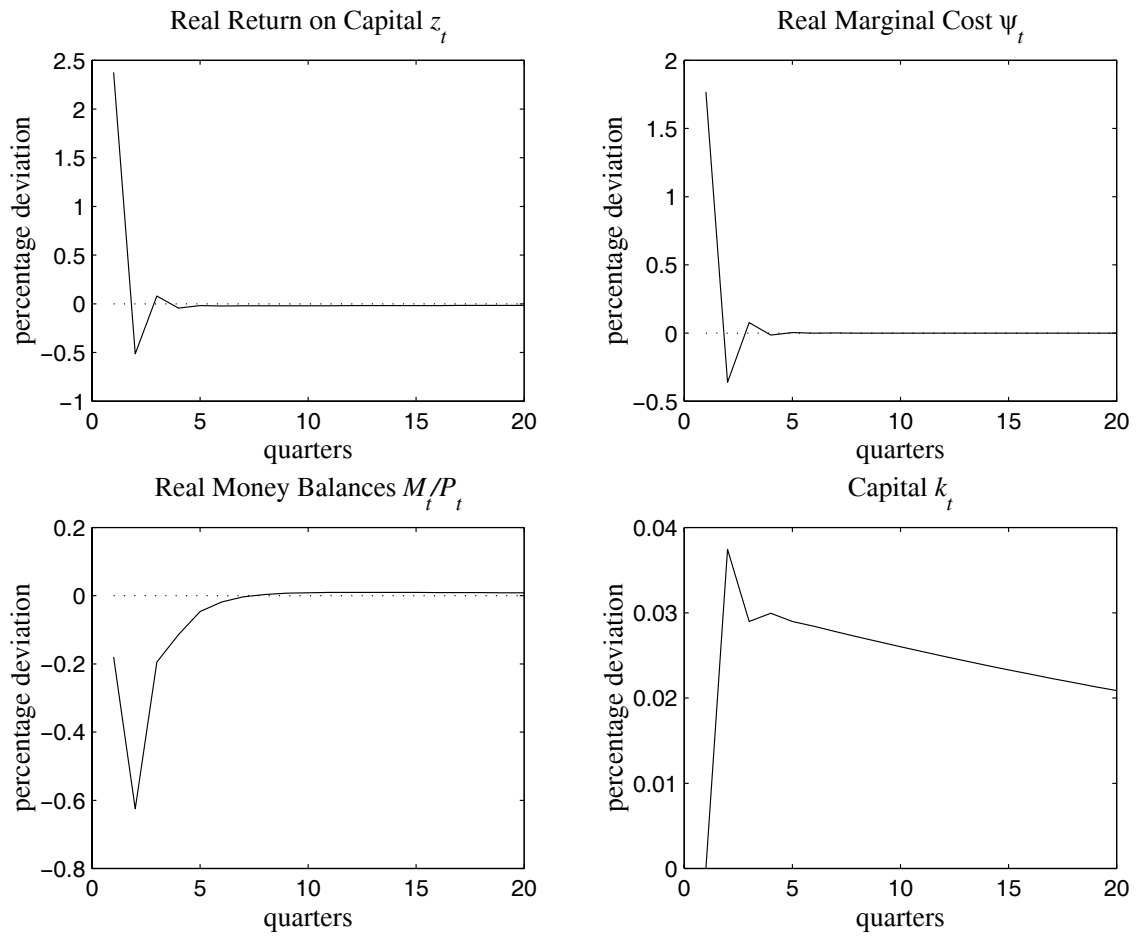


Figure 4.11: Impulse Response Functions for  $\hat{z}_t, \hat{\psi}_t, \hat{M}_t - \hat{P}_t, \hat{k}_t, b = 0$

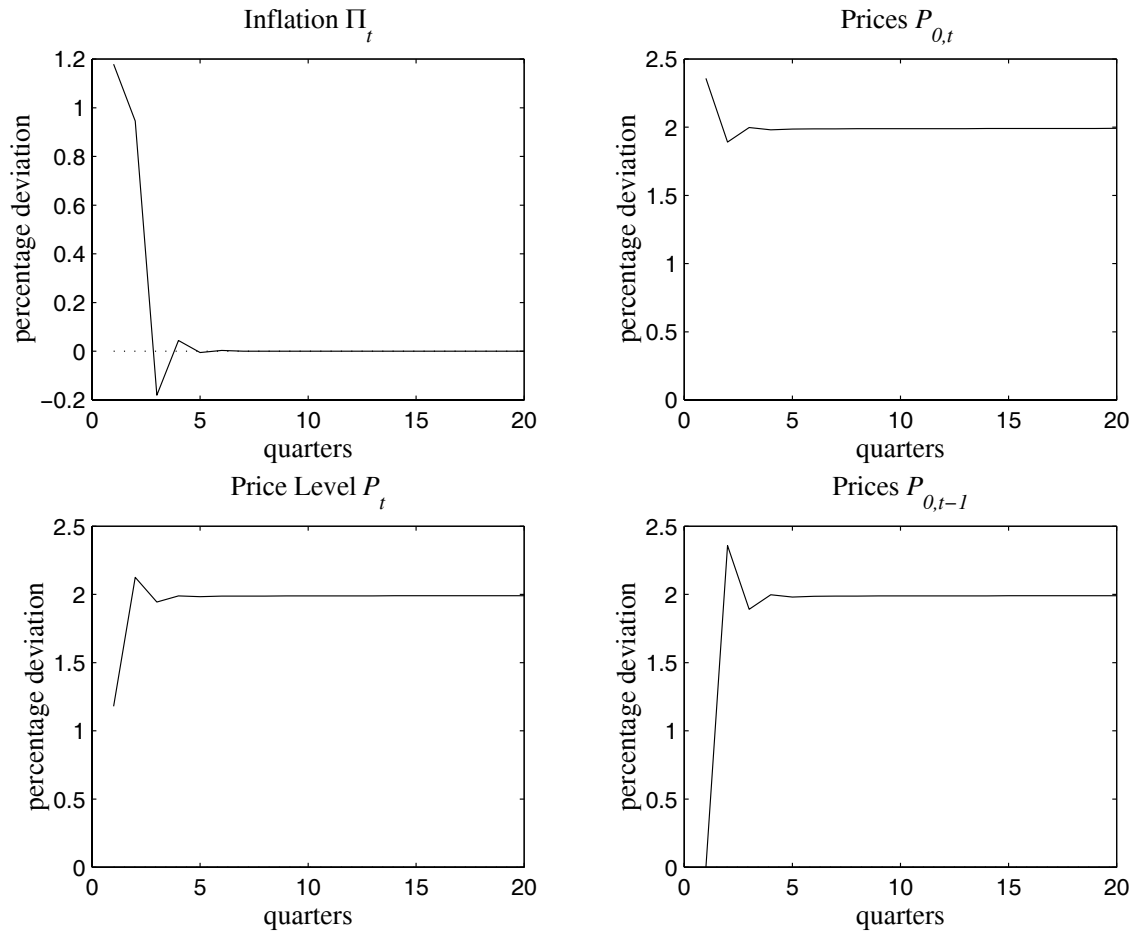


Figure 4.12: Impulse Response Functions for  $\hat{\Pi}_t, \hat{P}_{0,t}, \hat{P}_t, \hat{P}_{0,t-1}$ ,  $b = 0$



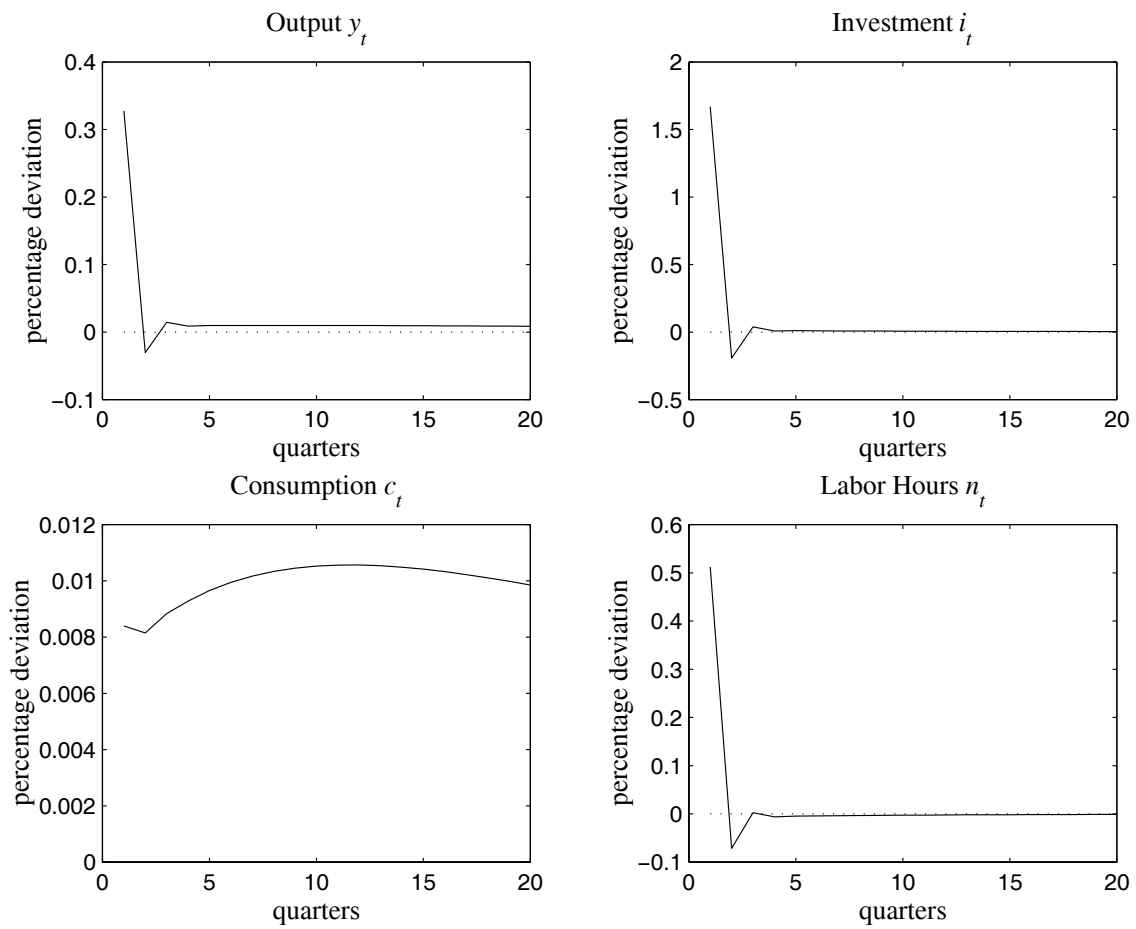


Figure 4.13: Impulse Response Functions for  $\hat{y}_t, \hat{i}_t, \hat{c}_t, \hat{n}_t, b = 1$

# Chapter 5

## Wage Staggering and Sticky Prices in a Monetary Stochastic Dynamic General Equilibrium Model with Labor and Capital

### 5.1 Introduction

Recently Ascari (2003*a*) has provided a unifying framework for the analysis of price and wage staggering in dynamic stochastic general equilibrium models. He simplifies the models so that an exact analytical solution can be obtained. He is therefore able to identify the influence of several specific model parameters on the persistence of a money growth shock. Bénassy also explores the implications of staggered prices and wages analytically (see Bénassy (2003*b*) and Bénassy (2003*a*)). Most other papers in the literature examine simulation results of calibrated versions of the models under investigation and provide some intuition deduced from simplified equilibrium conditions.<sup>1</sup>

Ascari (2003*a*) concludes that, first, labor immobility across sectors plays a key role in enabling both wage and price staggering models to exhibit persistence. Second, this channel is the more important the higher intertemporal elasticities of substitution are. Ascari considers both rigidities separately. In his wage models labor can be immobile because there are industry specific households organized as unions which have monopoly power since labor cannot move across industries (workers organized by skills). This kind of labor immobility is also analyzed in Ascari (2000).

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<sup>1</sup>Another exception is Andersen (2004).

Another way to model immobile labor is to assume that households organized as unions supply differentiated labor inputs to the firms (workers organized by industries). Huang and Liu (2002), Erceg (1997) and Gerke (2003) are examples for this research branch. The model in this chapter also belongs to this class. The approach of Bénassy is some kind of combination of both of Ascari's labor immobilities and is unique in the literature.

Unfortunately - and despite of Ascari's unifying paper - the models differ substantially in the way price and wage stickiness is rationalized and implemented. There are mainly two ways to incorporate sticky wages:<sup>2</sup> The first is the well known Calvo pricing scheme (see Calvo (1983)) where household unions face a fixed probability of being able to change their wage rate. The second are Taylor type wage contracts (see Taylor (1980)) where the unions set the wage for a specified period of time, e.g. 2 or 4 periods. The Calvo approach is used extensively in Bénassy's work. Woodford (2003*a*) also assumes Calvo pricing while Chari, Kehoe and McGrattan (2000), Huang and Liu (2002), Erceg (1997), Ascari (2000) and Gerke (2003) use Taylor contracts. In addition the approaches differ with respect to their specific assumptions about production functions, capital accumulation, implied or assumed money demand functions, utility functions, capital adjustment costs etc. Some include sticky prices, others do not. It is therefore only natural that results are likely to differ substantially. Some peculiarities are summarized below not to explore in detail the reasons but just to demonstrate the diversity and to point to the main differences in the assumptions.

On the one hand Huang and Liu (2002) find that wage staggering has a much higher potential to create empirically observed reactions of output to a money growth shock than price staggering while on the other hand Ascari (2000) concludes that high persistence is an unlikely outcome. The reasons for these different conclusions can be due to the steady state inflation rate. Ascari shows that in general the degree of persistence is lower the higher the steady state growth rate of money and thus the inflation rate.<sup>3</sup> Huang and Liu study a model with a zero steady state rate of money growth. So possibly their results break down once the model is generalized along these lines. Bénassy (2003*b*) can show that both output and employment can display a hump-shaped response. The most important parameters in his model are the probability of wage adjustment and the autocorrelation coefficient of the money

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<sup>2</sup>I will only refer to wage stickiness since this is the main focus of the present chapter.

<sup>3</sup>In a related paper Ascari (2003*b*) examines the influence of a positive inflation rate in a model with Calvo *price* staggering in a similar model as in Chari, Kehoe and McGrattan (2000). He can show that higher inflation now causes a *higher* persistence in output.

growth process. He concludes that the reason for the failure of Chari, Kehoe and McGrattan (2000) to produce persistence in output is caused by a too short duration of the price contracts of only a quarter. But Chari et al. consider Taylor contracts. Bénassy conjectures that Ascari's model fails to produce a hump because he uses Taylor contracts of only two quarters in conjunction with a random walk for money. Again this difference can be due to the inflation rate which is zero at the steady state in Bénassy's approach.

Edge (2002) compares Taylor wage and price staggering models. She can show that Taylor price staggering is equally able to explain output persistence when each firm  $j$  uses only the labor input of a specific household  $i$  where  $i = j$ . Both firms and households are assumed to have neither monopsony nor monopoly power, respectively. She rationalizes this assumption by thinking of 'each point on the unit interval continuum as identifying a single factor market whose participants include a large (though finite) number of identical households and firms' (Edge (2002), p. 577). But this assumption of specific factor markets appears to be too restrictive compared to the approach of Chari, Kehoe and McGrattan (2000) who consider firms that hire homogenous labor but an additional fixed specific factor such as land. Their model variant can create more persistence in output but less than in Edge's setup. Moreover the behavior of households and firms has to be modeled differently because they essentially have monopoly and monopsony power in this case, respectively.

Gerke (2003) studies several different versions of Taylor type wage staggering in a model with price adjustment costs as in Rotemberg (1982). Most of his versions fail to produce persistence. But he can create a hump-shaped output response in a variant with a utility function that allows for different values of the elasticity of substitution between consumption and real money balances. His model is one of the few exceptions which consider a positive steady state inflation rate. The paper of Erceg (1997) can be interpreted as an extension of the Chari, Kehoe and McGrattan (2000) model. He uses both Taylor type wage and price staggering combined with different assumptions about the way capital is used: it can be either fixed or factor specific at the firm level or mobile in the aggregate. His model incorporates firm specific adjustment costs of capital and can generate considerable persistence. In the language of Ascari (2003a) it belongs to the type of model with immobile workers organized by industries and is thus successful in creating a persistent output response. But Erceg's setup differs from Ascari's with regard to capital accumulation. It is virtually absent in Ascari's analysis, so it is again not

obvious which mechanism is responsible for Erceg's success.

This chapter tries to combine some aspects of the work in Gerke (2003) as well as in Erceg (1997). I study only two period Taylor type wage contracts - not four period contracts as in Gerke's work. But I use the price adjustment cost version of Rotemberg (1982) also incorporated in Gerke while Erceg investigates also Taylor type price contracts which also last for four periods. I depart from the assumption of four period contracts since I think that the longer these contracts last the higher is the possibility of a cyclical reaction of the variables since the order of the difference equations grows. I will not consider Calvo pricing – neither for wage nor for price setting – since Chapter 3 has already shown that Calvo contracts produce non-cyclical impulse responses. I consider adjustment costs of capital but assume that households accumulate capital and sell it to the firms while in Erceg each firm decides on its capital stock itself. There is a zero inflation steady state. The main result of the chapter confirms the finding of Ascari (2003a) that immobile labor leads to more persistence in output. But it turns out that capital adjustment costs contribute significantly to output persistence. Without them money growth shocks cannot stimulate a persistent reaction of output.

The chapter is organized as follows: Section 5.2 presents the model along with the main assumptions on household and firm behavior. In Section 5.3 the results are presented using impulse response functions and are related to other results in the literature while in Section 5.4 the business cycle implications will be discussed. Section 5.5 concludes.

## 5.2 The Model

### 5.2.1 The Labor Market Intermediary

The labor market intermediary buys in every period  $t$   $n_{i,t}$  units of labor at the nominal wage rate  $W_{i,t}$  from the household  $i \in [0, 1]$  in order to bundle them to the aggregate labor input  $n_t$ . Then he offers this labor aggregate to the firms. The production function is assumed to be a CES aggregator as in Dixit and Stiglitz (1977) with  $\epsilon_w > 1$ .

$$n_t = \left( \int_0^1 n_{i,t}^{(\epsilon_w-1)/\epsilon_w} di \right)^{\epsilon_w/(\epsilon_w-1)} \quad (5.1)$$

The bundler maximizes his profits over  $n_{i,t}$  given the above production function and given the aggregate nominal wage  $W_t$ . So the problem can be written as

$$\max_{n_{i,t}} \left[ W_t n_t - \int_0^1 W_{i,t} n_{i,t} di \right] \text{ s.t. } n_t = \left( \int_0^1 n_{i,t}^{(\epsilon_w-1)/\epsilon_w} di \right)^{\epsilon_w/(\epsilon_w-1)} \quad (5.2)$$

The first order conditions for each household  $i$  imply

$$n_{i,t}^d = \left( \frac{W_{i,t}}{W_t} \right)^{-\epsilon_w} n_t \quad (5.3)$$

where  $-\epsilon_w$  measures the constant wage elasticity of labor demand from each household  $i$ . It is assumed that households offer exactly this amount of labor demanded so that demand always equals supply:  $n_{i,t}^d = n_{i,t}^s =: n_{i,t}$ . Since the labor market intermediary operates under perfect competition profits are zero. Inserting the demand function into the profit function and imposing the zero profit condition reveals that the only wage rate  $W_t$  that is consistent with this requirement is given by

$$W_t = \left( \int_0^1 W_{i,t}^{(1-\epsilon_w)} di \right)^{1/(1-\epsilon_w)} \quad (5.4)$$

When wages are set for just two periods as explored in the next section the wage equation simplifies. With wages set for two periods half of the households adjust their wage in period  $t$  and half do not. Moreover all adjusting households choose the same wage. Define  $W_{i,s,t}$  as the nominal wage at time  $t$  of any household  $i$  who has set its wage  $t-s$  periods ago. Then the wage index  $W_t$  is given by

$$W_t = \left( \frac{1}{2} W_{i,0,t}^{1-\epsilon_w} + \frac{1}{2} W_{i,1,t}^{1-\epsilon_w} \right)^{1/(1-\epsilon_w)} \quad (5.5)$$

## 5.2.2 The Household

I consider a MIU-setup where the household  $i$  is assumed to have preferences over consumption  $c_{i,t}$ , leisure  $1 - n_{i,t}$  and real money balances  $M_{i,t}/P_t$ . In this model the household sets its wage rate. The household cannot decide on its labor supply because it supplies exactly what the labor market intermediary demands. So the instantaneous utility function is given by

$$\begin{aligned} & u \left( c_{i,t}, \frac{M_{i,t}}{P_t}, \left( \frac{W_{i,s,t}}{W_t} \right)^{-\epsilon_w} n_t, a_t \right) \\ &= \frac{\left[ a_t \left( \eta c_{i,t}^\nu + (1-\eta) \left( \frac{M_{i,t}}{P_t} \right)^\nu \right)^{\frac{1}{\nu}} \right]^{1-\sigma}}{1-\sigma} - \frac{a_t \Theta \left[ \left( \frac{W_{i,s,t}}{W_t} \right)^{-\epsilon_w} n_t \right]^{1+\gamma}}{1+\gamma} \end{aligned} \quad (5.6)$$

$\sigma$  is the degree of risk aversion while  $\eta$  is a share parameter and  $\nu$  determines the interest elasticity of the implied money demand function. In this function  $c_{i,t}$  and  $M_{i,t}/P_t$  are combined to a composite good via a CES aggregator. Labor is separable because households will differ according to their labor supply.<sup>4</sup>  $\Theta$  is a weighting parameter and  $a_t$  is a taste shock which is the same for all households.  $s$  can take the values 0 and 1 since the household sets its wage for two periods. This implies that  $W_{i,0,t} = W_{i,1,t+1}$ . Note that the steady state inflation rate is zero in this model so that there is no indexation of wages.

The household's budget constraint can be written as follows:

$$\begin{aligned} & c_{i,t} + i_{i,t} + \frac{M_{i,t}}{P_t} + \frac{B_{i,t}}{P_t} \\ = & \frac{W_{i,s,t}}{P_t} n_{i,t} + z_{i,t} k_{i,t-1} + \frac{M_{i,t-1}}{P_t} + (1 + R_{t-1}) \frac{B_{i,t-1}}{P_t} + \frac{M_{i,t}^s}{P_t} + \gamma_i \frac{\Xi_t}{P_t} \end{aligned} \quad (5.7)$$

where

$$\Xi_t = \int_0^1 \Xi_{j,t} dj \quad (5.8)$$

are the nominal profits of the intermediate goods producing firms and  $\gamma_i$  is the share household  $i$  receives from these profits. The uses of wealth are real consumption  $c_{i,t}$ , real investment  $i_{i,t}$ , holdings of real money balances  $M_{i,t}/P_t$  and real bonds  $B_{i,t}/P_t$ . There are several sources of the household's wealth. It earns money working in the market at the desired wage rate  $W_{i,s,t}$  supplying  $n_{i,t} = (W_{i,s,t}/W_t)^{-\epsilon_w} n_t$  units of labor. It can spend its money holdings carried over from the previous period  $M_{i,t-1}/P_t$ . It receives a capital income equal to  $z_{i,t} k_{i,t-1}$  by selling capital to the firms where  $z_{i,t}$  denotes the real return on capital  $k_{i,t}$ . There are also previous period bond holdings including the interest on them  $(1 + R_{t-1}) (B_{i,t-1}/P_t)$ . Finally, the household receives a monetary transfer  $M_{i,t}^s$  from the monetary authority and a share  $\gamma_i$  of profits from the intermediate goods firms, respectively. The transfer is equal to the change in money balances, i.e.

$$M_{i,t}^s = M_{i,t} - M_{i,t-1} \quad (5.9)$$

The capital stock increases according to the following law of motion:

$$k_{i,t} = (1 - \delta) k_{i,t-1} + \phi \left( \frac{i_{i,t}}{k_{i,t-1}} \right) k_{i,t-1} \quad (5.10)$$

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<sup>4</sup>Note that labor must be additively separable because there is no longer a continuum of identical households each supplying the same continuum of differentiated labor. See Woodford (2003a), p. 222 for more details on this point.

There are costs of adjusting the capital stock which are captured by the  $\phi$  function.  $\delta$  is the rate of depreciation. The detailed properties of  $\phi$  will be discussed in the calibration subsection. Because this equation cannot be explicitly solved for  $i_{i,t}$  a second Lagrange multiplier ( $\theta_{i,t}$ ) has to be introduced into the optimization problem of the household.

The Lagrangian is then given by:

$$\begin{aligned}
L_i = & E_0 \left[ \sum_{t=0}^{\infty} \beta^t u \left( c_{i,t}, m_{i,t}, \left( \frac{W_{i,s,t}}{W_t} \right)^{-\epsilon_w} n_t, a_t \right) \right. \\
& + \sum_{t=0}^{\infty} \beta^t \lambda_{i,t} \left( z_{i,t} k_{i,t-1} + \frac{W_{i,s,t}}{P_t} \left( \frac{W_{i,s,t}}{W_t} \right)^{-\epsilon_w} n_t + m_{i,t-1} \frac{P_{t-1}}{P_t} + m_{i,t}^s \right. \\
& \left. \left. + (1 + R_{t-1}) b_{i,t-1} \frac{P_{t-1}}{P_t} + \gamma_i \frac{\Xi_t}{P_t} - c_{i,t} - i_{i,t} - m_{i,t} - b_{i,t} \right) \right. \\
& \left. + \sum_{t=0}^{\infty} \beta^t \theta_{i,t} \left( (1 - \delta) k_{i,t-1} + \phi \left( \frac{i_{i,t}}{k_{i,t-1}} \right) k_{i,t-1} - k_{i,t} \right) \right] \quad (5.11)
\end{aligned}$$

Here small variables indicate real quantities, i.e. for example  $m_{i,t} = M_{i,t}/P_t$ . Households optimize over  $c_{i,t}$ ,  $W_{i,s,t}$ ,  $i_{i,t}$ ,  $k_{i,t}$ ,  $m_{i,t}$  and  $b_{i,t}$  taking prices and the initial values of the price level  $P_0$  and the capital stock  $k_{i,0}$  as well as the outstanding stocks of money  $M_{i,0}$  and bonds  $B_{i,0}$  as given.  $s$  can take the values 0 and 1 and the household sets the wage rate for two periods so that  $W_{i,0,t} = W_{i,1,t+1}$  as explained above. The first order conditions then read

$$\frac{\partial L_i}{\partial c_{i,t}} = \beta^t D_1 u(\cdot, t) - \beta^t \lambda_{i,t} = 0 \quad (5.12)$$

$$\begin{aligned}
\frac{\partial L_i}{\partial W_{i,0,t}} = & -\epsilon_w \beta^t D_3 u(\cdot, t) \left( \frac{W_{i,0,t}}{W_t} \right)^{-\epsilon_w - 1} \frac{n_t}{W_t} + \beta^t \lambda_{i,t} \left( \frac{W_{i,0,t}}{W_t} \right)^{-\epsilon_w} \frac{n_t}{P_t} \\
& - \epsilon_w \beta^t \lambda_{i,t} \frac{W_{i,0,t}}{P_t} \left( \frac{W_{i,0,t}}{W_t} \right)^{-\epsilon_w - 1} \frac{n_t}{W_t} \\
& + \beta^{t+1} E_t \left[ -\epsilon_w D_3 u(\cdot, t+1) \left( \frac{W_{i,0,t}}{W_{t+1}} \right)^{-\epsilon_w - 1} \frac{n_{t+1}}{W_{t+1}} \right. \\
& \left. + \lambda_{i,t+1} \left( \frac{W_{i,0,t}}{W_{t+1}} \right)^{-\epsilon_w} \frac{n_{t+1}}{P_{t+1}} \right. \\
& \left. - \epsilon_w \lambda_{i,t+1} \frac{W_{i,0,t}}{P_{t+1}} \left( \frac{W_{i,0,t}}{W_{t+1}} \right)^{-\epsilon_w - 1} \frac{n_{t+1}}{W_{t+1}} \right] = 0 \quad (5.13)
\end{aligned}$$

$$\frac{\partial L_i}{\partial i_{i,t}} = -\beta^t \lambda_{i,t} + \beta^t \theta_{i,t} \phi' \left( \frac{i_{i,t}}{k_{i,t-1}} \right) \left( \frac{1}{k_{i,t-1}} \right) k_{i,t-1} = 0 \quad (5.14)$$



$$\begin{aligned} \frac{\partial L_i}{\partial k_{i,t}} = & E_t \beta^{t+1} \lambda_{i,t+1} z_{i,t+1} - \beta^t \theta_{i,t} + E_t \beta^{t+1} \theta_{i,t+1} \left[ (1 - \delta) \right. \\ & \left. + \phi \left( \frac{i_{i,t+1}}{k_{i,t}} \right) + \phi' \left( \frac{i_{i,t+1}}{k_{i,t}} \right) \left( -\frac{i_{i,t+1}}{k_{i,t}^2} \right) k_{i,t} \right] = 0 \end{aligned} \quad (5.15)$$

$$\frac{\partial L_i}{\partial m_{i,t}} = \beta^t D_2 u(\cdot, t) - \beta^t \lambda_{i,t} + E_t \beta^{t+1} \lambda_{i,t+1} \frac{P_t}{P_{t+1}} = 0 \quad (5.16)$$

$$\frac{\partial L_i}{\partial b_{i,t}} = -\beta^t \lambda_{i,t} + E_t \beta^{t+1} \lambda_{i,t+1} (1 + R_t) \frac{P_t}{P_{t+1}} = 0 \quad (5.17)$$

$D_x u(\cdot, t + y)$  denotes the first partial derivative of the  $u$ -function with respect to the  $x$ -th argument evaluated at period  $t + y$ . The derivatives with respect to  $\lambda_{i,t}$  and  $\theta_{i,t}$  are omitted since they are equal to the intertemporal budget constraint and the capital accumulation condition respectively.  $\phi'$  denotes the derivative of the  $\phi$ -function with respect to the investment to capital ratio which is regarded as one argument. In addition the household's optimal choices must also satisfy the transversality conditions:

$$\lim_{t \rightarrow \infty} \beta^t \lambda_{i,t} x_{i,t} = 0 \quad \text{for } x = m, b, k \quad (5.18)$$

It is assumed that there exists a contingent claims market where all households can insure themselves against all idiosyncratic risks. This implies that the household's decisions for consumption, money, bonds, investment and capital are all identical. In addition the factor prices and the Lagrange multipliers will also be identical. All households who reset their wage in the same period face identical decision problems so that they choose the same wage rate. This means that the index  $i$  can be dropped (see Christiano, Eichenbaum and Evans (2003), p. 13, for a more thorough discussion).

Equation (5.13) determines the optimal wage rate of the household. Using (5.12) to replace  $\lambda_t$  by the marginal utility of consumption it can be rearranged yielding the following formula:

$$W_{0,t} = \frac{\epsilon_w}{1 - \epsilon_w} \frac{D_3 u(\cdot, t) n_t + \beta E_t D_3 u(\cdot, t + 1) \left( \frac{W_{t+1}}{W_t} \right)^{\epsilon_w} n_{t+1}}{D_1 u(\cdot, t) \frac{n_t}{P_t} + \beta E_t D_1 u(\cdot, t + 1) \left( \frac{W_{t+1}}{W_t} \right)^{\epsilon_w} \frac{n_{t+1}}{P_{t+1}}} \quad (5.19)$$

The households set their optimal nominal contract wage as a constant markup  $\epsilon_w / (\epsilon_w - 1)$  over some kind of marginal rate of substitution between consumption and leisure which is given by the ratio of some weighted marginal disutilities of labor to some weighted marginal utilities of consumption. These weights are given by  $n$  and  $n/P$  in the two periods for which the wage is set and the growth factor of the

aggregate nominal wage rate, respectively.<sup>5</sup> The formula is similar in its structure to the one that results in a model with price staggering for the intermediate goods producing firms.

The efficiency condition for bond holdings establishes a relation between the nominal interest rate and the price level. Rearranging terms yields

$$(1 + R_t) = E_t \left[ \frac{\lambda_t}{\lambda_{t+1}} \frac{1}{\beta} \frac{P_{t+1}}{P_t} \right] \quad (5.20)$$

Supposed the Fisher equation is valid the real interest rate  $r_t$  is implicitly defined as

$$(1 + r_t) = E_t \left[ \frac{\lambda_t}{\lambda_{t+1}} \frac{1}{\beta} \right] \quad (5.21)$$

because  $P_{t+1}/P_t$  equals one plus the rate of expected inflation which is approximated by the ex-post-inflation rate. The derivative with respect to money determines the endogenous money demand function. Combining the optimum conditions for consumption, bonds and money yields the following equation:

$$D_2u(\cdot, t) = D_1u(\cdot, t) \frac{R_t}{1 + R_t} \quad (5.22)$$

This specification allows to estimate an empirical money demand function. A detailed description will be presented in the calibration section. The efficiency conditions for investment implies that  $\lambda_t$  equals  $\theta_t$  times the change in adjustment costs.

$$\lambda_t = \theta_t \phi' \left( \frac{i_{i,t}}{k_{i,t-1}} \right) \quad (5.23)$$

### 5.2.3 The Finished Goods Producing Firm

The firm producing the final good  $y_t$  in the economy uses  $y_{j,t}$  units of each intermediate good  $j \in [0, 1]$  purchased at price  $P_{j,t}$  to produce  $y_t$  units of the finished good. The production function is assumed to be a CES aggregator as in Dixit and Stiglitz (1977) with  $\epsilon_p > 1$ .

$$y_t = \left( \int_0^1 y_{j,t}^{(\epsilon_p-1)/\epsilon_p} dj \right)^{\epsilon_p/(\epsilon_p-1)} \quad (5.24)$$

The firm maximizes its profits over  $y_{j,t}$  given the above production function and given the price  $P_t$ . So the problem can be written as

$$\max_{y_{j,t}} \left[ P_t y_t - \int_0^1 P_{j,t} y_{j,t} dj \right] \text{ s.t. } y_t = \left( \int_0^1 y_{j,t}^{(\epsilon_p-1)/\epsilon_p} dj \right)^{\epsilon_p/(\epsilon_p-1)} \quad (5.25)$$

<sup>5</sup>Ascari (2003a) derives the same formula in his ‘craft unions’ case.

The first order conditions for each good  $j$  imply

$$y_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{-\epsilon_p} y_t \quad (5.26)$$

where  $-\epsilon_p$  measures the constant price elasticity of demand for each good  $j$ . Because the firm operates under perfect competition profits are zero. Inserting the demand function into the profit function and imposing the zero profit condition reveals that the only price  $P_t$  that is consistent with this requirement is given by

$$P_t = \left( \int_0^1 P_{j,t}^{(1-\epsilon_p)} dj \right)^{1/(1-\epsilon_p)} \quad (5.27)$$

### 5.2.4 The Intermediate Goods Producing Firm

Intermediate good firms operate under a Cobb-Douglas-technology which is subject to an aggregate random productivity shock  $a_t$ .

$$y_{j,t} = a_t n_{j,t}^\alpha k_{j,t-1}^{1-\alpha} \quad (5.28)$$

Here  $n_{j,t}$  is the labor input employed in period  $t$  by a firm  $j$ , similarly  $k_{j,t-1}$  is the capital stock, and  $0 < \alpha < 1$  is labor's share.

Each intermediate goods producing firm faces costs of adjusting its price  $P_{j,t}$ . The adjustment costs can be measured in units of the final good and are given by

$$\frac{\phi_p}{2} \left[ \frac{P_{j,t}}{P_{j,t-1}} - 1 \right]^2 y_t \quad (5.29)$$

where  $\phi_p > 0$ . This equation captures both costs that stem from adjusting prices as well as costs that emerge through the misallocation of supply and demand, see Rotemberg (1982). These costs increase with greater price increases and also with the amount of the final good produced.<sup>6</sup>

Intermediate goods firms maximize their profits which are given by<sup>7</sup>

$$\Xi_{j,t} = P_{j,t} y_{j,t} - P_t w_t n_{j,t} - P_t z_t k_{j,t-1} - P_t \frac{\phi_p}{2} \left[ \frac{P_{j,t}}{P_{j,t-1}} - 1 \right]^2 y_t \quad (5.30)$$

where  $w_t$  is the aggregate real wage rate. Note that firms cannot supply more of the good  $j$  as is demanded by the final good firm. This demand is given in (5.26).

<sup>6</sup>This kind of modeling sticky prices is extensively used in the literature, see for example Dib and Phaneuf (2001), Gerke (2003) and Ireland (1997).

<sup>7</sup>The distinction between the producing and the pricing unit is not necessary in a model with price adjustment costs.

Inserting this restriction in the profit function as well as in the production function (5.28) allows to write down the intermediate goods firms' optimization problem which is a dynamic one due to the adjustment costs.

$$\begin{aligned} \max_{n_{j,t}, k_{j,t-1}, P_{j,t}} \quad & E_t \sum_{t=0}^{\infty} \beta^t \lambda_t \frac{\Xi_{j,t}}{P_t} \\ \text{s.t.} \quad & \left( \frac{P_{j,t}}{P_t} \right)^{-\epsilon_p} y_t = a_t n_{j,t}^\alpha k_{j,t-1}^{1-\alpha} \end{aligned} \quad (5.31)$$

$\beta^t \lambda_t / P_t$  is the pricing kernel. It is equal to the marginal value of an additional unit of profits to the household.<sup>8</sup> The Lagrangian for this problem can be written as follows:<sup>9</sup>

$$\begin{aligned} L_j = E_0 \left[ \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{P_t} \left[ P_{j,t} \left( \frac{P_{j,t}}{P_t} \right)^{-\epsilon_p} y_t - P_t w_t n_{j,t} - P_t z_t k_{j,t-1} \right. \right. \\ \left. \left. - P_t \frac{\phi_p}{2} \left( \frac{P_{j,t}}{P_{j,t-1}} - 1 \right)^2 y_t \right] \right. \\ \left. + \sum_{t=0}^{\infty} \beta^t \xi_t \left( a_t n_{j,t}^\alpha k_{j,t-1}^{1-\alpha} - \left( \frac{P_{j,t}}{P_t} \right)^{-\epsilon_p} y_t \right) \right] \end{aligned} \quad (5.32)$$

The first order conditions are given below:

$$\frac{\partial L_j}{\partial k_{j,t-1}} = -\beta^t \lambda_t z_t + \beta^t \xi_t (1 - \alpha) a_t n_{j,t}^\alpha k_{j,t-1}^{-\alpha} = 0 \quad (5.33)$$

$$\frac{\partial L_j}{\partial n_{j,t}} = -\beta^t \lambda_t w_t + \beta^t \xi_t \alpha a_t n_{j,t}^{\alpha-1} k_{j,t-1}^{1-\alpha} = 0 \quad (5.34)$$

$$\begin{aligned} \frac{\partial L_j}{\partial P_{j,t}} = \quad & \beta^t \lambda_t \left( \frac{P_{j,t}}{P_t} \right)^{-\epsilon_p} \frac{y_t}{P_t} - \epsilon_p \beta^t \lambda_t \left( \frac{P_{j,t}}{P_t} \right)^{-\epsilon_p} \frac{y_t}{P_t} \\ & - \beta^t \lambda_t \phi_p \left[ \frac{P_{j,t}}{P_{j,t-1}} - 1 \right] \frac{y_t}{P_{j,t-1}} \\ & + E_t \beta^{t+1} \lambda_{t+1} \phi_p \left[ \frac{P_{j,t+1}}{P_{j,t}} - 1 \right] \frac{P_{j,t+1}}{P_{j,t}^2} y_{t+1} \\ & + \beta^t \epsilon_p \xi_t \left( \frac{P_{j,t}}{P_t} \right)^{-\epsilon_p - 1} \frac{y_t}{P_t} = 0 \end{aligned} \quad (5.35)$$

The first two conditions can be rearranged to yield the familiar microeconomic conditions for profit maximization generalized to markup pricing:

$$\lambda_t z_t = \xi_t (1 - \alpha) a_t n_{j,t}^\alpha k_{j,t-1}^{-\alpha} \quad (5.36)$$

$$\lambda_t w_t = \xi_t \alpha a_t n_{j,t}^{\alpha-1} k_{j,t-1}^{1-\alpha} \quad (5.37)$$

<sup>8</sup>Formally it is given by  $\partial L_i / \partial \Xi_t$  in the household's optimization problem where  $\gamma_i = 1$ .

<sup>9</sup> $P_t w_t = W_t$  is the aggregate nominal wage rate which is the relevant wage for each intermediate goods firm. The same holds for  $z_t$  which is also not firm specific. In addition the Lagrange multiplier  $\xi_t$  is the same across all firms.

Dividing by  $\xi_t$  on both sides results in

$$\frac{\lambda_t}{\xi_t} z_t = (1 - \alpha) a_t n_{j,t}^\alpha k_{j,t-1}^{-\alpha} \quad (5.38)$$

$$\frac{\lambda_t}{\xi_t} w_t = \alpha a_t n_{j,t}^{\alpha-1} k_{j,t-1}^{1-\alpha} \quad (5.39)$$

where  $\lambda_t/\xi_t$  is the markup factor  $\mu_t$ . This reflects the market power of the firms since factor prices  $w_t, z_t$  are not just equal to the marginal products of labor and capital respectively.

In a symmetric equilibrium every firm will make the same choices so that

$$P_{j,t} = P_t, n_{j,t} = n_t, k_{j,t-1} = k_{t-1} \text{ for all } t \quad (5.40)$$

So (5.38) and (5.39) hold with all  $j$ 's eliminated. This means that the efficiency condition for the optimal price of the firms simplifies considerably because all ratios of  $P_{j,t}/P_t$  are then equal to one.

$$\begin{aligned} \lambda_t (1 - \epsilon_p) y_t - \lambda_t \phi_p \left[ \frac{P_t}{P_{t-1}} - 1 \right] \frac{P_t}{P_{t-1}} y_t \\ + E_t \beta \lambda_{t+1} \phi_p \left[ \frac{P_{t+1}}{P_t} - 1 \right] \frac{P_{t+1}}{P_t} y_{t+1} + \epsilon_p \xi_t y_t = 0 \end{aligned} \quad (5.41)$$

In case that there are no adjustment costs of prices, i.e.  $\phi_p = 0$ , the markup is constant and equal to  $\mu_t = \lambda_t/\xi_t = \mu = \lambda/\xi = \epsilon_p/(\epsilon_p - 1)$ .<sup>10</sup>

## 5.2.5 Market Clearing Conditions and Other Equations

The aggregate resource constraint is derived using the resource constraint of households, firms, the government and the monetary authority. Due to the adjustment costs of prices some resources have to be used to finance them so that the condition deviates from the standard one and is given by

$$y_t = c_t + i_t + \frac{\phi_p}{2} \left[ \frac{P_t}{P_{t-1}} - 1 \right]^2 y_t \quad (5.42)$$

where the assumption of a symmetric equilibrium has already been taken into account. It is well known that models like the one at hand imply multiple equilibria and sunspots because bonds are not determined. To escape this problem the household budget constraint is dropped and bonds are set to zero:  $b_t = 0$  for all  $t$ .

<sup>10</sup>In the steady state with zero inflation  $\mu$  is also equal to  $\epsilon_p/(\epsilon_p - 1)$  but irrespective of the value of  $\phi_p$ .

Real marginal cost  $\psi_t$  is just the reciprocal of the markup so that

$$\psi_t = \frac{1}{\mu_t} \quad (5.43)$$

From the definition of the markup  $\psi_t$  is thus linked to the Lagrange multipliers in the following way:

$$\psi_t = \frac{\xi_t}{\lambda_t} \quad (5.44)$$

The aggregate real wage is just the nominal wage divided by the price level:

$$w_t = \frac{W_t}{P_t} \quad (5.45)$$

### 5.2.6 The Monetary Authority

The model is closed by adding a monetary policy rule. Therefore an exogenous process for the money growth rate is considered. To achieve persistent but non permanent effects the level of money follows an AR(2)-process. Assume that money grows at a factor  $g_t$ :

$$M_t = g_t M_{t-1} \quad (5.46)$$

If  $\hat{g}_t$  follows an AR(1)-process  $\hat{g}_t = \rho_g \hat{g}_{t-1} + \epsilon_{g_t}$  then money will follow an AR(2)-process.<sup>11</sup> Note that inflation is zero at the steady state so also money growth is zero there ( $g = 1$ ).

There is another shock in the model, namely the productivity shock  $a_t$ . As mentioned above this shock can also act as a taste shock. So one can easily analyze the model's impulse responses to this productivity/ taste shock. Under these circumstances  $\hat{a}_t$  follows an AR(1)-process

$$\hat{a}_t = \rho_a \hat{a}_{t-1} + \epsilon_{a_t} \quad (5.47)$$

with  $\epsilon_{a_t}$  white noise and  $0 < \rho_a < 1$ .

### 5.2.7 The Steady State

Imposing the condition of constancy of the price level in the steady state ( $P_t = P_{t-1} = P$ ) on the nominal interest rate equation reveals the familiar condition from RBC models that  $\beta = 1/(1 + R)$ . In addition, as there is no steady state price inflation,  $R = r$ . The two period wage setting of the households implies  $W_0 = W_1$ . Using this in the wage index reveals that  $W_0 = W_1 = W$ . There is also no wage

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<sup>11</sup>A hat ( $\hat{\phantom{x}}$ ) represents the relative deviation of the respective variable from its steady state (see the Appendix).  $\rho_g$  lies between 0 and 1 and  $\epsilon_{g_t}$  is white noise.

inflation. Nevertheless the nominal wage rates of the households differ since in every period only half of the households adjust their wage while the other half is passive and cannot reoptimize.<sup>12</sup> The optimal steady state real wage rate of the optimizing households can be derived from (5.13).

$$\frac{W_0}{P} = -\frac{\epsilon_w}{\epsilon_w - 1} \frac{\partial u / \partial n}{\partial u / \partial c} \quad (5.48)$$

It is given by a constant markup  $\epsilon_w / (\epsilon_w - 1)$  over the marginal rate of substitution between consumption and labor  $-(\partial u / \partial n) / (\partial u / \partial c) = dc / dn$ . At the steady state each household's individual labor supply  $n_i$  is equal to aggregate labor supply  $n$  because  $n_i = (W_0 / W)^{\epsilon_w} n = n$  since  $W_0 = W$  for all  $i$ .

The capital accumulation equation tells us that  $\phi(i/k) = \delta$  at the steady state. It is assumed that  $\phi' = 1$  in steady state to ensure that Tobin's  $q$  is equal to one ( $q = 1/\phi'$ ). As a consequence of the requirement that the model with adjustment costs of capital should display the same steady state as the model without them  $i/k$  is equal to  $\phi(i/k)$ . Using this in the efficiency condition for capital it can be shown that the rental rate on capital is  $z = r + \delta$  as in a standard RBC model. With the help of (5.39) and the steady state for  $z$  it is possible to pin down  $k/n$  which amounts to

$$\frac{k}{n} = \left( \frac{r + \delta}{a} \frac{\mu}{1 - \alpha} \right)^{-1/\alpha} \quad (5.49)$$

Real marginal costs are determined by  $\psi = 1/\mu$  while  $\mu$  is given by the steady state of the efficiency condition for the optimal price (5.41). This results in  $\mu = \epsilon_p / (\epsilon_p - 1)$ .  $\psi$  can be used to calculate  $w$  using (5.39) as well:

$$w = \psi a \alpha \left( \frac{k}{n} \right)^{1-\alpha} \quad (5.50)$$

The calculation of the steady state value of consumption is tedious. From the production function one knows that labor productivity is given by

$$\frac{y}{n} = a \left( \frac{k}{n} \right)^{1-\alpha} \quad (5.51)$$

This productivity can be combined with the investment to capital ratio to calculate the investment share:

$$\frac{i}{y} = \left( \frac{i}{k} \frac{k}{n} \right) / \left( \frac{y}{n} \right) \quad (5.52)$$

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<sup>12</sup>Gerke (2003) considers a model with a positive steady state inflation rate which allows for further asymmetries. See the discussion later.

Now one can derive the consumption share using the aggregate resource constraint.

$$\frac{c}{y} = -\frac{i}{y} + 1 \quad (5.53)$$

Note that  $y = c + i$  at the steady state because  $P/P - 1 = 0$  in (5.42) so that the presence of adjustment costs does not have any influence. To get the level of  $c$  the level of  $y$  and  $i$  have to be determined:  $y = n \cdot y/n$ ,  $i = y \cdot i/y$ . Finally  $c = y - i$  is the consumption steady state value.

(5.48) can be used to calculate the preference parameter  $\Theta$  since  $n$  will be given exogenously. Using (5.22) the ratio of  $m$  over  $c$  depends only upon  $\beta, \eta$  and  $\nu$ .

$$m = c \left[ \frac{\eta}{1 - \eta} (1 - \beta) \right]^{\frac{1}{\nu - 1}} \quad (5.54)$$

In turn  $\Theta$  can be determined as a function of these parameters and  $c, m, w$  and  $n$  by solving (5.48).

$$\Theta = \frac{\epsilon_w - 1}{\epsilon_w} \left[ a (\eta c^\nu + (1 - \eta) m^\nu)^{\frac{1}{\nu}} \right]^{-\sigma} [\eta c^\nu + (1 - \eta) m^\nu]^{\frac{1}{\nu} - 1} \eta c^{\nu - 1} w n^{-\gamma} \quad (5.55)$$

### 5.2.8 Calibration

In order to compute impulse responses the parameters of the model have to be calibrated. It is possible to either specify  $\beta$  or  $r$  exogenously. Here  $\beta$  will be set to 0.99 implying a value of  $r$  of about 0.0101 per quarter which is in line with other values used for the real interest rate in the literature.  $\psi$  and  $\mu$  can be determined by fixing a value for the elasticity of the demand functions for the differentiated products,  $\epsilon_p$ . This elasticity being equal to 4 causes the static markup  $\mu = \epsilon_p / (\epsilon_p - 1)$  to be 1.33 which is in line with the study of Linnemann (1999) about average markups. The wage elasticity of the demand for the household's labor inputs  $\epsilon_w$  is given by 10, a value that is also used in Erceg (1997) as well as in Gerke (2003). In order to determine the steady state real wage  $w$  the productivity shock  $a$  has to be specified, along with calculating  $k/n$ , see below. As there is no information available about that parameter it is arbitrarily set at 10.  $n$  is specified to be equal to 0.25 implying that agents work 25 % of their non-sleeping time.

In the benchmark case,  $\sigma$ , the parameter governing the degree of risk aversion, is set to 2.  $\gamma$ , which is equal to the inverse of the intertemporal elasticity of labor supply, is chosen to be equal to 1, as in Gerke (2003). The parameters  $\nu$  and  $\eta$  are calibrated by estimating an empirical money demand function the form of which is implied by the efficiency conditions of the household. This functional form is



obtained by solving (5.22) for  $m_t$  and taking logarithms:

$$\ln m_t = \frac{1}{\nu - 1} \ln \frac{\eta}{1 - \eta} + \frac{1}{\nu - 1} \ln \left( \frac{R_t}{1 + R_t} \right) + \ln c_t \quad (5.56)$$

Estimates of Chari, Kehoe and McGrattan (2000) reveal that  $\eta = 0.94$  and  $\nu = -1.56$ . They use US data from Citibase covering 1960:1-1995:4 regressing the log of consumption velocity on the log of the interest rate variable  $R_t/(1 + R_t)$ . Since the focus is on the qualitative results of the model the money demand function is not estimated for specific German data. The implied value of  $\Theta$  is then equal to 0.0035 while  $m/c$  is given by 2.06.

As this model considers the role of capital accumulation several other technological parameters have to be calibrated. The most common one is the depreciation rate  $\delta$  which is set to 0.025 implying 10% depreciation per year. Labor's share  $\alpha$  is 0.64 whereas the elasticity of Tobin's  $q$  with respect to  $i/k$  is set to -0.5.<sup>13</sup> This value is also used in King and Wolman (1996). The presence of adjustment costs of capital dampens the volatility of investment and is a common feature in equilibrium business cycle models. Using  $r, \delta, a, \alpha$  and  $\psi$  the ratio  $k/n$  can be determined. The sensitivity parameter of the intermediate goods producing firms' adjustment cost function  $\phi_p$  is equal to 3.95 in the benchmark case, the same value used in Gerke (2003). Ireland (1997) estimates a value of 4.05 for  $\phi_p$  using US data and a maximum likelihood approach. The model studied here implies that the steady state costs of price adjustment are essentially zero because steady state inflation is zero.

For the exogenous money growth process  $\rho_g = 0.5$  is used. As the focus of the chapter is on the persistence effects of money growth shocks productivity shocks will not be considered. But they can be used to check whether the model displays reasonable impulse responses to technology shocks.

### 5.3 Impulse Response Functions

The solution is conducted using an extended version of the algorithm of King, Plosser and Rebelo (2002) which allows for singularities in the system matrix of the reduced model. The theoretical background of this algorithm is developed in King and Watson (1999) whereas computational aspects and the implementation are discussed in King and Watson (2002).

How is a monetary policy shock transmitted in this model? An intuition could be the following. A positive money growth shock leads to higher resources of the

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<sup>13</sup>It can be shown that this elasticity is given by  $-[\phi''/\phi' \cdot (i/k)]$ .

household. Thus household's demand for goods rises and this in turn causes a rise in the labor demand of the firms to enable them to increase production. Higher consumption reduces the marginal utility of consumption and higher labor demand lowers the marginal disutility of labor. In turn – as the household sets its optimal wage as a constant markup over the ratio of some weighted average of marginal utilities of labor to some weighted average of marginal utilities of consumption – it raises its wage rate.<sup>14</sup> At the same time the household takes into account that a higher wage would lower the demand for its specific labor since its relative wage would be higher than that of those households who will not change their wage.<sup>15</sup> This substitution effect together with the income effect - due to the reduced labor income - dampens the rise in the optimal wage of the household so that the wage rate will not rise proportionally with aggregate demand. Since firms set prices as a markup over marginal costs and since marginal costs are determined by the aggregate wage rate which itself is influenced by the optimal wage rate the rise in prices will be dampened as well. In addition prices will react weaker because firms face price adjustment costs.

Figures 5.1 – 5.4 show the impulse responses of selected variables to a one percent shock to the money growth rate. They overall confirm the intuition above. Figure 5.1 displays the reaction of output, consumption, investment and labor. The responses are strongest in the period of the shock and approach smoothly the steady state. They display considerable persistence compared to the model with Taylor price staggering in Chapter 3. The contract multiplier is equal to 0.52 which can be considered as relatively high compared with the results of Andersen (2004).<sup>16</sup> All these aggregates converge to their steady states from above showing no cyclical reaction. In Figure 5.2 real marginal costs react moderately (0.32% deviation from steady state) and show a hump. Unfortunately the nominal interest rate rises so that the model does not display the liquidity effect. Figure 5.3 reveals that inflation peaks in the initial period but also that the price level does not overshoot here. Due to the small increase in real marginal costs firms raise their prices only by a small amount which gives rise to a persistent reaction of the price level. The real wage is hump-shaped and countercyclical in this model version. It is the only variable having a cyclical impulse response. The household adjusts its optimal wage carefully causing a persistent wage response as well. As a consequence the wage index  $\widehat{W}_t$  rises smoothly. Real money balances rise and show a small hump 11 quarters after

<sup>14</sup>See (5.19) and (5.48) and note that  $\partial u/\partial n < 0$ .

<sup>15</sup>See (5.3).

<sup>16</sup>Remember that his values range between 0.55 and 0.87.

the money growth shock.

Three results are of particular importance. First, there is only a moderate response of real marginal costs although the inclusion of the capital stock changes the marginal cost function. It depends also on the rental of capital  $z_t$  so the dynamics are not only determined by the real wage rate  $w_t$ . As has already been discussed in Chapter 3  $\psi_t$  can be written as follows:

$$\psi_t = \left(\frac{w_t}{\alpha}\right)^\alpha \left(\frac{z_t}{1-\alpha}\right)^{1-\alpha} \frac{1}{a_t} \quad (5.57)$$

Here  $\widehat{w}_t$  declines by 0.19% due to the money growth shock because the price level rises stronger than the nominal wage rate. The rental rate  $\widehat{z}_t$  has a 1.23 percentage deviation in the initial period resulting in the 0.32% reaction of  $\widehat{\psi}_t$  (because of the weighting parameters  $\alpha$  and  $1-\alpha$  in (5.57)). Second, prices show in turn a smooth and also moderate reaction to increased marginal costs. There is no overshooting as usually observed in MIU-model specifications like those in Chapter 2 or 4. This must be due to the assumption of adjustment costs of prices. Manipulating (D.13) reveals that the dynamics of the price level are much simpler here than in the model with Taylor price staggering considered in Chapter 3.

$$\beta\mu\phi_p\widehat{\pi}_{t+1} = \beta\mu\phi_p\widehat{\pi}_t + \epsilon_p\widehat{\psi}_t \quad (5.58)$$

The structure of this equation is very similar to the New Keynesian Phillips curve (3.40) which explains the smooth response of prices.<sup>17</sup> The moderate increase in real marginal costs translates into a persistent increase in the price level and inflation. Third, the optimal reset wage of the household  $\widehat{W}_{0,t}$  is not cyclical as is the optimal reset price  $\widehat{P}_{0,t}$  of intermediate goods producing firms under Taylor staggering in Chapter 3. This result is confirmed by Kim (2003) in his Section 5. He can show in a simplified version of his model that under Taylor *wage* staggering only the real wage and output follow first order stochastic difference equations with a positive autoregressive coefficient while under Taylor *price* staggering only this parameter is always negative. In a model with *both* Taylor price and wage contracts output and prices are both cyclical in spite of the presence of wage staggering, see Figure 8 in Kim (2003), p. 49. This result can be confirmed but details are not presented here due to space considerations. Thus the success of the model at hand is also due to the assumption of a different mechanism of price stickiness, namely adjustment costs of prices.

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<sup>17</sup>Note that it is not *equal* to a New Keynesian Phillips curve.

Gerke (2003) considers a similar model with four-period wage staggering. The main differences between his model and the one studied here are - besides the length of the wage contracts - a positive steady state inflation rate and the absence of capital adjustment costs. In his benchmark model he uses a utility function that is additively separable in consumption and leisure as in Walsh (1998), p. 69, where consumption and money are aggregated by some kind of Cobb-Douglas function. The impulse responses in this model are cyclical for output, investment, labor and the optimal wage. In a sensitivity analysis he uses a different utility function that is very similar to the preference specification (5.6). In this case output displays a hump-shaped response in Gerke's model. This result does not hold here. In light of Ascari (2000) it may be conjectured that the reason is possibly the positive inflation rate in Gerke's model. Ascari finds that the negative relationship between persistence in output and the inflation rate is also affected by the intertemporal elasticity of substitution of labor and the elasticity of substitution between the differentiated goods in a *non-linear* way. So it depends on the specific values of these parameters used whether the model generates plausible persistent output reactions.

Gerke also reports that results are not sensitive with regard to the price adjustment cost parameter  $\phi_p$  in his benchmark model. In my model the opposite is true. Interestingly the model here can generate considerable persistence when  $\phi_p$  is very high implying high costs of price adjustment for the firms. Figure 5.5 shows the responses for  $\phi_p = 100$ . The contract multiplier rises to 0.80 compared to 0.52. But when adjustment costs of capital are zero as in Gerke a higher  $\phi_p$  does not increase but *decrease* persistence. This result is very interesting as it shows that there is a non-linear relationship between price and capital adjustment costs: Only for zero or low adjustment costs of capital a higher  $\phi_p$  leads to a lower contract multiplier and less persistence in output. For moderate and high capital adjustment costs a higher  $\phi_p$  causes a higher contract multiplier. Figure 5.6 represents the result for zero adjustment costs and  $\phi_p = 100$  where output is even cyclical. These results are similar to those in Gerke's benchmark model, see his Figures 6.2 and 6.3, especially concerning the relative strength of the reactions and the smoothness of consumption. The cyclicity emerges only under high price adjustment costs here. In Figure 5.7  $\phi_p$  is set to zero along with zero adjustment costs. This leads to a quite persistent output response. Consumption is even hump-shaped now but investment reacts too strongly relative to output. The contract multiplier is equal to 0.31. In the benchmark model with very high adjustment costs of capital (with an elasticity of Tobin's  $q$  with respect to the investment to capital ratio equal to -500) persistence can only

be increased by a small amount. Investment's reaction to a money growth shock is now extremely small, see the scale in Figure 5.8, while at the same time output does only have a slightly higher persistence with a contract multiplier of 0.57, compared to 0.52.

A higher intertemporal elasticity of substitution for consumption - a lower value for  $\sigma$  - also enhances the persistence of output, confirming Gerke's results. In this case output and consumption react stronger than investment and households raise their wage rate very strongly leading to an overshooting of  $\widehat{W}_{0,t}$ .

In an early paper on the role of wage staggering in a dynamic stochastic setting Erceg (1997) stresses the role of the wage elasticity of the demand for the households' differentiated labor inputs  $\epsilon_w$  and the form of the money demand function to create persistence in output. In his model both prices and wages are set in a staggered way for four periods. He argues that in a model with capital accumulation a high value of  $\epsilon_w$  is not sufficient to explain a persistent output reaction to a money growth shock. In addition the money demand function has to be income based with an income elasticity equal to one. But in a model like the one at hand the money demand function implied by (5.22) and given in (5.56) is consumption based and the implied income elasticity would be lower than one as consumption varies much less than output in response to money growth shock. Erceg proposes in turn a model with adjustment costs of capital at the firm level and claims that in this case output reacts with considerable persistence to a money growth shock. The difference to the model at hand is the assumption in Erceg that firms accumulate their own capital and not households. Moreover adjustment costs of capital are modeled differently: Firms operate using the effective stock of capital which is given by subtracting a quadratic term in new investment from  $k_{j,t}$ . The model at hand can generate a persistent output reaction *although* the money demand function is consumption based and for a moderate value of  $\epsilon_w$ , as Figure 5.1 reveals. There is no need for a higher elasticity of money demand as in Erceg. Variations of the wage elasticity of the demand for households' labor  $\epsilon_w$  change the benchmark results considerably. Using Erceg's value would correspond to  $\epsilon_w = 33.\bar{3}$  and results in a contract multiplier of 0.65. Real money balances are hump-shaped in this case. Interestingly output persistence is sensitive with regard to the price elasticity of the demand for intermediate goods  $\epsilon_p$ . With  $\epsilon_p = 1.1$  which implies a very low elasticity and an unrealistic high markup factor of 11 the model creates a persistent output impulse response, see Figure 5.9, with a contract multiplier equal to 0.84.

Andersen (2004) stresses the role of capital accumulation as an important prop-

agation mechanism. He develops a model that has an analytical solution and can show which technology and preference parameters are important for a persistent output reaction. His model does not assume that individual labor supply plays a decisive role for wage formation as in my model. Specifically he uses some kind of wage bargaining model where unions trade off wages and employment. He integrates the idea that involuntary unemployment plays an important role, especially in a European context, in an otherwise quite standard dynamic stochastic equilibrium model. His main result is that neither capital accumulation nor nominal contracts alone can generate plausible impulse responses but that the interaction of both mechanisms can strengthen persistence up to unit roots. The result that a higher capital accumulation parameter increases persistence does not hold here. Higher capital accumulation would imply a lower value of the depreciation rate  $\delta$ . Using  $\delta = 0.01$  generates an even less persistent output reaction with a contract multiplier of 0.48. It must be noted that Andersen's capital accumulation is very unusual. He employs a parameterized version of the adjustment cost function  $\phi(I/K)$  where  $\phi(I/K) = (I/K)^\delta$ ,  $K$  and  $I$  in levels. His equation for the evolution of the capital stock then reads

$$K_{t+1} = K_t \left( \frac{I_t}{K_t} \right)^\delta = K_t^{1-\delta} I_t^\delta \quad 0 \leq \delta \leq 1 \quad (5.59)$$

Andersen argues that higher capital accumulation is then associated with a higher  $\delta$ . But  $\delta = 1$  implies  $K_{t+1} = I_t$  which means full depreciation of the capital stock whereas  $\delta = 0$  implies  $K_{t+1} = K_t$  which is constancy of capital. So it is not clear why a *higher*  $\delta$  leads to stronger capital accumulation. Changing nevertheless  $\delta$  to 0.1 indeed leads to a higher contract multiplier of 0.58 compared to 0.52 in the benchmark case. Variations of his marginal value of wage income to unions cannot be conducted in the model at hand so that implications cannot be compared.

Huang and Liu (2002) study a model with both wage and price staggering and conclude that also in a model augmented by capital accumulation wage staggering has a much higher potential to generate persistence in output than price staggering. Their setup deviates with regard to capital adjustment costs. These are modeled in a similar way as the price adjustment costs in (5.29) without dependence on  $y_t$ . Huang and Liu again stress the influence of  $\epsilon_w$  on the model outcome. The higher the wage elasticity of household's labor supply the higher the contract multiplier. They yield a value as high as 0.56 for  $\epsilon_w = 6$  while in my model the value would be 0.46 for that case. This confirms the intuition from above: The higher the demand elasticity for the differentiated labor inputs the higher would be the loss in the demand for labor

of the specific household and the higher the stickiness of the optimal wage rate, the closer the reset wage to the existing one. In turn the higher will be the persistence of output.<sup>18</sup>

The model is sensitive to variations in  $\gamma$ , the inverse of the intertemporal elasticity of substitution of labor. For  $\gamma = 0$  this elasticity tends to infinity and *lowers* the contract multiplier of output to 0.38, see Figure 5.10. Interestingly, for very high values of  $\gamma$  and thus low intertemporal elasticities of substitution of labor the multiplier increases slightly up to 0.59 for  $\gamma = 100000000$ . This contradicts results of Ascari (2003a) who finds that the degree of persistence seems to be extremely insensitive to the value of the intertemporal elasticity of labor. Additionally, the standard direction of the influence of  $1/\gamma$  is reversed.

Results of Bénassy (2003b) concerning the ability to produce hump-shaped responses do not carry over to this model. Bénassy can derive a condition for a hump in employment that depends on the relation between the probability that households can adjust their wage and the autocorrelation coefficient of the money growth process  $\rho_g$ . This would require setting  $\rho_g > 0.5$ .<sup>19</sup> Varying  $\rho_g$  appropriately does not help in creating humps. Remember that Bénassy considers Calvo contracts which could be the reason for his result.

## 5.4 Business Cycle Properties

In order to explore the implications for the business cycle properties one has to specify the standard deviation of the AR(1)-process for money growth. Here the value estimated in Cooley and Hansen (1995), p.201, is used.<sup>20</sup> It implies a value of 0.0000792 for the variance  $\sigma_g^2$ . Table 5.1 shows the results for the benchmark model after HP-filtering with  $\lambda = 1600$ .<sup>21</sup>

$\sigma_{\hat{x}}$  again denotes the percentage standard deviation of  $\hat{x}$  whereas  $\sigma_{\hat{x}}/\sigma_{\hat{y}}$  measures the respective standard deviation relative to that of output  $\hat{y}$ . The next two columns report the autocorrelations for one and two lags of the respective aggregate. The

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<sup>18</sup>See also Ascari (2003a) on this point. He can show analytically that for  $\epsilon_w \rightarrow \infty$  output has a unit root.

<sup>19</sup>Otherwise the contract length would have to be changed complicating the model setup considerably.

<sup>20</sup>It is not intended to take the model explicitly to the data because of its overwhelming simplicity. This justifies the use of Cooley and Hansen's parameter values.

<sup>21</sup>Remember that all values in the tables have been rounded using the computer output. So it is possible that the relative standard deviations deliver a different value when using the values in the table.

Table 5.1: Moments in the Benchmark Wage Staggering Model

$\hat{x}_t$			autocorrelation		cross correlation of $\hat{x}_t$ with $\hat{y}$ in				
	$\sigma_{\hat{x}}$	$\sigma_{\hat{x}}/\sigma_{\hat{y}}$	1	2	$t-2$	$t-1$	$t$	$t+1$	$t+2$
$\hat{y}_t$	0.84	1.00	0.38	0.06	0.06	0.38	1.00	0.38	0.06
$\hat{i}_t$	2.08	2.48	0.38	0.06	0.05	0.37	1.00	0.39	0.07
$\hat{c}_t$	0.54	0.65	0.38	0.06	0.07	0.39	1.00	0.37	0.05
$\hat{n}_t$	1.32	1.58	0.38	0.05	0.03	0.36	1.00	0.39	0.08
$\hat{w}_t$	0.24	0.29	0.12	-0.09	0.54	0.55	-0.50	-0.34	-0.22
$\hat{\mu}_t$	0.40	0.48	0.68	0.24	-0.31	-0.73	-0.90	-0.30	0.00
$\hat{R}_t$	0.95	1.13	0.37	0.10	0.09	0.36	1.00	0.39	0.07
$\hat{\psi}_t$	0.40	0.48	0.68	0.24	0.31	0.73	0.90	0.30	-0.00
$\hat{\Pi}_t$	0.82	0.98	0.53	0.15	0.17	0.54	0.98	0.36	0.04
$\hat{P}_t$	1.85	2.21	0.90	0.70	0.47	0.40	0.16	-0.27	-0.43

remaining columns display the cross correlations with output. A variable  $\hat{x}$  is leading  $\hat{y}$  if the absolute value of the correlation  $\rho(\hat{x}_t, \hat{y}_{t+i})$  is highest for  $i > 0$ . Accordingly a variable  $\hat{x}$  is lagging  $\hat{y}$  if the absolute value of the correlation  $\rho(\hat{x}_t, \hat{y}_{t+i})$  has a maximum for  $i < 0$ . In case that this correlation is positive one speaks of a procyclical variable while it is called anticyclical if it is negative. If the maximum correlation occurs at lag 0 ( $i = 0$ ) the variable is moving with the cycle. The model performs quite well along the lines of relative standard deviations of consumption and investment. Output's absolute volatility is 0.84 which is higher than in the Calvo staggering model of Chapter 3. The nominal rate has the lowest absolute and relative variability of all models considered. It is even lower than in the MIU-model with GHH preferences and a high labor supply elasticity and lower than empirically observed. With the exception of the real wage rate all aggregates are persistent with positive autocorrelations.  $\hat{w}_t$  is contemporaneously negatively correlated with output but has a tendency to lag procyclically with one lag (0.55). Maußner (1994), p. 22, finds that the real wage lags procyclically with three quarters so that the model can explain this behavior quite good. Real marginal costs are again strongly correlated with output. Unfortunately this model produces also perfect correlations between consumption, investment, labor and output which is counterfactual. Even the nominal rate is perfectly correlated with  $\hat{y}_t$ . In sum, this model does quite a good job in replicating business cycle stylized facts with respect to absolute and relative standard deviations as well as autocorrelations especially when taking into account its simplicity. Cross correlations are not matched well. The results suggest



that money growth shocks can account at least partly for the business cycle.

## 5.5 Conclusions

A stochastic dynamic general equilibrium model has been proposed to explain persistent reactions of output and inflation to a money growth shock. Wages are set in a staggered way for two periods while there are also adjustment costs of prices at the firm level. The results confirm the finding from the literature that sticky wages have a higher potential in explaining persistence in output than sticky prices. The wage elasticity of the demand for differentiated labor inputs in conjunction with the assumption of immobile labor plays an important role for creating this result.

But the chapter also demonstrates that sticky prices can contribute to persistence when prices are costly to adjust for the firms. In the presence of adjustment costs of capital price adjustment costs intensify the persistence of output. When there are no costs of adjusting the capital stock there is no persistence even in a model with wage staggering.

In addition, the chapter shows that a model with Taylor *wage* staggering does not produce cyclical impulse responses as a model with Taylor *price* staggering. This result is also confirmed by Kim (2003). Moreover sticky prices that emerge through adjustment costs appear to have a higher potential to generate persistence in models with costly capital adjustment. In models with Taylor price staggering of two periods capital adjustment costs do not enhance the persistence of output to a money growth shock, as demonstrated in Chapter 3.

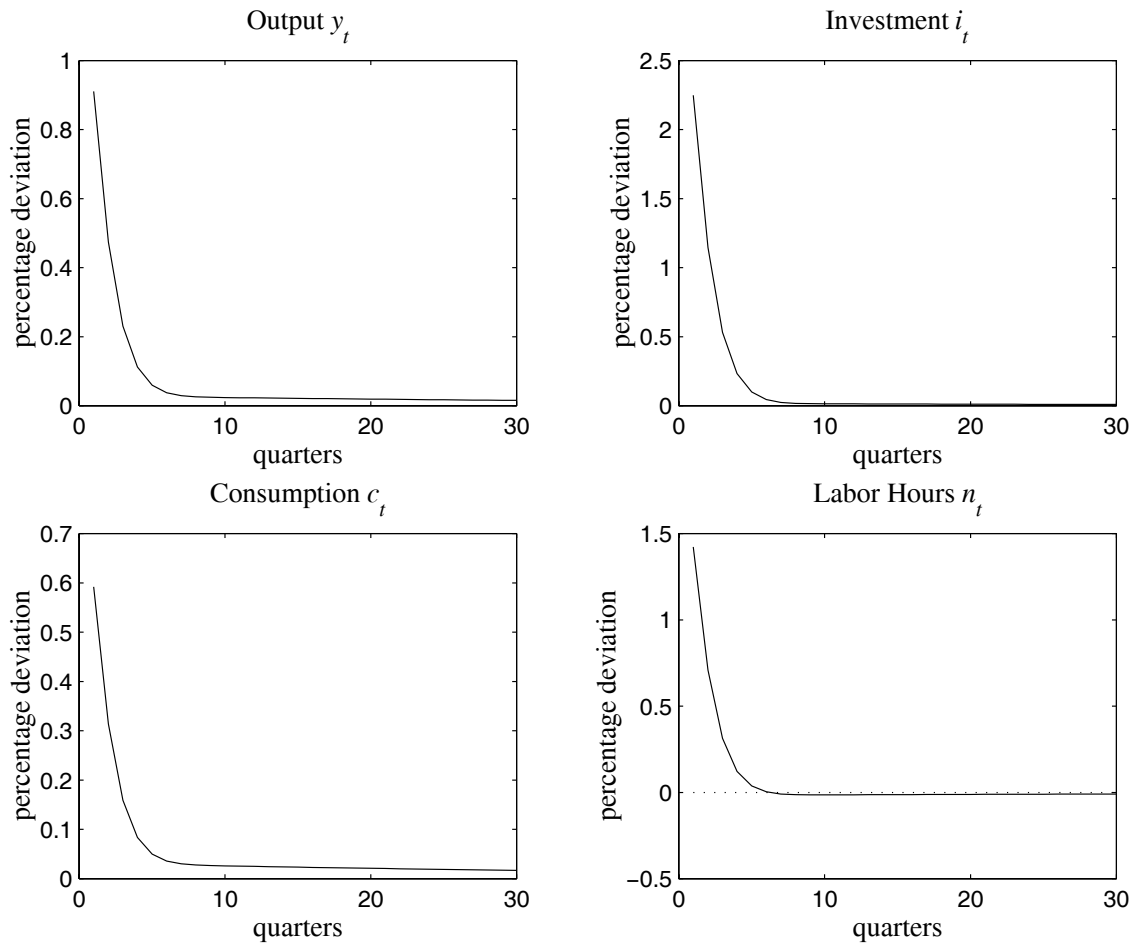


Figure 5.1: Impulse Response Functions for  $\hat{y}_t, \hat{i}_t, \hat{c}_t, \hat{n}_t$

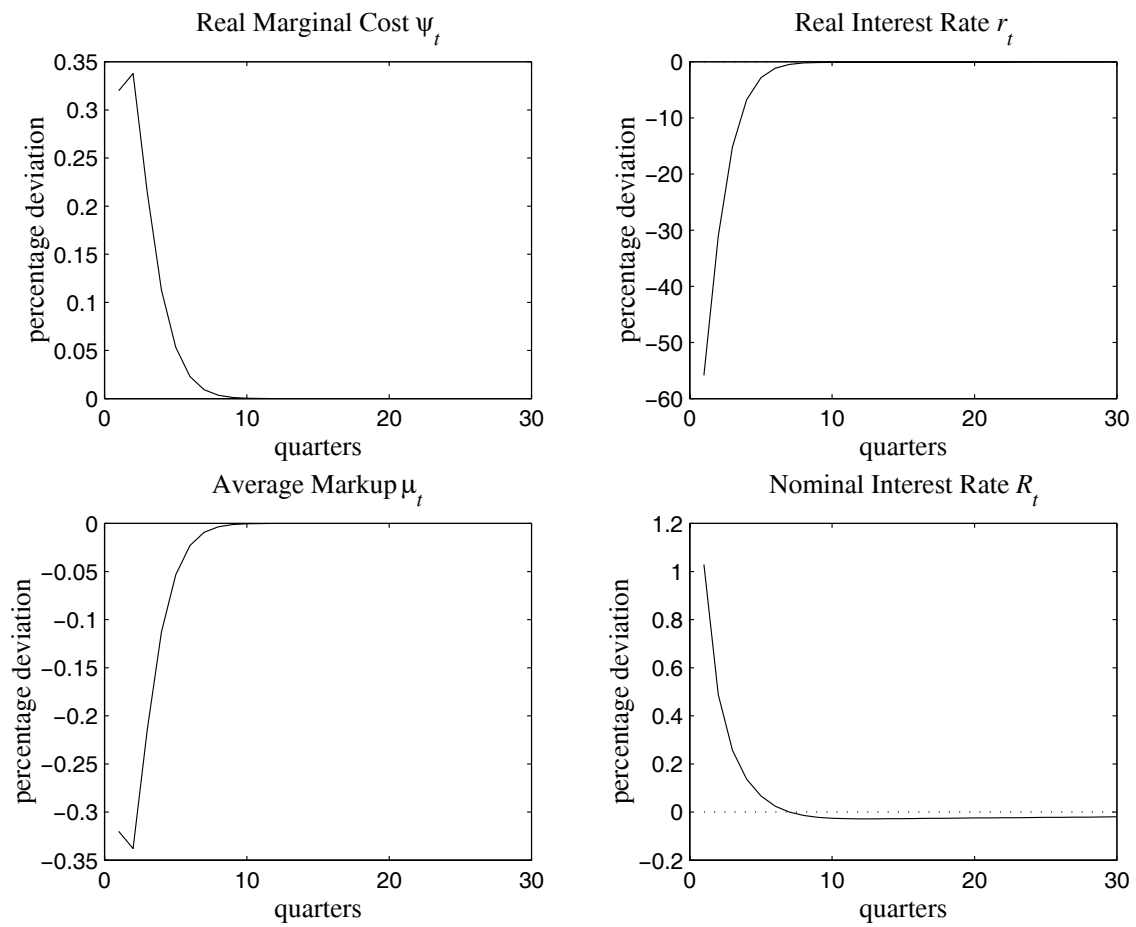


Figure 5.2: Impulse Response Functions for  $\hat{\psi}_t, \hat{r}_t, \hat{\mu}_t, \hat{R}_t$

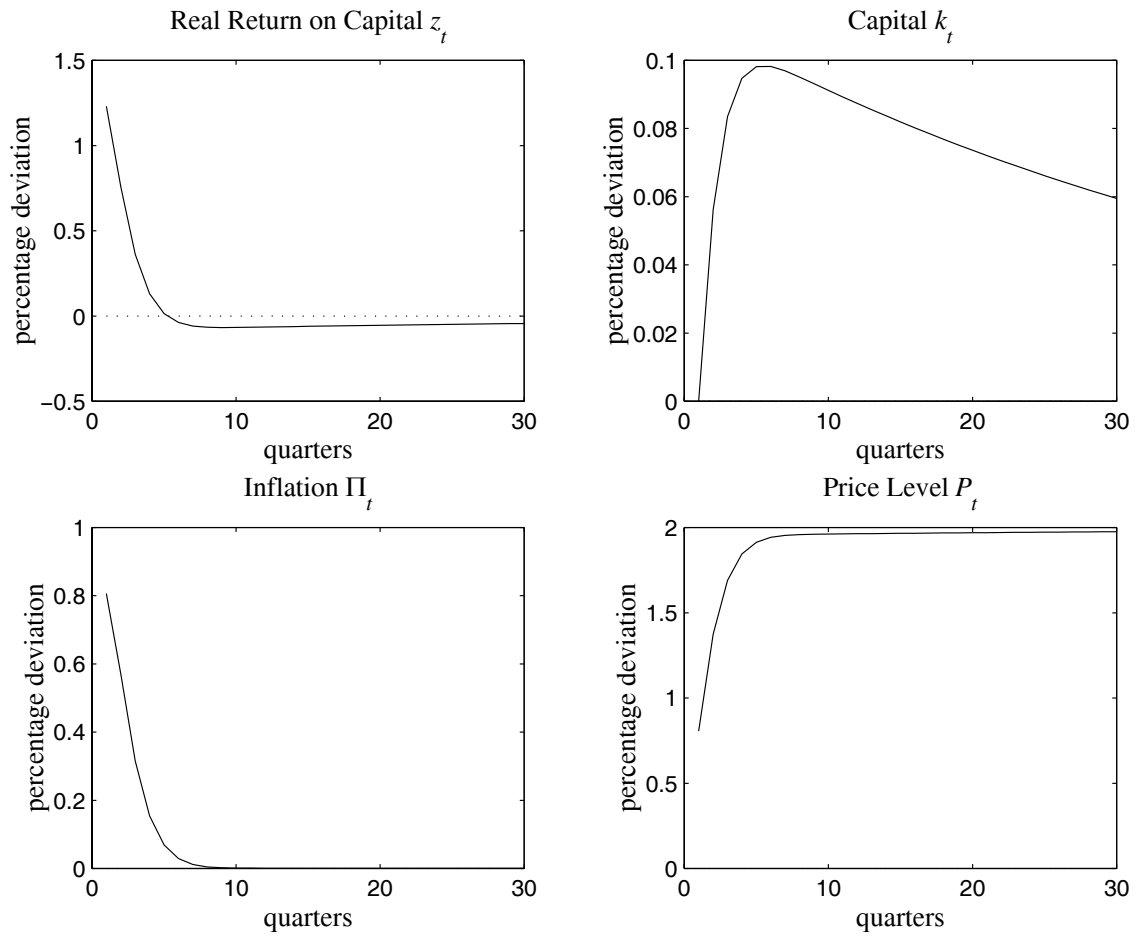


Figure 5.3: Impulse Response Functions for  $\hat{z}_t, \hat{k}_t, \hat{\Pi}, \hat{P}_t$

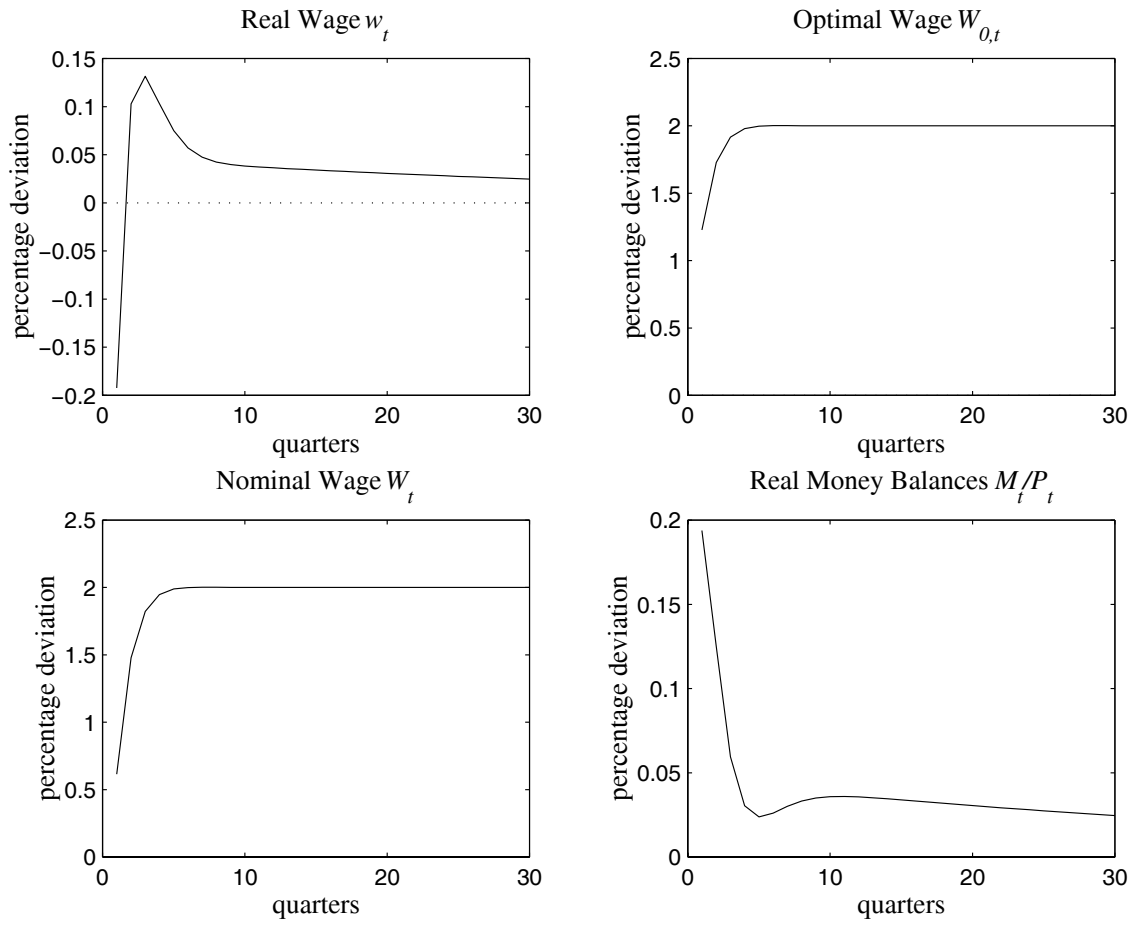


Figure 5.4: Impulse Response Functions for  $\widehat{w}_t, \widehat{W}_{0,t}, \widehat{W}_t, \widehat{M}_t - \widehat{P}_t$

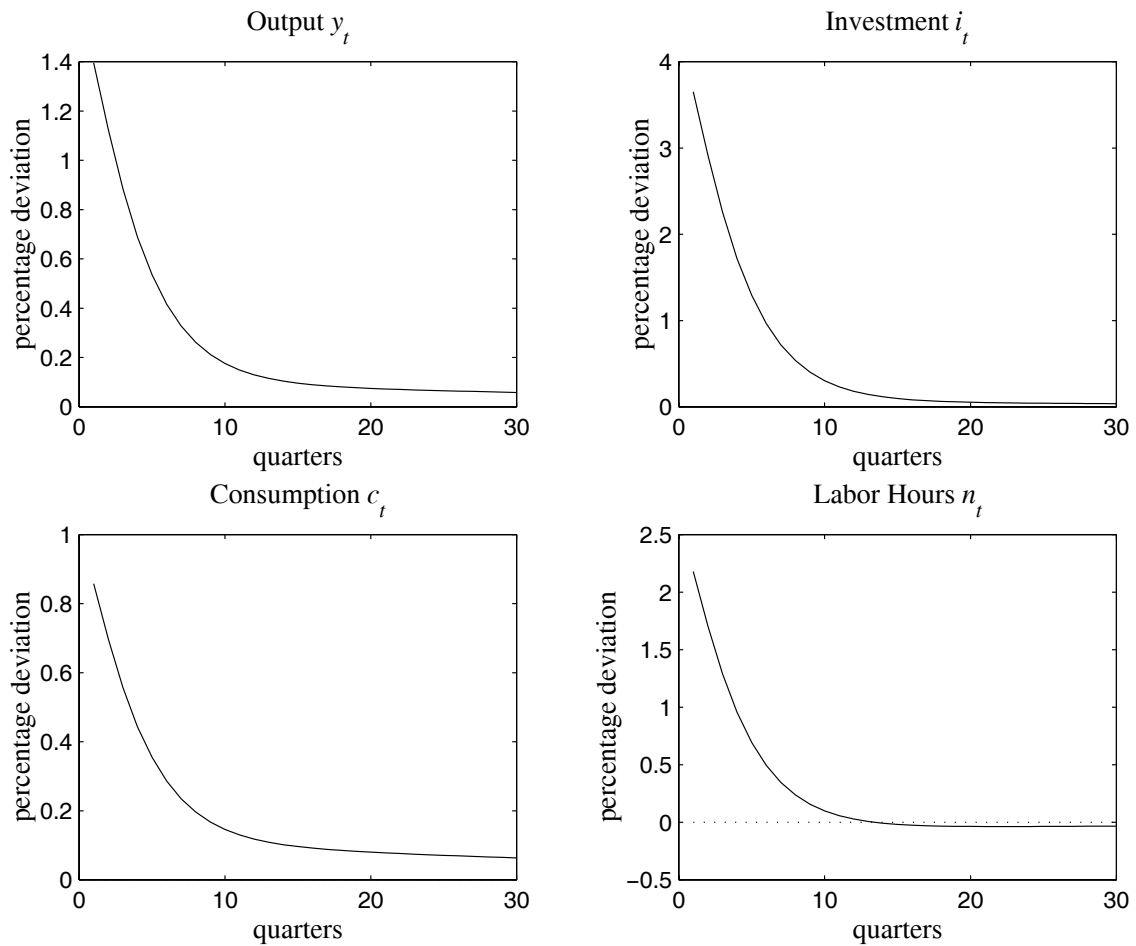


Figure 5.5: Impulse Response Functions for  $\hat{y}_t, \hat{i}_t, \hat{c}_t, \hat{n}_t$ , very high price adjustment costs ( $\phi_p = 100$ ) and benchmark capital adjustment costs (Tobin's  $q$  elasticity of -0.5)

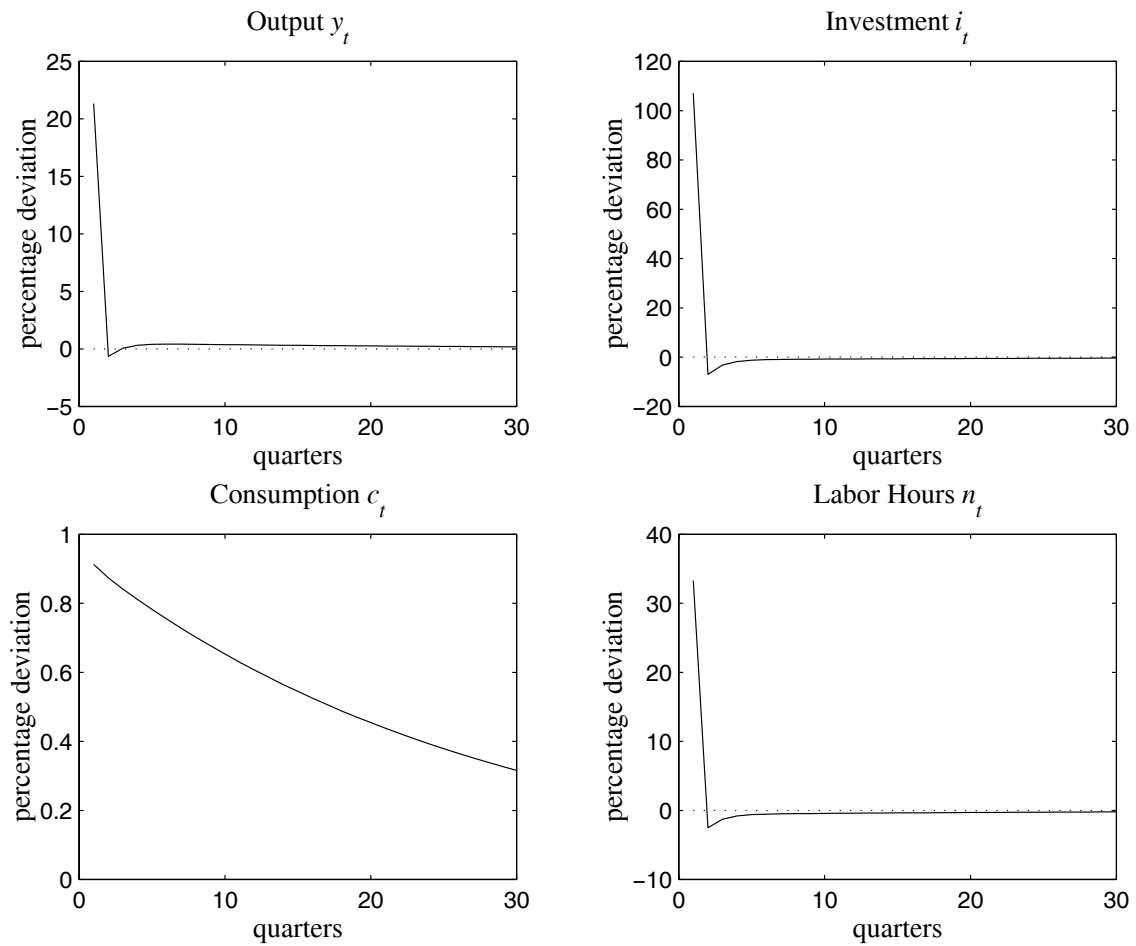


Figure 5.6: Impulse Response Functions for  $\hat{y}_t, \hat{i}_t, \hat{c}_t, \hat{n}_t$ , very high price adjustment costs ( $\phi_p = 100$ ) and zero capital adjustment costs (Tobin's  $q$  elasticity of 0)

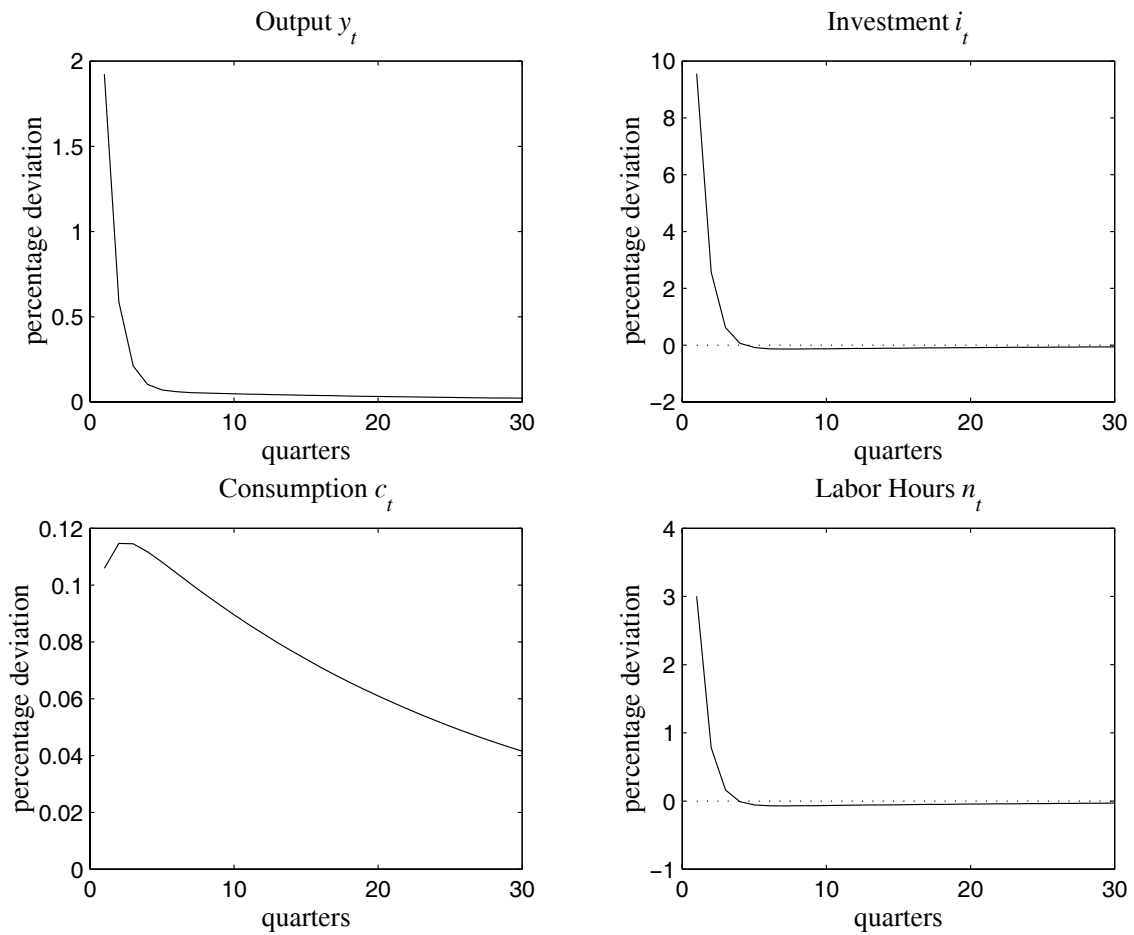


Figure 5.7: Impulse Response Functions for  $\hat{y}_t, \hat{i}_t, \hat{c}_t, \hat{n}_t$ , zero price adjustment costs ( $\phi_p = 0$ ) and zero capital adjustment costs (Tobin's  $q$  elasticity of 0)



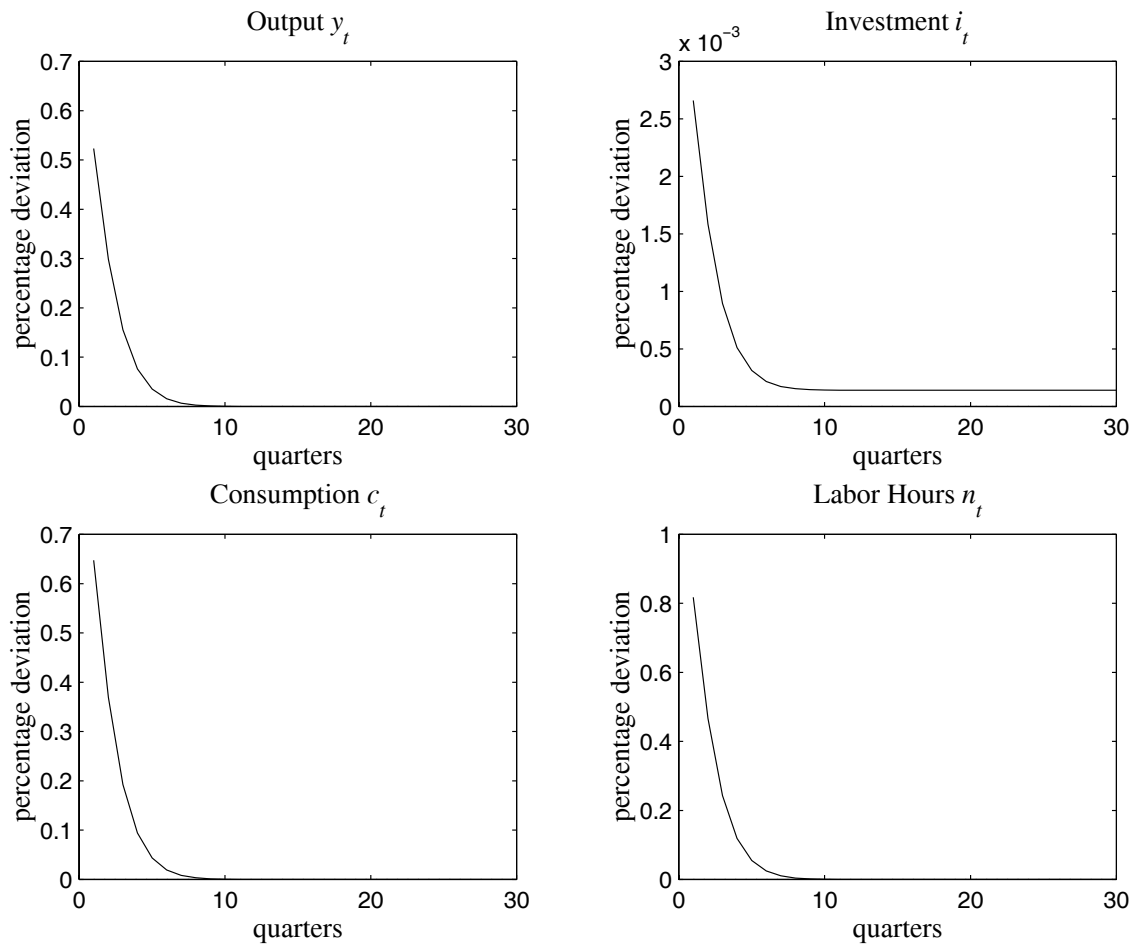


Figure 5.8: Impulse Response Functions for  $\hat{y}_t, \hat{i}_t, \hat{c}_t, \hat{n}_t$ , benchmark price adjustment costs ( $\phi_p = 3.95$ ) and high capital adjustment costs (Tobin's  $q$  elasticity of -500)

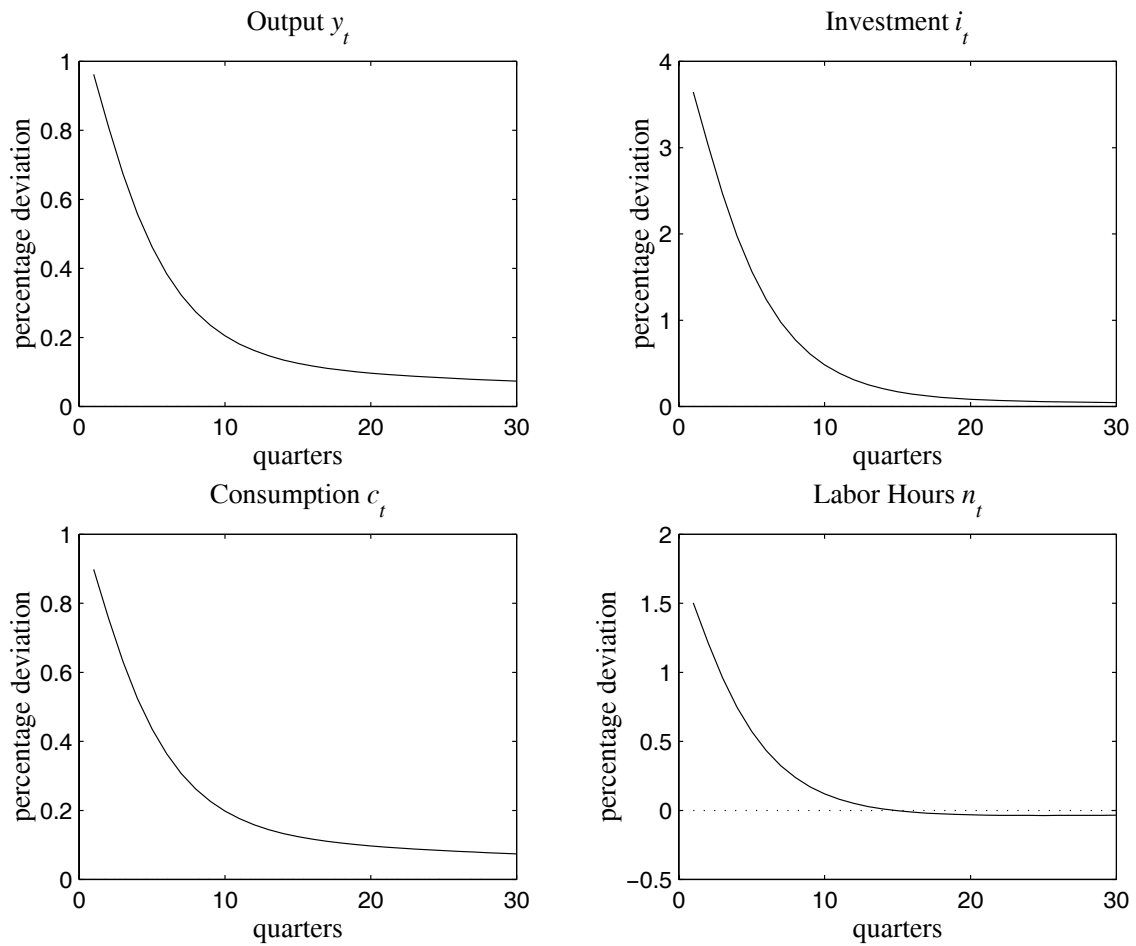


Figure 5.9: Impulse Response Functions for  $\hat{y}_t, \hat{i}_t, \hat{c}_t, \hat{n}_t$ , very low price elasticity ( $\epsilon_p = 1.1$ )

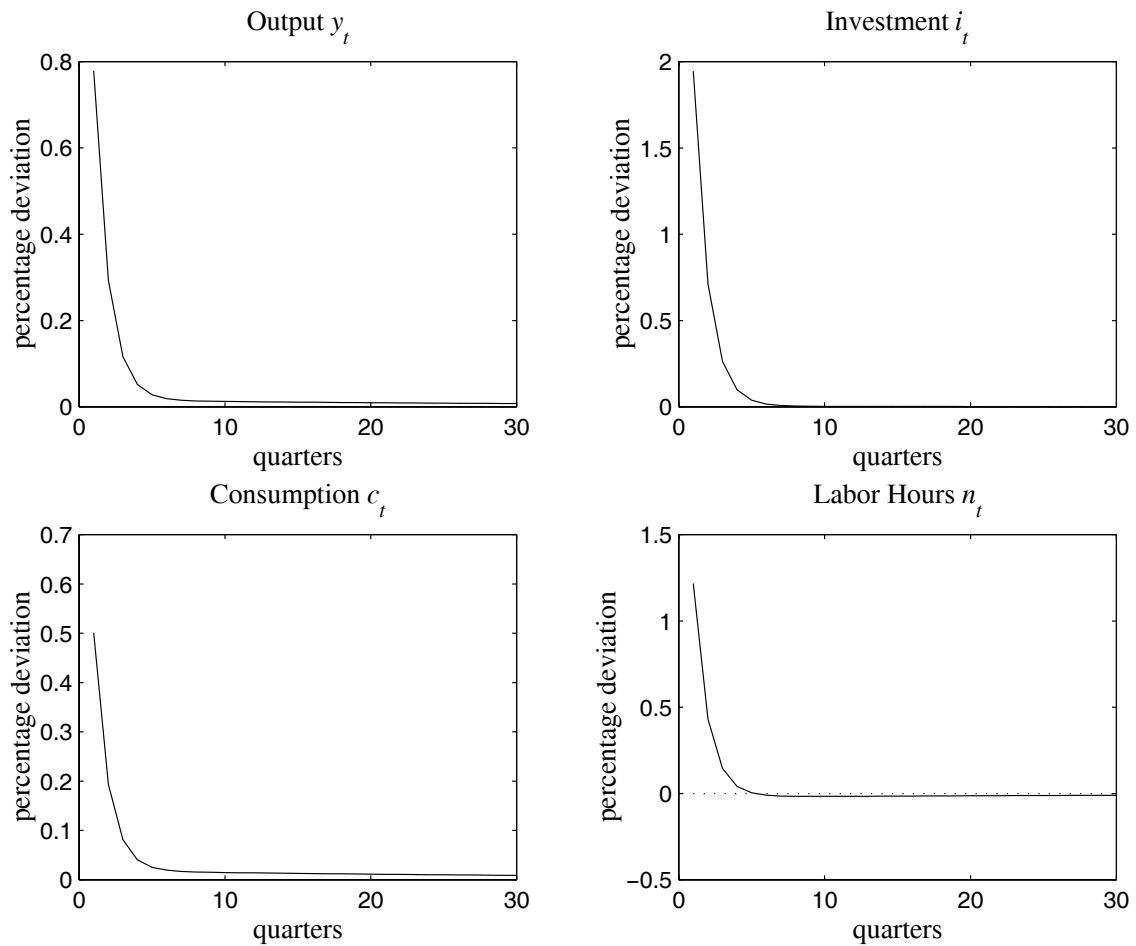


Figure 5.10: Impulse Response Functions for  $\hat{y}_t, \hat{i}_t, \hat{c}_t, \hat{n}_t$ , infinite labor supply elasticity ( $\gamma = 0$ )

## Chapter 6

# Optimal Monetary Policy in a Monetary Stochastic Dynamic General Equilibrium Model with Price Staggering

### 6.1 Introduction

In the last two or three years macroeconomists have intensified their interest in analyzing monetary policy. This is mainly due to the adoption of inflation targeting in several countries in the world, among them the United Kingdom and Sweden. These countries have been particularly successful in driving down their inflation rates in the 1990s. Svensson (1999) gives an excellent overview of the literature on that topic.

The task of the monetary authority in these models is to regulate aggregate demand to stabilize output and inflation. Output stabilization is necessary because sticky prices deteriorate aggregate demand causing ‘Okun gaps’. High and variable inflation is generally viewed as resulting in increased relative price volatility and in other costs of production or exchange and thus has to be avoided. In order to determine how the central bank will balance the ‘Okun gaps’ against the costs of inflation a loss function in these two arguments is assumed. The specific optimal monetary policy rule depends on the specific form of this loss function and on the detailed structure of the economy. In general the policy cannot completely eliminate fluctuations in output and inflation.

In the model analyzed here the central bank focuses just on the stabilization

of the price level. This policy is optimal although the macroeconomic equilibrium is inefficient because firms have market power and ‘Okun gaps’ can arise through price stickiness. The model combines two strands of research: the public finance approach to policy analysis and features of the ‘New Keynesian’ macroeconomics. This combination is quite new to the macroeconomic literature.

In general the public finance approach concentrates on identifying distortions and on measuring the resulting costs to individuals, sometimes called ‘Harberger triangles’. So far ‘Okun gaps’ have not been analyzed using the public finance approach because they were considered not to be caused by microeconomic distortions.<sup>1</sup> This is fundamentally different here. Making use of two central New Keynesian features, namely the optimizing approach to sticky prices, as e.g. in Calvo (1983), and the modeling of firms in an imperfect competition environment, as e.g. in Rotemberg (1987), as extensively used in the previous chapters, and embedding this into a dynamic general equilibrium model of the form used in the Real Business Cycle literature Okun gaps in fact arise from microeconomic distortions. This ‘New Neoclassical Synthesis’ makes it possible to use Harberger-type analysis to identify distortions and to characterize optimal policy.

So the two main building blocks of the model are a stochastic dynamic general equilibrium model similar to those analyzed in the previous chapters and a Ramsey problem which is solved by the central bank where the objective function of the monetary authority is given by the utility function of the representative household.

The model at hand cannot (yet) be used to answer questions like ‘What is the trade-off between inflation variability and output variability under alternative specifications of an interest rate rule?’. For this purpose the structure of the model has to be improved upon. So far the only exogenous disturbances are productivity shocks. To produce a reasonable outcome some other shocks as energy or government spending shocks have to be included. What the model can answer are questions concerning the response of the optimal policy to a productivity disturbance and its influence on output, inflation and interest rates.

The result of King and Wolman (1999) that the central bank can achieve a complete stabilization of the price level does not hold in this version of the model. Their result is mainly due to the assumed utility function which implies the absence of any substitution between consumption and leisure.

The chapter proceeds as follows: Section 6.2 describes the model and its underlying structure in detail. Section 6.3 discusses the policy problem as a social planner

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<sup>1</sup>See Tobin (1977): “It takes a heap of ‘Harberger triangles’ to fill an ‘Okun gap’.”

exercise of the central bank. Section 6.4 demonstrates on a theoretical basis why prices will not be constant here. The model is calibrated and impulse response functions will be analyzed focusing on their optimal character. Section 6.5 concludes and gives some suggestions for future research.<sup>2</sup>

## 6.2 The Model

In the model monopolistically competitive firms are assumed to set final product prices optimally. Supply satisfies demand at these prices. Firms do so in a staggered manner: each firm sets its price for two periods with half of the firms adjusting each period.<sup>3</sup> So far the model is in line with Taylor (1980). Stickiness in individual prices causes stickiness in the price level and therefore there is room for monetary policy to combat this nonneutrality.

The model can be viewed as representative for a class of models in the spirit of the ‘New Neoclassical Synthesis’ (see also Goodfriend and King (1997) for a detailed description of this new approach). It combines the above mentioned New-Keynesian-style price stickiness with an otherwise neoclassical business cycle model in the tradition of the RBC literature. To facilitate the analysis the model abstracts from capital accumulation considerations. The production functions will therefore all be constant returns to scale in the single production factor labor. There will be no money demand distortions caused by positive nominal interest rates in order to focus the analysis completely on the effects of monetary policy (money supply side) that operate through sticky prices. This is justified here since empirically money nearly bears an interest equal to other assets so that there is no distortion from holding money for the representative household. The model does not consider fiscal policy. Changes in the money supply are thus offset by transfers to or lump-sum taxes from the household.

### 6.2.1 The Household

Consumers are assumed to have preferences over consumption  $c_t$  and leisure  $(1 - n_t)$  given by the utility function

$$\sum_{t=0}^{\infty} \beta^t u(c_t, n_t, a_t) \quad (6.1)$$

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<sup>2</sup>This Chapter has already been published in Gail (2002a).

<sup>3</sup>The analysis can be easily extended to multi-period price setting.

The momentary utility function used by King and Wolman (1999) is given by

$$u(c_t, n_t, a_t) = \ln \left( c_t - \frac{a_t \theta}{1 + \gamma} n_t^{1+\gamma} \right) \quad (6.2)$$

Here  $a_t$  is a preference shock that also acts like a productivity shock.  $\theta$  and  $\gamma$  are positive parameters,  $\beta$  is the discount factor. This function is familiar from the analysis of Greenwood, Hercowitz and Huffman (1988) and has been labeled GHH preferences. It has the special property that hours worked only depend upon the real wage and not upon consumption. It is a simplified version of the preference specification (2.1) with  $\sigma = 1$ .

The utility function analyzed in this chapter is the standard CRRA function also used in the CIA-setup of Chapter 2.  $\sigma$  governs the degree of risk aversion and  $\zeta$  measures the relative weight of consumption for the representative agent.

$$u(c_t, n_t, a_t) = \frac{\left[ a_t c_t^\zeta (1 - n_t)^{1-\zeta} \right]^{1-\sigma} - 1}{1 - \sigma} \quad (6.3)$$

It should be noted that in contrast to the standard use of this utility function there is a disturbance  $a_t$  acting like a preference shock.<sup>4</sup>

The intertemporal optimization problem for the household is to maximize lifetime utility subject to an intertemporal budget constraint. The household is assumed to have access to a bond market and to hold money. Its budget constraint is therefore given by

$$P_t c_t + M_t + B_t + P_t w_t h \left( \frac{M_t}{P_t c_t} \right) = P_t w_t n_t + \Xi_t + (1 + R_{t-1}^M) M_{t-1} + (1 + R_{t-1}) B_{t-1} \quad (6.4)$$

where

$$\Xi_t = \int_0^1 \Xi_{j,t} dj \quad (6.5)$$

are the nominal profits of the intermediate goods producing firms. The uses of wealth are nominal consumption  $P_t c_t$ , holdings of money balances  $M_t$  and bonds  $B_t$ .  $h(M_t/(P_t c_t))$  is the time spent on transactions activity, i.e. for purchasing goods while the real wage  $w_t$  is considered to be the opportunity cost of a unit of time

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<sup>4</sup>King and Wolman (1999) argue that it is necessary in (6.2) to have  $a_t$  affecting equally production and preferences in order to achieve balanced growth. This is doubtful because the model does not explicitly account for growth aspects as, e.g., in King, Plosser and Rebelo (1988). The use of  $a_t$  in (6.3) affecting preferences is a new feature not analyzed in the literature in the context of optimal monetary policy before.

spent shopping.<sup>5</sup> The household has several sources of its wealth. It earns labor income  $P_t w_t n_t$  working in the market at the real wage rate  $w_t$ . As money is assumed to be interest bearing it can spend its money holdings carried over from the previous period augmented by the interest on these money balances  $(1 + R_{t-1}^M) M_{t-1}$ . There are previous period bond holdings including the interest on them  $(1 + R_{t-1}) B_{t-1}$ . The interest rate on bonds  $R_{t-1}$  is endogenous while the one on money  $R_{t-1}^M$  is set exogenously by the monetary authority and is slightly below the bond rate. Finally the household receives profits from the intermediate goods firms  $\Xi_t$ .

The Lagrangian for the household (index  $H$ ) can be written as follows:

$$\begin{aligned} L_H = & \sum_{t=0}^{\infty} \beta^t u(c_t, n_t, a_t) \\ & + \sum_{t=0}^{\infty} \beta^t \lambda_t \left[ w_t n_t + (1 + R_{t-1}^M) \frac{P_{t-1}}{P_t} m_{t-1} + \frac{\Xi_t}{P_t} \right. \\ & \left. + (1 + R_{t-1}) \frac{P_{t-1}}{P_t} b_{t-1} - c_t - m_t - b_t - w_t h\left(\frac{m_t}{c_t}\right) \right] \end{aligned} \quad (6.6)$$

Here small variables indicate real quantities, i.e. for example  $b_t = B_t/P_t$ . This function is optimized over  $c_t, n_t, m_t$  and  $b_t$ . The first order conditions will be important for the optimal policy of the central bank so they are reported below.

$$\frac{\partial L_H}{\partial c_t} = \beta^t \frac{\partial u(c_t, n_t, a_t)}{\partial c_t} - \beta^t \lambda_t + \beta^t \lambda_t w_t \frac{m_t}{c_t^2} h'\left(\frac{m_t}{c_t}\right) = 0 \quad (6.7)$$

$$\frac{\partial L_H}{\partial n_t} = \beta^t \frac{\partial u(c_t, n_t, a_t)}{\partial n_t} + \beta^t \lambda_t w_t = 0 \quad (6.8)$$

$$\frac{\partial L_H}{\partial m_t} = -\beta^t \lambda_t \left[ 1 + \frac{w_t}{c_t} h'\left(\frac{m_t}{c_t}\right) \right] + E_t \left[ \beta^{t+1} \lambda_{t+1} (1 + R_t^M) \frac{P_t}{P_{t+1}} \right] = 0 \quad (6.9)$$

$$\frac{\partial L_H}{\partial b_t} = -\beta^t \lambda_t + E_t \left[ \beta^{t+1} \lambda_{t+1} (1 + R_t) \frac{P_t}{P_{t+1}} \right] = 0 \quad (6.10)$$

The third condition defines implicitly the money demand function.  $h'(\cdot)$  is the derivative of  $h$  with respect to  $m/c$ . Combining this equation with (6.10) allows to analyze the nature of money demand in this model.

$$\lambda_t \left[ \frac{w_t}{c_t} h'\left(\frac{m_t}{c_t}\right) \right] = \beta E_t \left[ \lambda_{t+1} (R_t^M - R_t) \frac{P_t}{P_{t+1}} \right] \quad (6.11)$$

When the rate on money approaches the rate on bonds ( $R_t^M \cong R_t$ ) real costs of holding money go to zero. This implies that  $h'(\cdot)$  is zero. Since only the derivative

<sup>5</sup>For a detailed discussion of the shopping-time approach see King and Wolman (1996).



of a constant is zero real money holdings per unit of consumption must be constant:  $m_t/c_t = k$ . Hence money demand is given by

$$m_t = kc_t \Leftrightarrow M_t = kP_t c_t \quad (6.12)$$

$k$  represents the satiation level of cash balances. Money supply always satisfies the demand for cash. As there are lump-sum taxes and transfers available for the household they can be used to offset changes in the money supply. The efficiency condition for bond holdings establishes a relation between the nominal interest rate (on bonds) and the price level. Rearranging terms yields

$$(1 + R_t) = E_t \left[ \frac{\lambda_t}{\lambda_{t+1}} \frac{1}{\beta} \frac{P_{t+1}}{P_t} \right] \quad (6.13)$$

Supposed the Fisher equation is valid the real interest rate  $r_t$  is implicitly defined as

$$(1 + r_t) = E_t \left[ \frac{\lambda_t}{\lambda_{t+1}} \frac{1}{\beta} \right] \quad (6.14)$$

because  $E_t [P_{t+1}/P_t]$  equals one plus the rate of expected inflation.

Combining the first two efficiency conditions and remembering that  $h'(\cdot) = 0$  reveals that the marginal rate of substitution between consumption and labor is equal to the real wage.

$$-\frac{\partial u(c_t, n_t, a_t) / \partial n_t}{\partial u(c_t, n_t, a_t) / \partial c_t} = \frac{dc_t}{dn_t} = w_t \quad (6.15)$$

## 6.2.2 The Finished Goods Producing Firm

The firm producing the final good  $c_t = y_t$  in the economy uses  $c_{j,t}$  units of each intermediate good  $j \in [0, 1]$  purchased at price  $P_{j,t}$  to produce  $c_t$  units of the finished good. The production function is assumed to be a CES aggregator as in Dixit and Stiglitz (1977) with  $\epsilon > 1$ .

$$c_t = \left( \int_0^1 c_{j,t}^{(\epsilon-1)/\epsilon} dj \right)^{\epsilon/(\epsilon-1)} \quad (6.16)$$

The firm maximizes its profits over  $c_{j,t}$  given the above production function and given the price  $P_t$ . So the problem can be written as

$$\max_{c_{j,t}} \left[ P_t c_t - \int_0^1 P_{j,t} c_{j,t} dj \right] \text{ s.t. } c_t = \left( \int_0^1 c_{j,t}^{(\epsilon-1)/\epsilon} dj \right)^{\epsilon/(\epsilon-1)} \quad (6.17)$$

The first order conditions for each good  $j$  imply

$$c_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{-\epsilon} c_t \quad (6.18)$$

where  $-\epsilon$  measures the constant price elasticity of demand for each good  $j$ . Since the firm operates under perfect competition it does not make any profits. Inserting the demand function into the profit function and imposing the zero profit condition reveals that the only price  $P_t$  that is consistent with this requirement is given by

$$P_t = \left( \int_0^1 P_{j,t}^{(1-\epsilon)} dj \right)^{1/(1-\epsilon)} \quad (6.19)$$

In case that prices are fixed for just two periods and assuming that all price adjusting producers in a given period choose the same price the consumption aggregate can be written as

$$c_t = c(c_{0,t}, c_{1,t}) = \left( \frac{1}{2} c_{0,t}^{(\epsilon-1)/\epsilon} + \frac{1}{2} c_{1,t}^{(\epsilon-1)/\epsilon} \right)^{\epsilon/(\epsilon-1)} \quad (6.20)$$

where  $c_{j,t}$  can then be interpreted as the quantity of a good consumed in period  $t$  whose price was set in period  $t - j$ . Similarly in the two period price setting case to be explored in detail in the next section the price equation simplifies. With prices set for two periods half of the firms adjust their price in period  $t$  and half do not. Moreover all adjusting firms choose the same price. Then  $P_{j,t}$  is the nominal price at time  $t$  of any good whose price was set  $j$  periods ago and  $P_t$  is the price index at time  $t$  and is given by

$$P_t = \left( \frac{1}{2} P_{0,t}^{1-\epsilon} + \frac{1}{2} P_{1,t}^{1-\epsilon} \right)^{1/(1-\epsilon)} \quad (6.21)$$

### 6.2.3 The Intermediate Goods Producing Firm

Intermediate good firms can be considered to consist of a producing and a pricing unit. The producing unit operates under a technology that is linear in labor  $n_{j,t}$  and subject to random productivity shocks  $a_t$ .<sup>6</sup>

$$y_{j,t} = c_{j,t} = a_t n_{j,t} \quad (6.22)$$

Here  $n_{j,t}$  is the labor input employed in period  $t$  by a firm who set the price in period  $t - j$ . Firms always meet the demand for their product, that is  $y_{j,t} = c_{j,t}$ . Those who do not adjust their prices in a given period can be interpreted as passive while those who do adjust do so optimally.

<sup>6</sup>There are no diminishing returns to labor.

The pricing unit sets prices to maximize the present discounted value of profits whereas the producing unit chooses labor to minimize costs.<sup>7</sup> In case of the models considered here there is no capital so the costs are solely given by the wage bill. Thus minimizing  $P_t w_t n_{j,t}$  with respect to  $n_{j,t}$  subject to the production function implies for the total cost function  $TC_{j,t}$ <sup>8</sup>

$$TC_{j,t} = \frac{P_t w_t c_{j,t}}{a_t} \quad (6.23)$$

With only one factor of production one can just express the labor input by manipulating the production function so that  $n_{j,t} = c_{j,t}/a_t$  and insert this into the wage bill equation. It is useful for further calculations to define nominal marginal cost as  $\Psi_t$  which is equal to  $(\partial TC_{j,t}/\partial c_{j,t}) = P_t w_t/a_t$ . Thus real marginal costs are given by  $\psi_t = w_t/a_t$ . With a relative price defined by  $p_{j,t} = P_{j,t}/P_t$  real profit  $\xi_{j,t} = \Xi_{j,t}/P_t$  for a firm of type  $j$  is equal to

$$\xi_{j,t} = p_{j,t} c_{j,t} - w_t n_{j,t} \quad (6.24)$$

Using the demand function for the intermediate goods ( $c_{j,t} = p_{j,t}^{-\epsilon} c_t = a_t n_{j,t}$ ) and the definition of real marginal costs given above the profit function can be rewritten as

$$\xi_{j,t} = \xi(p_{j,t}, c_t, \psi_t) = p_{j,t} c_{j,t} - \psi_t c_{j,t} = c_{j,t} (p_{j,t} - \psi_t) = p_{j,t}^{-\epsilon} c_t (p_{j,t} - \psi_t) \quad (6.25)$$

When prices are fixed for two periods the firm has to take into account the effect of the price chosen in period  $t$  on current and future profits. The price in period  $t+1$  will be affected by the gross inflation rate  $\Pi_{t+1}$  between  $t$  and  $t+1$  ( $\Pi_{t+1} = P_{t+1}/P_t$ ).

$$p_{1,t+1} = \frac{p_{0,t}}{\Pi_{t+1}} \quad (6.26)$$

The optimal relative price has to balance the effects due to inflation between profits today and tomorrow. This intertemporal maximization problem is formally given by

$$\begin{aligned} \max_{p_{0,t}} E_t & \left[ \xi(p_{0,t}, c_t, \psi_t) + \beta \frac{\lambda_{t+1}}{\lambda_t} \xi(p_{1,t+1}, c_{t+1}, \psi_{t+1}) \right] \\ \text{s.t.} \quad & p_{1,t+1} = \frac{p_{0,t}}{\Pi_{t+1}} \end{aligned} \quad (6.27)$$

The term  $\lambda_{t+1}/\lambda_t$  is equal to the ratio of future to current marginal utility of labor and the respective real wage ratio (derived in the household's optimization problem)

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<sup>7</sup>The model deviates in this respect from the standard textbook model in which profits are maximized over the quantity.

<sup>8</sup>Remember that the wage is perfectly flexible in a competitive input market. So there is no index  $j$  for  $w_t$  and  $P_t$  which means that they are not firm-specific.

and considered to be - in conjunction with  $\beta$  - the appropriate discount factor for real profits.<sup>9</sup> The efficiency condition for this problem is given by

$$0 = \frac{\partial \xi(p_{0,t}, c_t, \psi_t)}{\partial p_{0,t}} + \beta E_t \left( \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial \xi(p_{1,t+1}, c_{t+1}, \psi_{t+1})}{\partial p_{1,t+1}} \frac{1}{\Pi_{t+1}} \right) \quad (6.28)$$

Multiplying this equation by  $p_{0,t}$  and  $\lambda_t$  produces a more symmetric form of the efficiency condition that will be used in the next section to derive the model solution.

$$0 = \lambda_t p_{0,t} \frac{\partial \xi(p_{0,t}, c_t, \psi_t)}{\partial p_{0,t}} + \beta E_t \left( \lambda_{t+1} p_{1,t+1} \frac{\partial \xi(p_{1,t+1}, c_{t+1}, \psi_{t+1})}{\partial p_{1,t+1}} \right) \quad (6.29)$$

Using (6.25) one can solve this condition for the optimal price to be set in period  $t$ . This yields a forward-looking form of the price equation and is in that respect similar to the one in Taylor (1980).

$$p_{0,t} = \frac{\epsilon}{\epsilon - 1} \frac{\lambda_t c_t \psi_t + \beta E_t \lambda_{t+1} (P_{t+1}/P_t)^\epsilon c_{t+1} \psi_{t+1}}{\lambda_t c_t + \beta E_t \lambda_{t+1} (P_{t+1}/P_t)^{\epsilon-1} c_{t+1}} \quad (6.30)$$

The optimal relative price  $p_{0,t}$  depends upon the current and future real marginal costs, the gross inflation rate, current and future consumption as well as today's and tomorrow's interest rates captured by the  $\lambda$ 's.

#### 6.2.4 Constraints of the Monetary Authority

The objective of the monetary authority is to maximize welfare which means here maximizing the utility of the representative agent. In the absence of any distortions any rate of inflation would coincide with an optimal policy. But in this setup there are monopolistic competition and staggered prices. So the authority has to offset - in principle - the effects of these two frictions. It is constraint by technology and resource conditions as well as the price setting behavior of the firms.

It is assumed that the central bank follows an optimal plan under commitment. Fiscal policy instruments are not available so a first best allocation cannot be achieved. The purpose is to isolate the characteristics of an optimal monetary policy without a discussion of fiscal issues.

Three resource conditions have to be considered. Consumption of a good whose price was set  $j$  periods ago cannot exceed production of that good.

$$c_{j,t} \leq a_t n_{j,t} \quad \text{for } j = 0, 1 \quad (6.31)$$

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<sup>9</sup>See also Chapter 2 on this point again.

The consumption aggregator for a firm setting its price for two periods is given by (6.20) and repeated here.

$$c_t \leq \left( \frac{1}{2} c_{0,t}^{(\epsilon-1)/\epsilon} + \frac{1}{2} c_{1,t}^{(\epsilon-1)/\epsilon} \right)^{\epsilon/(\epsilon-1)} \quad (6.32)$$

The agent's time endowment is  $n_t$  and can be used for production of goods whose prices were set in period  $t$  and  $t - 1$ .

$$n_t = \frac{1}{2} n_{0,t} + \frac{1}{2} n_{1,t} \leq 1 \quad (6.33)$$

The household equally splits its time for producing goods whose prices were set in the actual period and the period before.<sup>10</sup>

The quantities the monetary authority chooses must be consistent with those of the monopolistic price setting firms. Formally this is achieved via an implementation constraint. The central bank must make sure that the firms will in fact set their quantities as the optimal plan implies. It has to induce the firms to choose those quantities which are consistent with an optimal monetary policy. So it takes into account the optimality condition in (6.29) which is repeated here for convenience.

$$\lambda_t p_{0,t} \frac{\partial \xi_{0,t}}{\partial p_{0,t}} + \beta E_t \lambda_{t+1} p_{1,t+1} \frac{\partial \xi_{1,t+1}}{\partial p_{1,t+1}} = 0 \quad (6.34)$$

This condition can be expressed in a more compact way making use of the function  $x$  which only depends on real quantities.

$$x(c_{0,t}, c_t, n_t, a_t) + \beta E_t x(c_{1,t+1}, c_{t+1}, n_{t+1}, a_{t+1}) = 0 \quad (6.35)$$

Using the demand function  $c_{j,t} = p_{j,t}^{-\epsilon} c_t$  one can eliminate relative prices. Real marginal costs are substituted by  $w_t/a_t$  and the real wage  $w_t$  is eliminated by use of the equality with the rate of substitution between consumption and leisure (see (6.15)). This yields:

$$x(c_{j,t}, c_t, n_t, a_t) = \lambda_t c_t \left[ (1 - \epsilon) \left( \frac{c_{j,t}}{c_t} \right)^{1-1/\epsilon} + \epsilon \frac{1 - \zeta}{\zeta} \frac{1}{a_t} \frac{1}{1 - n_t} \left( \frac{c_{j,t}}{c_t} \right) c_t \right] \quad (6.36)$$

As  $\lambda_t$  is equal to the marginal utility of consumption in period  $t$  it is in that respect also a function of  $c_t, n_t$  and  $a_t$ . This formula deviates from the one in King and Wolman (1999) in that there is also a direct influence of  $a_t$  in the second term in the bracketed expression.

<sup>10</sup>The factor 0.5 shows up because  $n_{j,t}$  is labor hired per  $j$ -type firm and half the firms are of each type.

### 6.3 The Policy Problem

The determination of the optimal monetary policy is conducted in a two-step procedure. First, the optimal choices for the real variables in the model are derived by solving the policy problem of the monetary authority acting as a social planner. Second, the implications for the nominal variables such as prices and interest rates are determined by using these optimal decision functions and by combining them with those of the household's problem. While unusual in macroeconomics this practice is common in public finance and other areas of applied general equilibrium analysis and is getting more and more standard in dynamic macroeconomic models with distortions. Expectations are not considered here so that the solution is derived under a certainty equivalence perspective.

The Lagrangian of the central bank (index  $C$ ) can be written as follows:

$$\begin{aligned}
L_C = & \sum_{t=0}^{\infty} \beta^t u(c_t, n_t, a_t) \\
& + \sum_{t=0}^{\infty} \beta^t \phi_t [x(c_{0,t}, c_t, n_t, a_t) + \beta x(c_{1,t+1}, c_{t+1}, n_{t+1}, a_{t+1})] \\
& + \sum_{t=0}^{\infty} \beta^t \Lambda_t [c(c_{0,t}, c_{1,t}) - c_t] \\
& + \sum_{t=0}^{\infty} \beta^t \Omega_t \left[ n_t - \frac{1}{2} n_{0,t} - \frac{1}{2} n_{1,t} \right] \\
& + \sum_{t=0}^{\infty} \beta^t [\rho_{0,t} (a_t n_{0,t} - c_{0,t}) + \rho_{1,t} (a_t n_{1,t} - c_{1,t})]
\end{aligned} \tag{6.37}$$

This function is optimized over  $c_{0,t}, c_{1,t}, c_t, n_{0,t}, n_{1,t}, n_t$  and of course with respect to the Lagrange multipliers  $\phi_t, \Lambda_t, \Omega_t, \rho_{0,t}$  and  $\rho_{1,t}$ .

#### 6.3.1 Optimality Conditions

Defining an artificial multiplier  $\phi_{-1}$  at date  $t = 0$  the optimality conditions can be written in the time-invariant form below. The multiplier will be discussed more thoroughly in the next section.

The first order condition with respect to each firm's labor input is given by

$$\frac{\partial L_C}{\partial n_{j,t}} = \beta^t \left( -\frac{1}{2} \Omega_t + \rho_{j,t} a_t \right) = 0 \quad \text{for } j = 0, 1 \tag{6.38}$$

The optimal choice of consumption levels from each type of firm is determined by

$$\frac{\partial L_C}{\partial c_{j,t}} = \beta^t \left( \phi_{t-j} \frac{\partial x(c_{j,t}, c_t, n_t, a_t)}{\partial c_{j,t}} + \Lambda_t \frac{\partial c(c_{0,t}, c_{1,t})}{\partial c_{j,t}} - \rho_{j,t} \right) = 0 \quad \text{for } j = 0, 1 \quad (6.39)$$

Aggregate consumption must obey

$$\begin{aligned} \frac{\partial L_C}{\partial c_t} &= \beta^t \left( \frac{\partial u(c_t, n_t, a_t)}{\partial c_t} + \phi_t \frac{\partial x(c_{0,t}, c_t, n_t, a_t)}{\partial c_t} \right) \\ &+ \beta^t \left( \phi_{t-1} \frac{\partial x(c_{1,t}, c_t, n_t, a_t)}{\partial c_t} - \Lambda_t \right) = 0 \end{aligned} \quad (6.40)$$

whereas for aggregate labor the condition is

$$\begin{aligned} \frac{\partial L_C}{\partial n_t} &= \beta^t \left( \frac{\partial u(c_t, n_t, a_t)}{\partial n_t} + \phi_t \frac{\partial x(c_{0,t}, c_t, n_t, a_t)}{\partial n_t} \right) \\ &+ \beta^t \left( \phi_{t-1} \frac{\partial x(c_{1,t}, c_t, n_t, a_t)}{\partial n_t} + \Omega_t \right) = 0 \end{aligned} \quad (6.41)$$

In addition the constraints have to hold with equality, that is the derivatives of the Lagrangian with respect to the multipliers.

$$\frac{\partial L_C}{\partial \rho_{j,t}} = \beta^t (a_t n_{j,t} - c_{j,t}) = 0 \quad \text{for } j = 0, 1 \quad (6.42)$$

$$\frac{\partial L_C}{\partial \Lambda_t} = \beta^t [c(c_{0,t}, c_{1,t}) - c_t] = 0 \quad (6.43)$$

$$\frac{\partial L_C}{\partial \phi_t} = \beta^t [x(c_{0,t}, c_t, n_t, a_t) + \beta x(c_{1,t+1}, c_{t+1}, n_{t+1}, a_{t+1})] = 0 \quad (6.44)$$

$$\frac{\partial L_C}{\partial \Omega_t} = \beta^t \left[ n_t - \frac{1}{2} n_{0,t} - \frac{1}{2} n_{1,t} \right] = 0 \quad (6.45)$$

It should be noted that the multiplier  $\phi_{-1}$  is not present in (6.37). It is introduced to have a simple representation of optimal policy in a world of commitment of the central bank. The multiplier guarantees that the efficiency conditions take the same form irrespective of the period the monetary authority is optimizing.

### 6.3.2 General Implications of the Optimality Conditions

In most cases the optimality conditions differ from those of an unrestricted representative agent. This is due to the effect of the implementation constraint on the social planner's behavior.

The conditions for the firms' labor input  $n_{j,t}$  just equate the utility-denominated price of a unit of each type of good ( $\rho_{j,t}$ ) to the utility-denominated value

of labor ( $\Omega_t$ ) divided by productivity ( $a_t$ ) which is the same under purely flexible prices. But the efficiency conditions for aggregate consumption and labor differ from those of an unrestricted planner.  $\phi_t$  is the shadow price of decreasing a price settings firm's marginal present discounted profits with respect to relative price and it is negative here because the planner wants the firms to have positive marginal profits.<sup>11</sup> In comparison to the decentralized problem the central bank values a marginal unit of consumption, measured by  $\Lambda_t$ , higher. This is because the derivative of  $x(c_{j,t}, c_t, n_t, a_t)$  with respect to aggregate consumption  $c_t$  is negative so that  $\Lambda_t$  is higher than marginal utility of consumption ( $\partial u(c_t, n_t, a_t) / \partial c_t$ ). For similar reasons a marginal unit of labor  $\Omega_t$  is valued higher here than under perfect competition.

The first order conditions for  $c_{j,t}$  do not have such a straightforward analogue in the competitive model. But it can be shown that the monetary authority equates appropriately chosen marginal rates of substitution and transformation (see King and Wolman (1999), p. 376).

The multiplier  $\phi_t$  appears not only in the current period  $t$ , but also in lagged form  $\phi_{t-1}$  as can be seen in the conditions for aggregate labor and consumption. This is a consequence of the fact that changes in future consumption affect the price setting behavior of firms in period  $t - 1$ . Recall that from (6.30)  $p_{0,t}$  depends not only on current period consumption but also on future consumption  $c_{t+1}$ . (6.30) is a forward-looking constraint. It must be clear that the efficiency conditions are valid for all periods, including  $t = 0$ , whereas in the Lagrangian  $\phi_{-1}$  is not present. This lagged multiplier in  $t = 0$  is introduced to make sure that an optimal plan is feasible. It allows the use of standard fixed-coefficient linear rational expectations solution methods.

If the monetary authority is allowed to reformulate its policy on a period-by-period basis, then there will be the problem of time-inconsistency of the optimal plan as in Kydland and Prescott (1977) and Barro and Gordon (1983). In that case the optimal policy problem cannot be formulated in the way just described. But here it is assumed that the central bank is required to commit to the state-contingent plan in the initial period and to stick to it through time. The introduction of  $\phi_{-1}$  prohibits the study of an optimal policy dependent on the effects of an initial start-up period. Therefore  $\phi_{-1}$  is set to the steady state value of  $\phi$  and not to zero. The use of the steady state of  $\phi$  reflects that the central bank has been following an

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<sup>11</sup>This can be shown when introducing a zero bound on marginal profits which means rewriting the implementation constraint as

$$-\sum_{t=0}^{\infty} \beta^t \phi_t [0 - x(c_{0,t}, c_t, n_t, a_t) - \beta x(c_{1,t+1}, c_{t+1}, n_{t+1}, a_{t+1})].$$



optimal plan for a long time.<sup>12</sup>

### 6.3.3 The Steady State

Looking at (6.38) in the steady state reveals that  $\rho_0 a = \rho_1 a = (1/2)\Omega$  so that  $\rho_1 = \rho_0 = (1/2)(\Omega/a)$ . A conjecture for the steady state values of consumption is that they are all equal which means  $c_0 = c_1 = c$ . In order to determine whether this is possible one has to check whether the other optimality conditions are consistent with the conjecture. For this end it is helpful to look at first at equations (6.39). Dividing (6.39) for  $j = 0$  by (6.39) for  $j = 1$  results in the following term:

$$\frac{\phi \frac{\partial x(c_0, c, n, a)}{\partial c_0} + \Lambda \frac{\partial c(c_0, c_1)}{\partial c_0}}{\phi \frac{\partial x(c_1, c, n, a)}{\partial c_1} + \Lambda \frac{\partial c(c_0, c_1)}{\partial c_1}} = \frac{\rho_0}{\rho_1} \quad (6.46)$$

As just shown the right hand side of this expression is unity in the steady state. For (6.46) to be satisfied the left hand side must also be equal to unity. Calculating the derivative of the consumption aggregator and imposing  $c_0 = c_1$  reveals that  $\partial c(c_0, c_1)/\partial c_0 = 1/2 = \partial c(c_0, c_1)/\partial c_1$ . Moreover, the effect of consumption related to today's price setting firms on today's implementation constraint  $\partial x(c_0, c, n, a)/\partial c_0$  is just the same as the effect of consumption related to yesterday's price setting firms on yesterday's implementation constraint  $\partial x(c_1, c, n, a)/\partial c_1$ . As  $\phi_t = \phi_{t-1} = \phi$  in the steady state and as  $\Lambda$  is constant the left hand side is equal to unity. Along a similar line of argument the derivatives of  $x(c_0, c, n, a)$  and  $x(c_1, c, n, a)$  with respect to  $c$  are identical so that (6.40) can be written as

$$\frac{\partial u(c, n, a)}{\partial c} + 2\phi \frac{\partial x(c_0, c, n, a)}{\partial c} - \Lambda = 0 \quad (6.47)$$

The condition for aggregate labor (6.41) reduces to

$$\frac{\partial u(c, n, a)}{\partial n} + 2\phi \frac{\partial x(c_0, c, n, a)}{\partial n} + \Omega = 0 \quad (6.48)$$

Using the results for (6.39) it suffices to use one of the conditions of (6.38) to get

$$\phi \frac{\partial x(c_0, c, n, a)}{\partial c_0} + \frac{1}{2}\Lambda - \frac{1}{2} \frac{\Omega}{a} = 0 \quad (6.49)$$

These three equations form a linear system in the three remaining Lagrange multipliers. The solution determines uniquely steady state values for  $\Lambda$ ,  $\Omega$  and  $\phi$ .

Closer inspection of (6.42) - (6.45) reveals that all labor inputs are equal ( $n_0 = n_1 = n = c/a$ ), and that marginal profits should be zero in any period:  $x(c_0, c, n, a) = x(c_1, c, n, a) = 0$ .

<sup>12</sup>This is what Woodford (1999) calls the *timeless perspective* approach to precommitment.

The optimal steady state inflation rate is equal to zero. This can be seen by calculating (6.18) at the steady state. Since  $c_j = c$  one gets  $P_j = P$ . The price of a firm setting the price in  $t$  is just equal to the overall price level and equal to the price firms set in period  $t - 1$ . Accordingly the gross inflation rate  $\Pi_t = P_t/P_{t-1}$  is equal to unity and inflation is zero.

## 6.4 Optimal Monetary Policy

In this section the solution to the policy problem is analyzed in detail. Having determined the steady state of all endogenous variables one can start taking linear approximations of the efficiency conditions around it. These equations are given in detail in Appendix E. The model has still to be augmented by equations for the nominal variables (nominal interest rate, price level, inflation rate and money demand) to analyze the implications of optimal monetary policy.

The solution is conducted using an extended version of the algorithm of King, Plosser and Rebelo (2002) which allows for singularities in the system matrix of the reduced model. The theoretical background of this algorithm is developed in King and Watson (1999) whereas computational aspects and the implementation are discussed in King and Watson (2002).

### 6.4.1 Implications of the Model Solution

One can use the resulting decision functions to calculate optimal responses to a productivity shock (impulse responses). In contrast to King and Wolman (1999) prices and labor inputs fluctuate and are not constant. The equality result for all relative prices even in the dynamic context is a very special one and due to the specific utility function used by these authors which has very strong implications for the substitution effects at work. It will be demonstrated which mechanisms are at work to stimulate fluctuating prices.

According to King and Wolman (1999) the labor input must not fluctuate in response to a productivity shock in order to make sure that marginal profits do not deviate from zero, that is  $dx_{j,t}(\cdot) = 0$ . This constancy is necessary to guarantee that a productivity shock does not induce optimal price variation. Under their preference specification labor does indeed not respond to a technology shock as long as the markup is constant. The markup  $\mu_t$  is the reciprocal of real marginal cost and is given by  $\mu_t = a_t/w_t = 1/\psi_t$ . They prove this to be possible and consistent with the linearized first order conditions with the help of the conjecture that  $\hat{\phi}_t$  does

not respond to productivity either and show that this is in fact the case and that it is compatible with the solution of the model.

This result is very special and does not hold in the model considered here. The utility function used in this model specification features a much richer set of substitution effects between consumption and labor. Refer to (6.3) and analyze the relationship between  $c$  and  $n$  in the household's optimization problem. The first order conditions of this problem imply - as shown above - the equality of the real wage with the rate of substitution between consumption and leisure. In the steady state this relationship is given by

$$n = 1 - \frac{c}{w} \frac{1 - \zeta}{\zeta} \quad (6.50)$$

Labor increases in the real wage as in case of utility function (6.2). But there is also a direct influence of consumption. With a positive technology shock, consumption will be increased while at the same time labor will be decreased. The impact of this favorable shock will be used completely to reduce working effort and to raise consumption. So the dynamics of consumption influence the dynamics of labor. Note that  $a$  does not appear in (6.50) as opposed to King and Wolman (1999). The respective steady state relation in their model is

$$n = \left(\frac{w}{a\theta}\right)^{1/\gamma} \quad (6.51)$$

This is the reason why they have to show that the markup  $\mu = a/w$  is constant. Constancy of the markup implies constancy of real marginal cost. Using this in (6.30) one can show that firms will not adjust their price  $p_{0,t}$  if the price level will be constant, i.e.  $P_t = P_{t-1}$ . This is because one can factor out  $\psi$  of the numerator. Then the remaining expression and the denominator cancel. To demonstrate that a constant price level is really a consequence of the optimal monetary policy they need zero response of the shadow price of real marginal profits  $\hat{\phi}_t$  to the technological shock and responses of consumption levels  $\hat{c}_t, \hat{c}_{j,t}$  exactly equal to productivity dynamics. This is proved by just imposing the conjecture of constancy on the linearized equations. But here  $\hat{\phi}_t$  *does* respond to  $\hat{a}_t$  as will be shown in the next subsection with the help of impulse responses.<sup>13</sup> In addition the symmetry of the responses of the consumption levels of the firms adjusting prices in  $t$  and  $t - 1$  vanishes. They also cease to be exactly the same as productivity dynamics.

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<sup>13</sup>It is not clear whether it is possible to show this analytically.

## 6.4.2 Impulse Response Functions and Optimal Monetary Policy

To compute impulse responses the parameters of the model have to be calibrated. Going back to section 6.3.3 one can see that  $a$  and either  $n$  or  $c$  have to be set exogenously. Because more information is available about hours worked,  $n$  is specified to be equal to 0.25 implying that agents work 25% of their non-sleeping time. The steady state value of the productivity shock is arbitrarily chosen to be 10.<sup>14</sup> It is assumed that productivity follows an AR(1)-process with  $\rho_a = 0.8$ . The discount factor  $\beta$  will be 0.99 and  $\sigma$ , the parameter governing the degree of risk aversion, is set to 2.<sup>15</sup> The elasticity of demand  $\epsilon$  is 4 causing the average static markup  $\mu = \epsilon/(\epsilon - 1)$  to be 1.33.<sup>16</sup> All remaining parameters can be calculated with the help of these specified values: the steady state consumption levels  $c = an$ , the real wage  $w = a/\mu$ , the real and nominal interest rates  $r = R = (1 - \beta)/\beta$ , and the preference parameter  $\zeta = c/(w + c - wn)$ . For  $\zeta$  this implies 0.3077, a value that is reasonably in line with other studies.

Figure 6.1 shows the impulse responses of consumption and labor caused by a one percent productivity shock. As mentioned above the response of  $\hat{c}$  does not exactly track the reaction of  $\hat{a}$  as in King and Wolman (1999). Aggregate consumption reacts a bit weaker than productivity. Moreover it is weakly influenced by  $\hat{\phi}$  which is itself hit by  $\hat{a}$ . The shadow price of marginal profits as well as aggregate labor input fall but the fall in  $\hat{\phi}$  is a persistent one whereas labor  $\hat{n}$  shows an interesting cyclical movement that is not very long lasting. Figure 6.2 gives some more detailed insight in the mechanisms at work. Here one can see that labor used by firms setting their price in period  $t$  ( $\hat{n}_0$ ) rises slightly while at the same time firms who set prices one period earlier will reduce labor input  $\hat{n}_1$ . This reduction is more pronounced than the expansion so that overall aggregate labor decreases. In the following period the picture changes. Now  $\hat{n}_1$  rises and  $\hat{n}_0$  falls. The effect is due to the shadow price of real marginal profits. Its relatively strong decline in the initial period of the shock drops to about only half of this magnitude. It should be noted that the

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<sup>14</sup>In contrast to the well known basic neoclassical model of King, Plosser and Rebelo (1988) there is no escape from specifying parameters such as  $a$  at the steady state. The system cannot be reduced until only deep parameters remain to be calibrated.

<sup>15</sup>Model results are qualitatively *not* sensitive to this value. Especially the result of a fluctuating price level also holds for log-linear preferences with  $\sigma = 1$ . Quantitative results for other values of  $\sigma$  will be discussed below.

<sup>16</sup>This formula can be deduced by combining (6.30) with the price index (6.21) evaluated at the steady state, i.e. for zero inflation.

fluctuations in labor are small compared with the respective values for consumption and  $\hat{\phi}$ . In addition, there is also a different reaction of  $\hat{c}_1$  and  $\hat{c}_0$ . Because the impulse responses look quite similar the differences of the respective functions are plotted. This reveals the stronger reaction of  $\hat{c}_0$  so that the difference is negative.  $\hat{c}_1$  responds a bit weaker causing a positive difference. Notice that both graphs are mirror images of each other.<sup>17</sup>

Figure 6.3 shows the reaction of prices. As the shadow price of real marginal profits falls the central bank as the social planner has to optimally induce firms to reduce their prices. The reaction of  $\hat{P}_0$  resembles much the behavior of  $\hat{\phi}$  but not that strong. Note that  $\hat{P}_0$  falls only by -0.005% whereas  $\hat{\phi}$  drops by nearly -0.11%. Firms who have set their prices one period before react with a lag of one period so that the impulse response for  $\hat{P}_1$  is the same as the one for  $\hat{P}_0$  just shifted one period ahead. This results in an overall variation in the price level. The central bank optimally induces a disinflation in order reach its goals: to maximize the utility of the representative agent in an environment of monopolistically competitive firms fixing nominal prices for two periods. Due to this two period price setting behavior there is a kink in  $\hat{P}$  that translates into a small inflationary period beginning two periods after the initial productivity shock but lasting only for just ten quarters. Although there are substitution effects at work (see (6.50)) these are small and cause only a weak inflationary bias. But nevertheless the monetary authority does not succeed in stabilizing the price level completely.

Figure 6.4 gives a graphical impression of the nominal and real interest rate. They are no longer equal. Since the price level is not constant there is inflation resulting in a slightly stronger reaction of the nominal rate to a productivity shock. The fall in the interest rate is quite strong: about 1.8% on an annual basis. It is more than four times the reaction in the King and Wolman (1999) model version with preference specification (6.2). Because the coefficients are even closer in value than those of the consumption levels in Figure 6.2 again the difference between the nominal and real rate is plotted. The nominal rate initially decreases stronger so that the difference is negative.<sup>18</sup> Note the interesting ‘cyclical’ character of this curve. There is a highly nonlinear relation between the response of the two rates due to the richer internal dynamics of the model. The response of money demand is nearly equal to that of consumption. This is due to the fact that the price level -

<sup>17</sup>The respective coefficients in the decision functions for  $\hat{c}_0$ ,  $\hat{c}_1$  and  $\hat{c}$  are 1.0052, 0.9851 and 0.9951.

<sup>18</sup>This is because the negative reaction of the nominal rate is stronger than that of the real rate so that the sign gets negative.

which is the difference between the cyclical components of money and consumption - does not react very strong to a productivity shock. (See the respective graphs in Figure 6.4 and refer to (E.19).)

The result that real and nominal rate differ can also be demonstrated analytically. A typical Taylor-rule would link the real and nominal rate according to<sup>19</sup>

$$R_t = r_t + f (\ln P_t - \ln \bar{P}) \quad (6.52)$$

where  $f$  would be a positive coefficient and  $\bar{P}$  the target price level.  $r_t$  would be determined from the real interest rate  $\hat{r}_t$  of the model solution. In King and Wolman (1999) the price level is constant under optimal policy so they reach the strong result that the central bank should just set the nominal rate equal to the real rate:  $R_t = r_t$ , which also implies the equality of the cyclical components:  $\hat{R}_t = \hat{r}_t$ . Here  $P$  fluctuates so that the term in parentheses ceases to be zero. Hence  $f$  would be different from zero. In addition it is no longer possible to write down the policy rule as in (6.52) because the fact that  $\hat{R}_t$  is different from the real rate makes it depend upon several state variables.

Varying the degree of risk aversion has no qualitative consequences for the model results regarding the fact that prices cannot be completely stabilized. But there are interesting quantitative effects. For small values of  $\sigma$  (smaller than 2) the reactions of prices, inflation, labor and consumption decline while for higher values the cyclical variation gets stronger. The higher the degree of relative risk aversion which corresponds to a lower degree of the elasticity of intertemporal substitution the stronger the effects of productivity shocks. The strengthening of the cyclical character is most probably due to the two period price setting of the firms. Because very risk averse households care more for today than for tomorrow they frequently change their demand for the differentiated consumption goods and their labor supply. So firms react stronger in setting their prices so that the central bank is less successful in stabilizing the price level and inflation. Figure 6.5 illustrates this result for prices and inflation.

A second important factor for the success of the central bank to stabilize prices is the length of the price setting period. To explore the consequences the model is solved assuming that every period a constant fraction of 20% of the firms can change their price with all adjusting firms choosing the same price (5-period price setting). As Figure 6.6 depicts this has the consequence of strengthening the persistence effects of the productivity shock. Optimal pricing now implies a very smooth

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<sup>19</sup>This formulation ignores a term for the output gap.

development of  $\widehat{P}_{0,t}$  which translates into an even smoother curve for the price level. Note that the intensity of the disinflation gets smaller compared to the case with a high degree of relative risk aversion (see Figure 6.5). The pronounced peak of the gross inflation rate in the sixth period is due to the assumption of equal fractions of firms adjusting every period. This causes the marginal adjustment probability to be zero all the time and one in every fifth period. Relaxing that assumption and using a vector of declining fractions gives the central bank the opportunity to smooth the productivity shock even more. Figure 6.7 is an example for the vector of fractions given by [0.30 0.25 0.20 0.15 0.10] meaning that in the first period 30% of the firms do adjust prices, in the second 25% and so on. This causes the inflationary period to start earlier but it also allows the monetary authority to dampen the peak and therefore to stabilize prices more efficiently.

The success of the central bank in stabilizing the price level depends also to a great degree upon the specific productivity process at work. So far an AR(1)-process has been assumed. The decision functions of the model change sharply when considering an AR(2)-process for productivity. In order to be able to compare results to those of King and Wolman (1999) the structure of the process is assumed to be

$$\widehat{a}_t = \rho_{a_1} \widehat{a}_{t-1} + \rho_{a_2} \widehat{a}_{t-2} + \epsilon_{\widehat{a}_t} \quad (6.53)$$

Using  $\rho_{a_1} = 1.3$  and  $\rho_{a_2} = -0.3$  one in fact has an ARIMA(1,1,0)-process for productivity. With  $\rho_{a_1}$  unchanged and  $\rho_{a_2} = -0.4$  it follows a stationary AR(2)-process. Figure 6.8 shows the impulse responses under this AR(2) specification. The graph for the nominal interest rate looks very much like that of King and Wolman, but the central bank has to raise this rate by more than twice the value of their study. Note that  $\widehat{R}_t$  rises initially as opposed to the decline under the AR(1)-process for the productivity shock.<sup>20</sup> This also implies a longer lasting inflationary bias. For difference stationary productivity Figure 6.9 gives a graphical impression of the impact of a technology shock. Again the reaction of the nominal rate is more than twice as large as in King and Wolman's study. Surprisingly the central bank can nearly completely eliminate the inflationary bias but not the initial disinflation. Prices decline permanently due the permanent character of the shock.

As has been shown prices are not constant in this model. Accordingly there is some room for the analysis of other policy rules. Even the optimal policy does not produce a constant price level. It may be that some other type of rule performs better. To evaluate the performance of alternative rules one has to add the rule

<sup>20</sup>It should be mentioned that the nominal rate also rises for very small degrees of risk aversion.

under consideration to the model and compare the resulting life-time-utility (6.1) to the optimal one. Technically the rule would be a function for the nominal interest rate replacing the equation that derives  $R_t$  in the optimal policy from the household's optimality conditions (see (6.13)). Such calculations are conducted in Henderson and Kim (1999). They analyze models with *one-period* wage and price contracts where exact solutions can be obtained so that it is also possible to derive exact welfare levels. The Pareto optimal welfare level can be reached in any situation so that all other policies focusing only on the stabilization of prices, the output gap or nominal income are suboptimal. This special result is due to the one-period contract structure. Erceg, Henderson and Levin (2000) extend the framework to *staggered* price and wage contracts. They show that under these circumstances the Pareto optimal welfare level cannot be achieved so that the policymaker always faces a tradeoff between wage inflation, price inflation and the output gap. However their claim that an optimal stabilization of prices is feasible if only prices are staggered is not supported by the results in this chapter. Their expectational Phillips curve depends on the utility function used which is additively separable in consumption and leisure. It would probably have a different form under CRRA preferences implying similar results to those obtained here.

## 6.5 Conclusions

This chapter has considered a version of the King and Wolman (1999) model of optimal monetary policy in a 'New Neoclassical Synthesis' environment. It has turned out that their result of complete price level stabilization is a very special one that depends - at least to a great extent - on the specific preference specification with zero substitution between consumption and labor. Prices fluctuate optimally under a more general utility function so that inflation will not be constant through time. Nevertheless these fluctuations are quite small.

Future research should focus on a richer production structure including capital accumulation considerations. Also the pricing structure can be extended to allow for state-dependent pricing, as opposed to time-dependent pricing assumed here. Dotsey, King and Wolman (1999) have begun studying the implications.

It would also be interesting to consider welfare losses associated with the inflationary bias. This could be done by comparing the approximated expected life-time-utility under CRRA preferences with that under King and Wolman's GHH specification. The approximation method proposed by Erceg, Henderson and Levin



(2000) can be used to answer that question. Yet in a recent article Kim and Kim (2003) have shown that sometimes the method of log-linearization around the steady state can lead to spurious welfare reversals.<sup>21</sup> Woodford (2003*b*) derives some conditions for the validity of this method in models like the one considered here. Whether the model at hand satisfies these conditions remains an open question since the price level fluctuates and is not constant through time.

Finally the model could also be used to analyze the business cycle implications of optimal monetary policy. It can answer, for example, questions about the variability of output and consumption as well as inflation and money. Further it implies certain correlation patterns between real and nominal variables. This line of research has not been pursued in this kind of literature on monetary policy.

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<sup>21</sup>They propose a quite complicated method very recently developed by Kim et al. (2003).

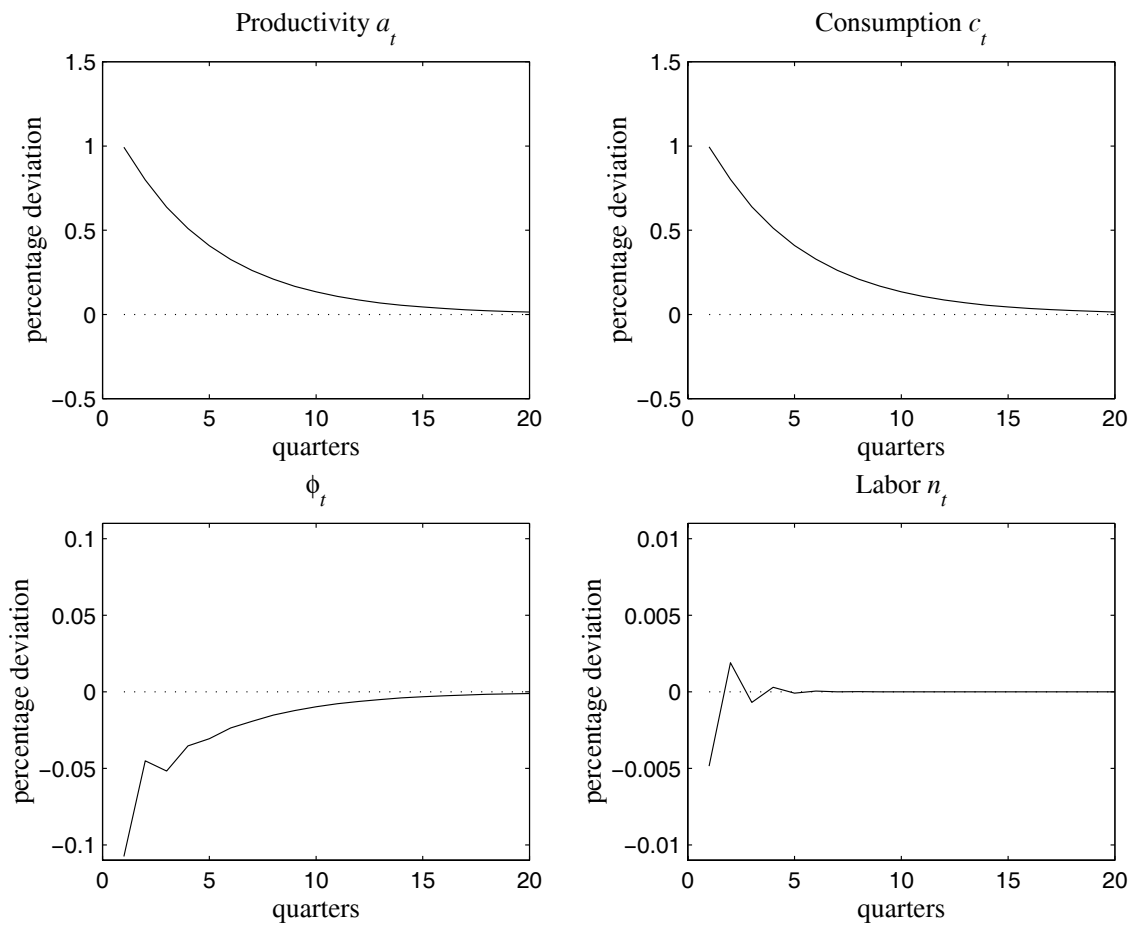


Figure 6.1: Impulse Response Functions for  $\hat{a}_t, \hat{c}_t, \hat{\phi}_t, \hat{n}_t$

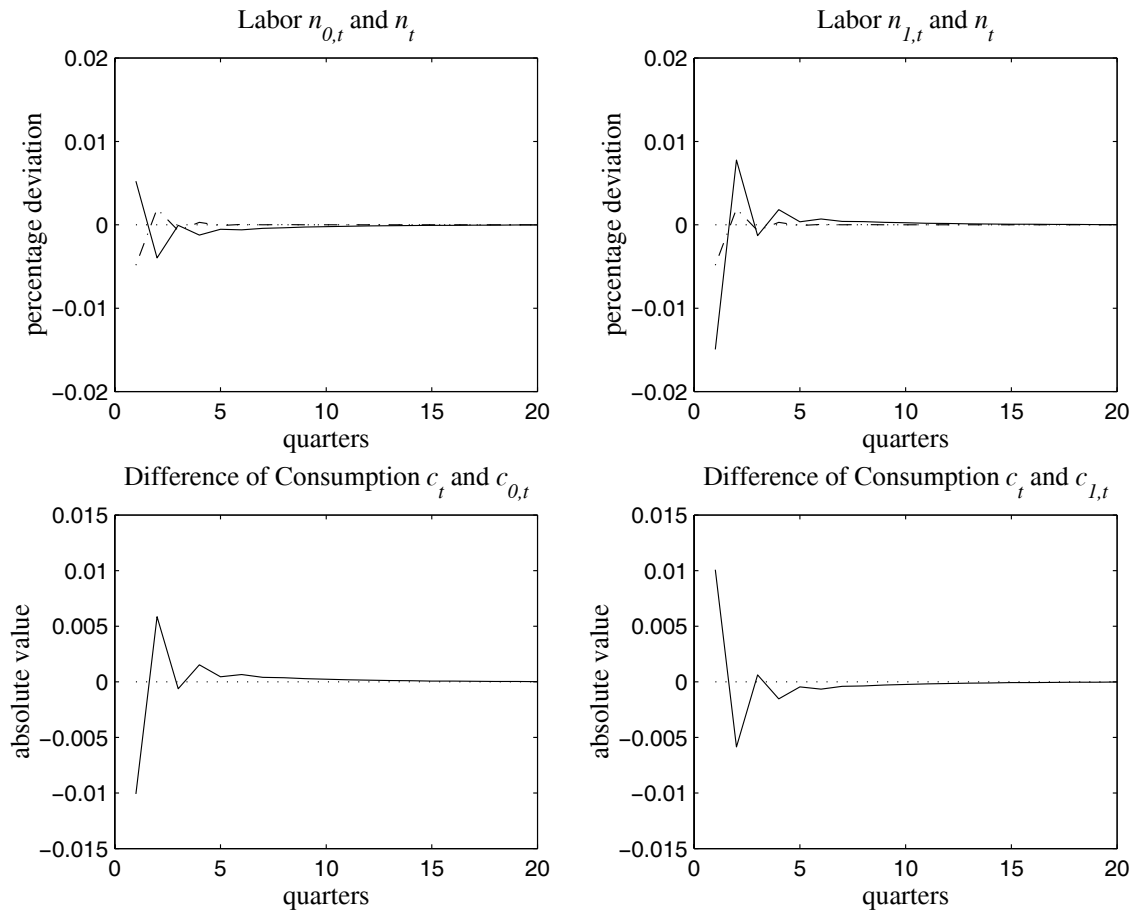


Figure 6.2: Impulse Response Functions for  $\hat{n}_{0,t}, \hat{n}_{1,t}, \hat{c}_{0,t}, \hat{c}_{1,t}$

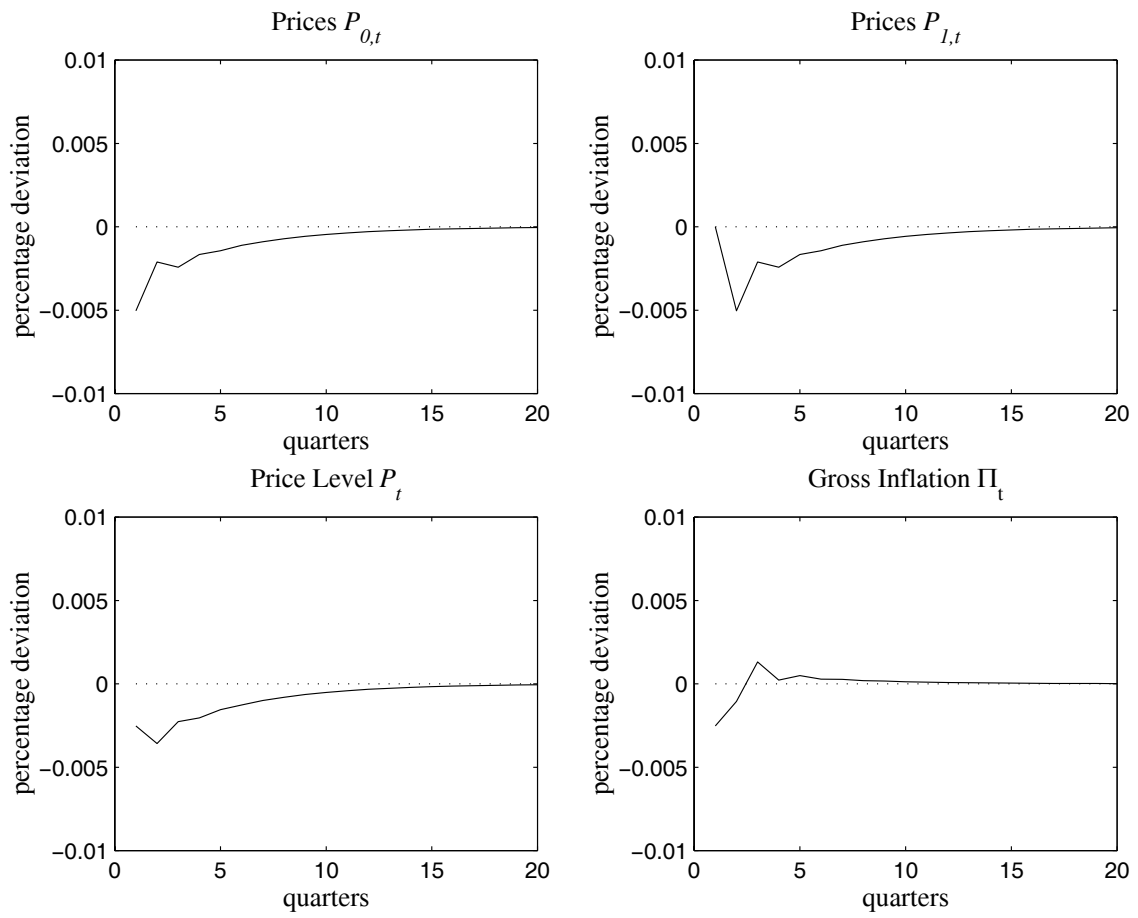


Figure 6.3: Impulse Response Functions for  $\hat{P}_{0,t}$ ,  $\hat{P}_{1,t}$ ,  $\hat{P}_t$ ,  $\hat{\Pi}_t$

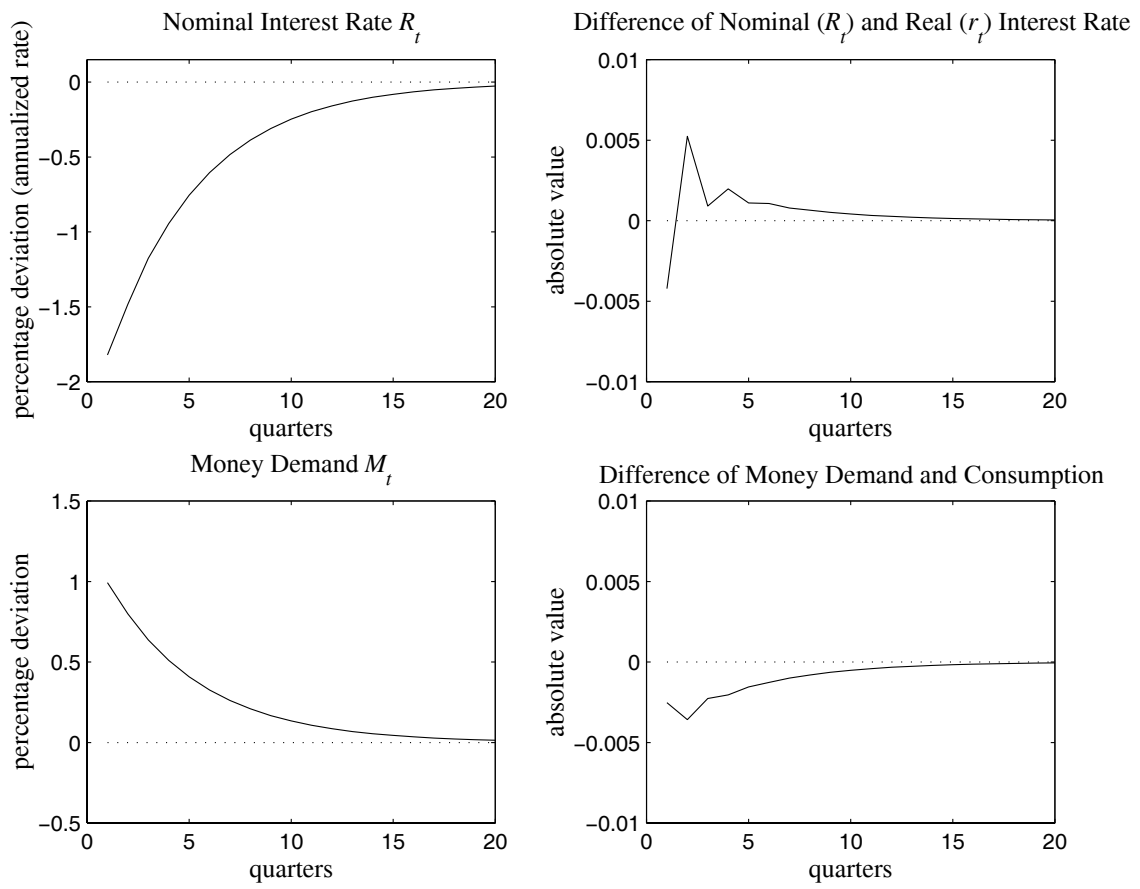


Figure 6.4: Impulse Response Functions for  $\widehat{R}_t, \widehat{r}_t, \widehat{M}_t$

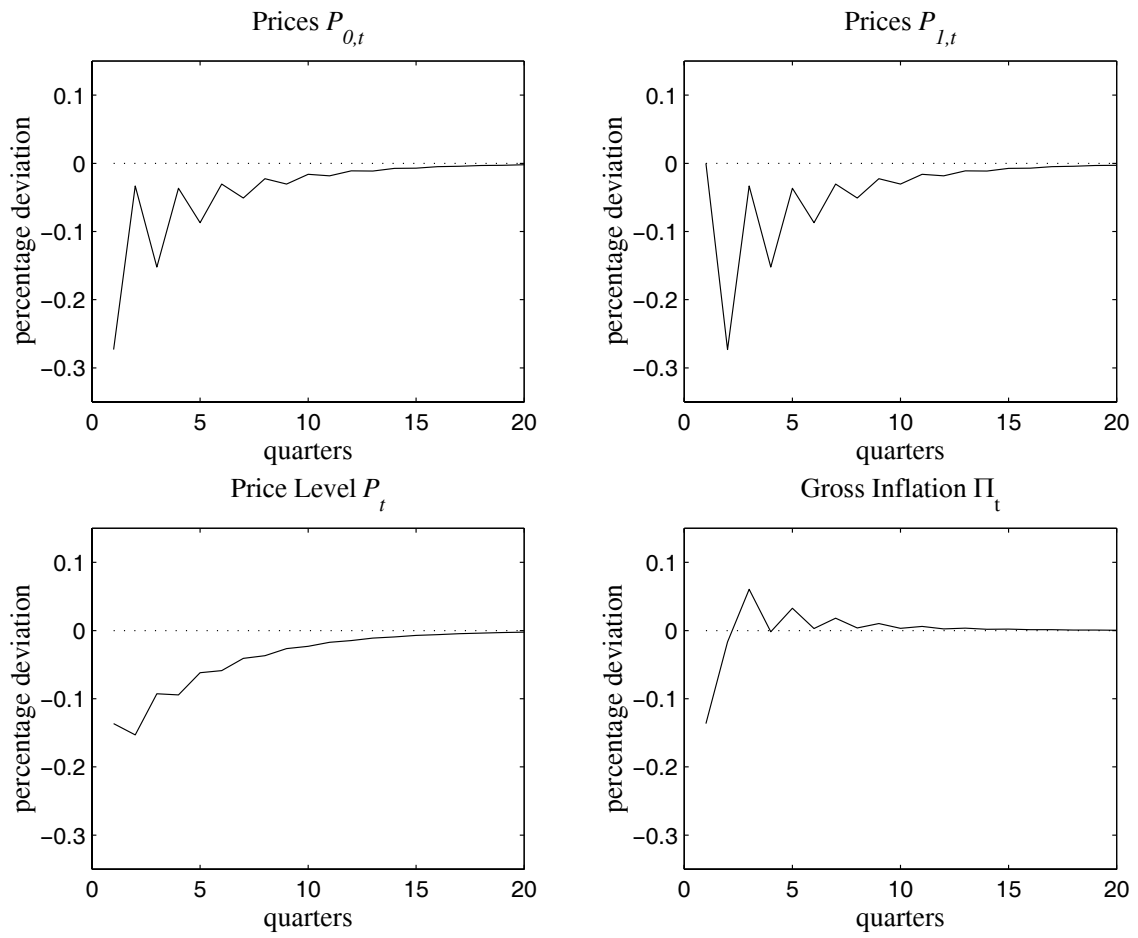


Figure 6.5: Impulse Response Functions for  $\hat{P}_{0,t}, \hat{P}_{1,t}, \hat{P}_t, \hat{\Pi}_t, \sigma=10$

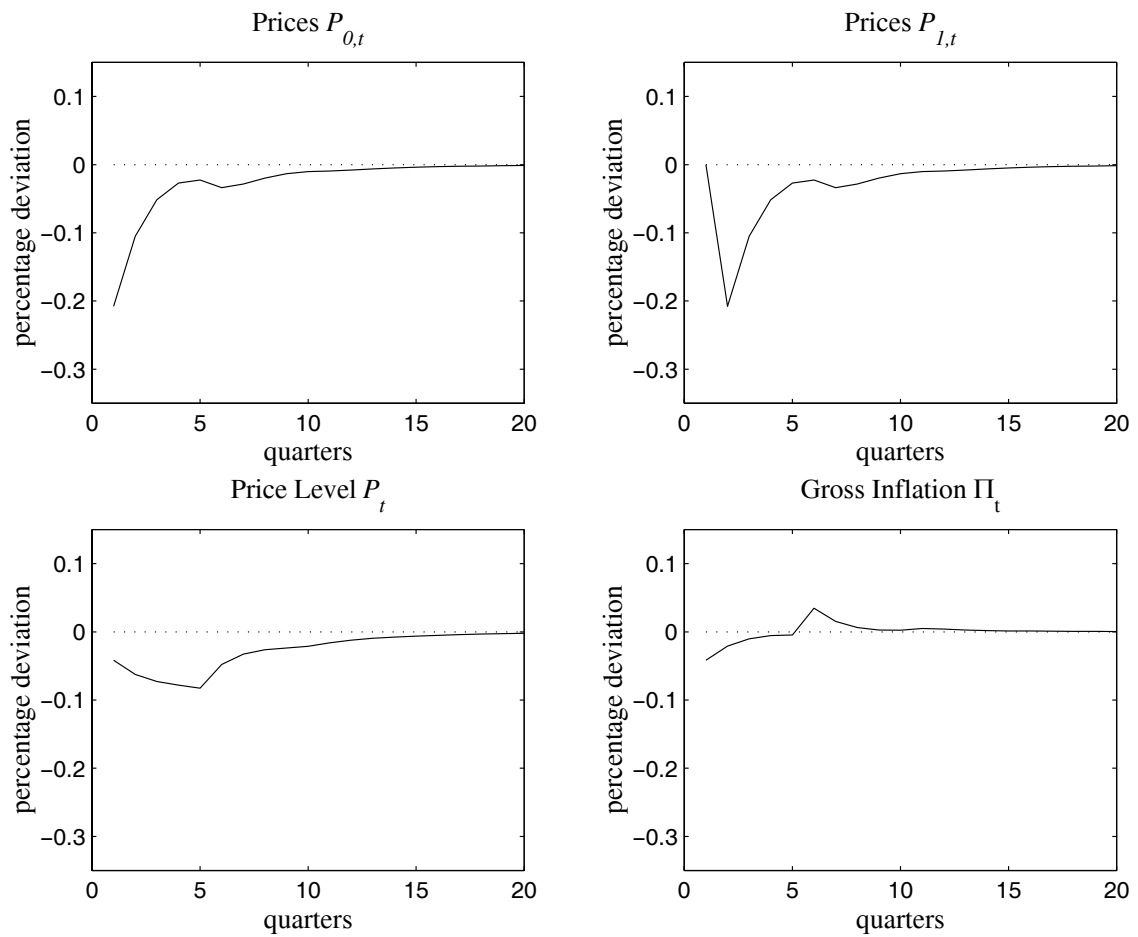


Figure 6.6: Impulse Response Functions for  $\widehat{P}_{0,t}$ ,  $\widehat{P}_{1,t}$ ,  $\widehat{P}_t$ ,  $\widehat{\Pi}_t$ , 5-period price setting, equal fractions

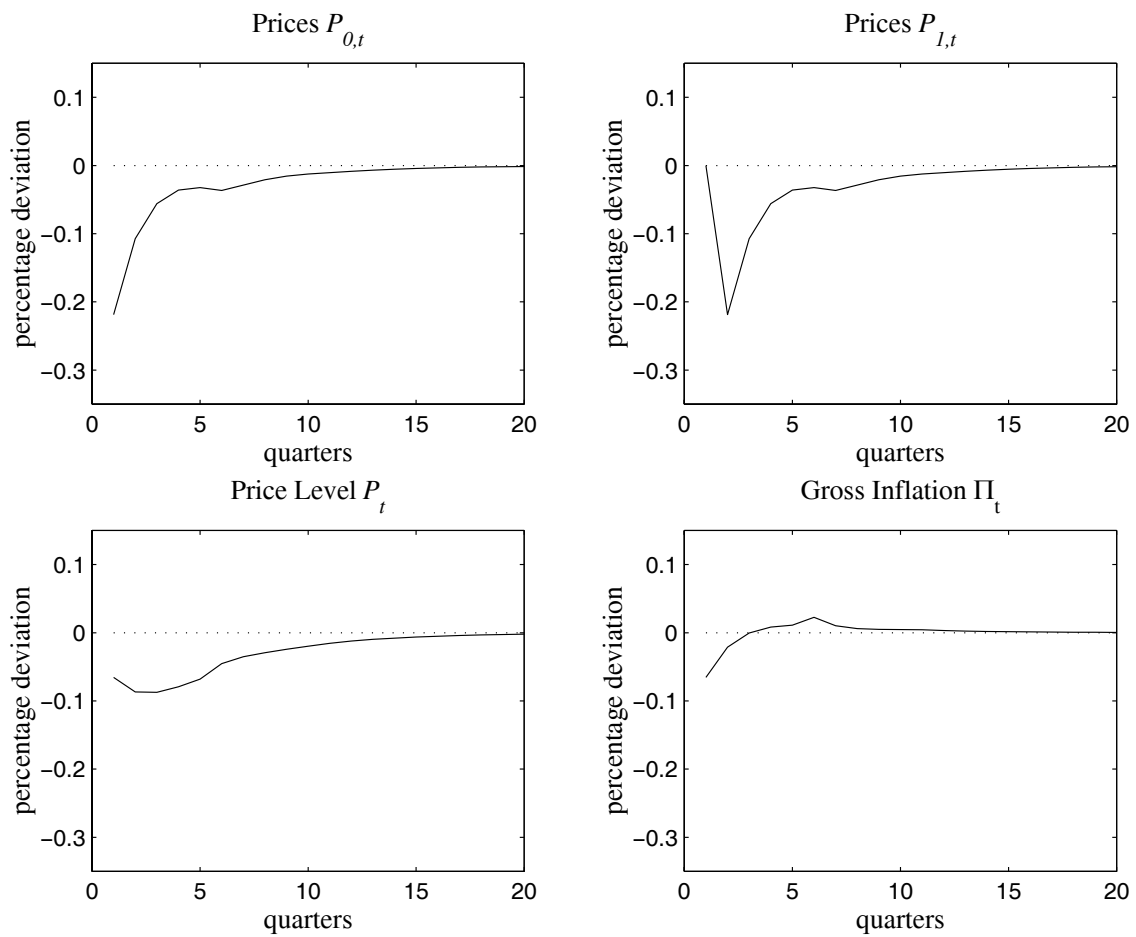


Figure 6.7: Impulse Response Functions for  $\hat{P}_{0,t}$ ,  $\hat{P}_{1,t}$ ,  $\hat{P}_t$ ,  $\hat{\Pi}_t$ , 5-period price setting, different fractions



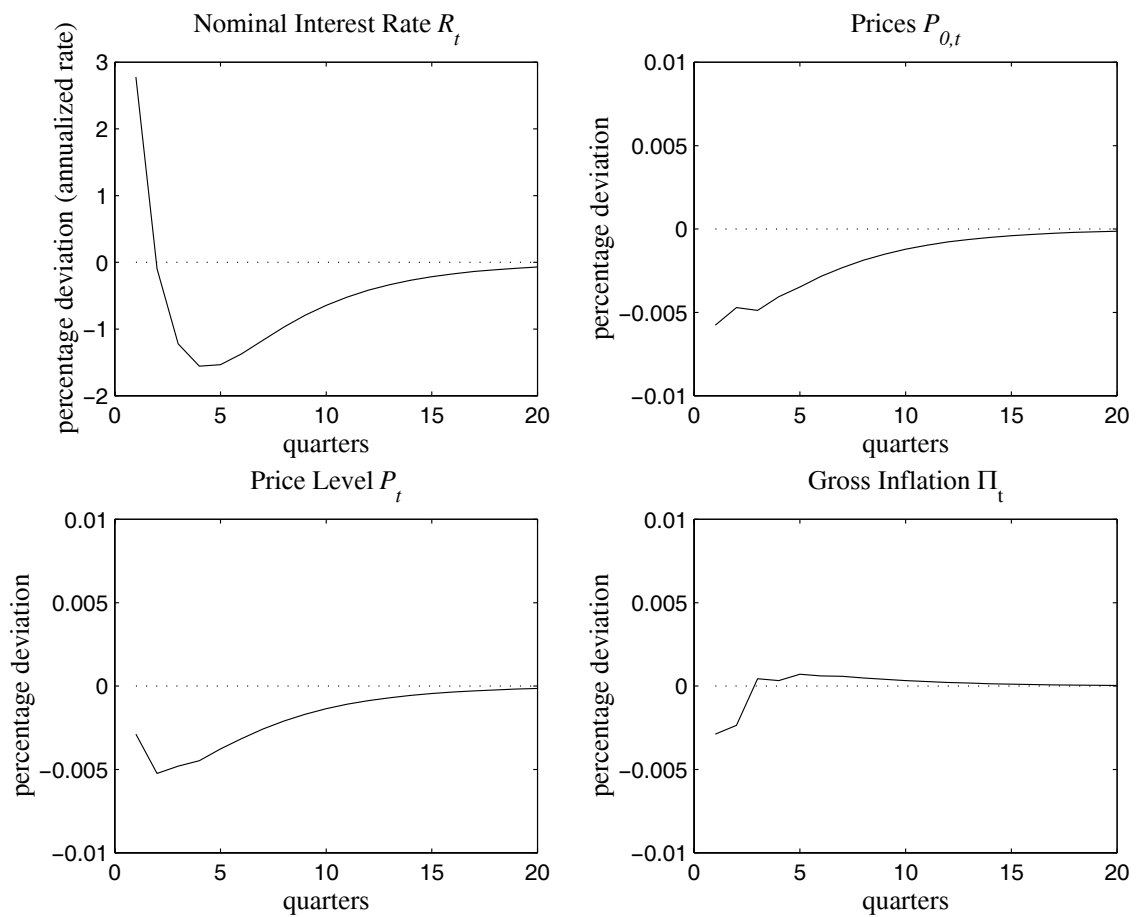


Figure 6.8: Impulse Response Functions for  $\widehat{R}_t, \widehat{P}_{0,t}, \widehat{P}_t, \widehat{\Pi}_t$ , AR(2) productivity shock

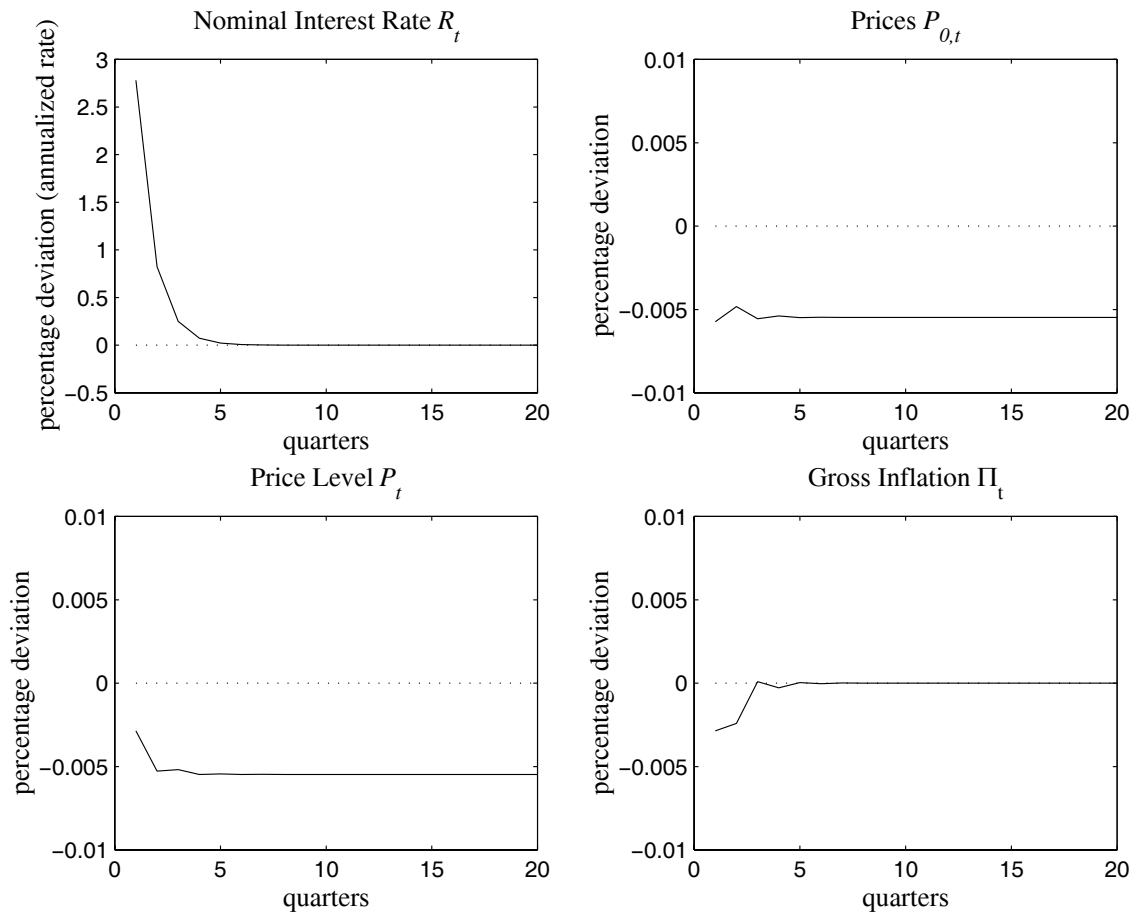


Figure 6.9: Impulse Response Functions for  $\widehat{R}_t, \widehat{P}_{0,t}, \widehat{P}_t, \widehat{\Pi}_t$ , ARIMA(1,1,0) productivity shock

# Chapter 7

## Final Remarks

This study has analyzed various versions of a monetary stochastic dynamic general equilibrium model. The question was whether such models are able to account for the observed persistent responses of output, consumption, investment, prices and other variables to a money growth shock. In addition, a DGE model of optimal monetary policy has been proposed in order to explain the behavior of a central bank in a closed economy. The main results can be summarized as follows:

1. Persistence is higher in a CIA-model with CRRA preferences than in a MIU-setup under GHH preferences.
2. Calvo price staggering can better account for persistence than Taylor price staggering.
3. Habit formation in consumption can explain only the persistence in consumption.
4. Taylor wage staggering in conjunction with adjustment costs of price changes and costly capital adjustment are important features to account for the observed persistence in output.
5. A central bank cannot completely stabilize the price level.

Monetary stochastic DGE models are still in an early stage. They are not yet a common workhorse of central bankers because they are still too simple to answer complicated questions of optimal policy in a more realistic setup. Woodford (2003*c*) puts it this way: ‘The development of realistic models with optimizing foundations that can be used for quantitative policy evaluation is currently an active area of research, but one that is sure to develop further in the next few years, so any

announcement of the correct quantitative specification is likely to be outdated by the time this book is published.<sup>1</sup> So what is the specific contribution of this literature? Isn't it a purely academic discussion of whether Calvo or Taylor price staggering is a more appropriate assumption? The answer is a clear no. Although these – admittedly very simple – models cannot account for what we call the stylized facts of the empirical business cycle they can guide policy makers such as central bankers in setting up more complicated models making use of the qualitative results obtained in these model economies. Large scale econometric models which are intensively used in central banks can e.g. be augmented by the New Keynesian Phillips curve and other forward looking equations in order to improve their empirical fit. This helps in interpreting results because the parameters are functions of the underlying preference and technology parameters and not ad hoc and thus merely atheoretical.

In my view there are two important research areas that have already delivered promising results: The first area deals with modern versions of the AS-AD or AS-IS-LM approach while the second is concerned with the analysis of optimal monetary policy in a similar way as in Chapter 6.

McCallum has contributed extensively to the first branch. The basics are laid down in McCallum and Nelson (1999b).<sup>2</sup> Casares and McCallum (2000) generalize the framework to include capital accumulation. The strength of this approach is that the AS- and AD-curves are derived from optimizing behavior of households and firms. In most cases there is an IS-curve derived from the consumption Euler equation and an AS-curve which is the New Keynesian Phillips curve under Calvo pricing. Monetary policy can then be given by a Taylor rule. When there are not more than these three equations even analytical solutions can be obtained. Casares and McCallum (2000) have an IS sector (that is, more than one equation), calibrate their model and study the results using impulse response functions. In McCallum and Nelson (1999a) the setup is extended to an open economy.<sup>3</sup> They find that nominal income targeting performs well compared to inflation targeting or the Taylor rule. McCallum and Nelson (2001) generalize their setup further in assuming that imports are not treated as finished consumer goods but as raw-material inputs in the home country's productive process. They show that this assumption leads to more realistic inflation dynamics.

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<sup>1</sup>See Woodford (2003c), p. 321.

<sup>2</sup>A good survey is also given by King (2000).

<sup>3</sup>Analysis of monetary policy in an open economy builds upon the work of Obstfeld and Rogoff (1995) for two-country models. See the excellent survey in Walsh (2003), p. 269-315.

The second branch seems to be even more fruitful. Here the setup builds upon that laid down in Chapter 6. Woodford has made important contributions in this area. Of special interest is the analysis of the implications of alternative monetary policies for welfare. In Woodford (2003*b*) he has shown that there is a close link between the utility of a representative agent and a loss function typically assumed in models of inflation targeting that is quadratic in the inflation rate and some measure of the output gap.<sup>4</sup> With the help of this criterion one can analyze alternative specifications of the monetary policy rule such as Taylor rules by comparing the welfare levels which are feasible under these rules. Preference will then be given to the rule that yields the highest welfare level or the smallest loss in welfare, respectively. This is of special interest since the timeless perspective precommitment policy rule analyzed in Chapter 6 is not the solution to the policy problem under optimal commitment. This solution would be given by setting  $\phi_{-1} = 0$ . Thus it is possible that the implicit interest rate rule for this case may lead to a higher welfare level in the economy.<sup>5</sup> Another important aspect concerns the objective function of the central bank. When there are no stochastic disturbances that influence the price setting of firms as in King and Wolman (1999) a central bank that maximizes the utility of the representative agent will automatically ensure that output will be kept equal to the flexible-price equilibrium level so that a policy of price stability is the appropriate objective. Once price setting is disturbed price stability is no longer feasible as Chapter 6 has demonstrated. When there are other distortions the objective function changes. This is exposed in Khan, King and Wolman (2000). They include a monetarist friction of costly exchange of wealth in their model by assuming that there are credit as well as cash goods whereby money is needed to purchase the latter. As a consequence price stability is no longer optimal. Instead the optimal rate of inflation is negative but less than the rate that would yield a zero nominal interest rate as proposed by Friedman (1969). In various simulations they find that this monetary inefficiency is small so that the optimal policy would be near to a policy that maintains price stability.<sup>6</sup> In an economy with price and wage staggering results differ. Erceg, Henderson and Levin (2000) have provided an approximation of the utility of the representative household for this case. The general insight from their analysis is that with both rigidities neither wage nor price stability can be achieved. There are always trade offs between either goal. It is how-

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<sup>4</sup>See also the summary in Walsh (2003), p. 519.

<sup>5</sup>See also Dennis (2001) on this point.

<sup>6</sup>Adão, Correia and Teles (2003) come to similar conclusions. They find that the Friedman rule is optimal when prices are set only one period in advance.

ever possible and also desirable to stabilize wages when wages are sticky and vice versa. That is, with sticky prices and flexible wages the nominal wage can adjust in response to productivity shocks in order to ensure the labor market equilibrium. Thus the optimal policy should keep prices stable. But Goodfriend and King (2001) argue that the simultaneous presence of wage and price stickiness does not undermine the case for price stability. They find that the reason for this result is that the labor market is typically characterized by long-term relationships where workers and firms have various opportunities to neutralize the allocative effects of sticky wages while in product markets spot transactions are dominant so that the effects of sticky prices cannot be neutralized in the same way. Whether this conclusion is justified remains an open question, especially in a world experiencing a large increase in the price level in the postwar period.

# Appendix A

## Price Staggering in a Monetary Stochastic Dynamic General Equilibrium Model with Labor

### A.1 Household's Equations: CIA-Model

The efficiency condition for aggregate consumption results in

$$\begin{aligned} & -D_1u(c, n, a) \widehat{P}_{t+1} + nD_{12}u(c, n, a) \widehat{n}_{t+1} + cD_{11}u(c, n, a) \widehat{c}_{t+1} \\ & = D_1u(c, n, a) \widehat{\lambda}_t - D_1u(c, n, a) \widehat{P}_t - aD_{13}u(c, n, a) \widehat{a}_{t+1} \end{aligned} \quad (\text{A.1})$$

using  $\Omega_t$  from the derivative with respect to  $m_t$ .

A hat ( $\widehat{\cdot}$ ) represents the relative deviation of the respective variable from its steady state ( $\widehat{a}_t = (a_t - a)/a$ ).  $D_iu(\cdot)$  denotes the first partial derivative of the  $u$ -function with respect to the  $i$ -th argument. Similarly  $D_{ij}u(\cdot)$  denotes the partial derivative of  $D_iu(\cdot)$  with respect to the  $j$ -th argument, all evaluated at the steady state. For aggregate labor one gets

$$\begin{aligned} 0 = & nD_{22}u(c, n, a) \widehat{n}_t + cD_{21}u(c, n, a) \widehat{c}_t \\ & - D_2u(c, n, a) \widehat{\lambda}_t - D_2u(c, n, a) \widehat{w}_t + aD_{23}u(c, n, a) \widehat{a}_t \end{aligned} \quad (\text{A.2})$$

The cyclical behavior of money demand can be deduced from (2.45).

$$\widehat{M}_t = \widehat{c}_t + \widehat{P}_t \quad (\text{A.3})$$

The nominal interest rate follows, according to (2.16),

$$-\widehat{P}_{t+1} + \widehat{\lambda}_{t+1} = -\widehat{P}_t - \frac{R}{1+R} \widehat{R}_t + \widehat{\lambda}_t \quad (\text{A.4})$$

in the approximated form, with  $R$  (respective  $r$  for the real rate) as the steady state values. The real rate  $r_t$  was deduced via the Fisher equation (see (2.17)) so that the approximated equation is given by

$$\widehat{\lambda}_{t+1} = -\frac{r}{1+r}\widehat{r}_t + \widehat{\lambda}_t \quad (\text{A.5})$$

## A.2 Household's Equations: MIU-Model

In the MIU-model the following three equations replace the first three in Appendix A.1.

$$\begin{aligned} 0 = & -mD_{12}u(c, m, n, a)\widehat{P}_t + nD_{13}u(c, m, n, a)\widehat{n}_t \\ & + cD_{11}u(c, m, n, a)\widehat{c}_t - D_1u(c, m, n, a)\widehat{\lambda}_t \\ & + mD_{12}u(c, m, n, a)\widehat{M}_t + aD_{14}u(c, m, n, a)\widehat{a}_t \end{aligned} \quad (\text{A.6})$$

Optimal labor is determined by

$$\begin{aligned} 0 = & nD_{33}u(c, m, n, a)\widehat{n}_t + cD_{31}u(c, m, n, a)\widehat{c}_t \\ & - D_3u(c, m, n, a)\widehat{\lambda}_t - D_3u(c, m, n, a)\widehat{w}_t \\ & + mD_{32}u(c, m, n, a)\widehat{M}_t + aD_{34}u(c, m, n, a)\widehat{a}_t \\ & - mD_{32}u(c, m, n, a)\widehat{P}_t \end{aligned} \quad (\text{A.7})$$

The efficiency condition for money now determines the respective money demand function. So one gets

$$\begin{aligned} & \beta D_1u(c, m, n, a)\widehat{P}_{t+1} - \beta D_1u(c, m, n, a)\widehat{\lambda}_{t+1} \\ = & cD_{21}u(c, m, n, a)\widehat{c}_t + mD_{22}u(c, m, n, a)\widehat{M}_t \\ & + nD_{23}u(c, m, n, a)\widehat{n}_t - D_1u(c, m, n, a)\widehat{\lambda}_t \\ & + [\beta D_1u(c, m, n, a) - mD_{22}u(c, m, n, a)]\widehat{P}_t \\ & + aD_{24}u(c, m, n, a)\widehat{a}_t \end{aligned} \quad (\text{A.8})$$

The equations for the nominal and real interest rate stay the same.

## A.3 Finished Goods Firm's Equations

It is possible to combine the demand functions for the differentiated products  $c_0$  and  $c_1$  (see (2.27)) to arrive at

$$\widehat{P}_{0,t} = -\frac{1}{\epsilon}\widehat{c}_{0,t} + \frac{1}{\epsilon}\widehat{c}_{1,t} + \widehat{P}_{1,t} \quad (\text{A.9})$$



The consumption aggregator (2.29) implies

$$0 = \frac{1}{2}\widehat{c}_{0,t} + \frac{1}{2}\widehat{c}_{1,t} - \widehat{c}_t \quad (\text{A.10})$$

The price level is uniquely determined since  $P_{1,t}$  is predetermined and  $P_{0,t}$  is given by (A.9). Using (2.30) one gets

$$0 = \frac{1}{2}\widehat{P}_{0,t} + \frac{1}{2}\widehat{P}_{1,t} - \widehat{P}_t \quad (\text{A.11})$$

## A.4 Intermediate Goods Firm's Equations

In contrast to the household's conditions the equations of the firms do not change under different utility functions. The production functions for the differentiated goods must obey

$$0 = \widehat{n}_{0,t} - \widehat{c}_{0,t} + \widehat{a}_t \quad (\text{A.12})$$

$$0 = \widehat{n}_{1,t} - \widehat{c}_{1,t} + \widehat{a}_t \quad (\text{A.13})$$

As discussed earlier firms are unable to change their prices for two periods so  $P_{0,t-1} = P_{1,t}$ . The Taylor approximation for this condition is given by

$$0 = -\widehat{P}_{0,t-1} + \widehat{P}_{1,t} \quad (\text{A.14})$$

The condition for optimal two period pricing is given in (2.41). Its Taylor approximation can be written as

$$\begin{aligned} & \beta [\epsilon\psi - (\epsilon - 1)] \widehat{\lambda}_{t+1} + \beta [\epsilon^2\psi - (\epsilon - 1)^2] \widehat{P}_{t+1} + \beta [\epsilon\psi - (\epsilon - 1)] \widehat{c}_{t+1} \\ & + \beta\epsilon\psi\widehat{\psi}_{t+1} = (\epsilon - 1)(1 + \beta)\widehat{P}_{0,t} + [(\epsilon - 1) - \epsilon\psi] \widehat{\lambda}_t \\ & + [(\epsilon - 1)^2 - \epsilon^2\psi] \widehat{P}_t + [(\epsilon - 1) - \epsilon\psi] \widehat{c}_t - \epsilon\psi\widehat{\psi}_t \end{aligned} \quad (\text{A.15})$$

Real marginal cost  $\psi_t$  is given by the ratio of the real wage  $w_t$  over the productivity shock  $a_t$ . Since the markup  $\mu_t$  is determined by the ratio of price over nominal marginal cost ( $\mu = P/(P\psi)$ ) and as there is no inflation it follows that  $\mu_t = a_t/w_t$ . So the Taylor approximations can be written as

$$0 = \widehat{\mu}_t + \widehat{w}_t - \widehat{a}_t \quad (\text{A.16})$$

$$0 = \widehat{\mu}_t + \widehat{\psi}_t \quad (\text{A.17})$$

The Taylor approximation of the labor market clearing condition amounts to

$$0 = \widehat{n}_t - \frac{1}{2}\widehat{n}_{0,t} - \frac{1}{2}\widehat{n}_{1,t} \quad (\text{A.18})$$

## A.5 Monetary Authority's and Other Equations

To close the model one needs to assume some exogenous process for the money supply. Here it will be assumed that money  $\widehat{M}_t$  follows an AR(2)-process (see the discussion in the main text). This implies that the growth rate of  $\widehat{M}_t$  follows an AR(1)-process. In order to model this properly one has to add the equation

$$0 = \widehat{M}_t - \widehat{g}_{M_t} \quad (\text{A.19})$$

where  $\widehat{g}_{M_t}$  is the exogenous stochastic process that will have the same characteristics as  $\widehat{M}_t$ , that is, follows the same AR(2)-process.

As it is interesting to study the implications for the inflation rate  $\Pi$  this equation is further added to the system:

$$0 = -\widehat{\Pi}_t + \widehat{P}_t - \widehat{P}_{t-1} \quad (\text{A.20})$$

There are now 19 variables

$\widehat{c}_{0,t}, \widehat{c}_{1,t}, \widehat{c}_t, \widehat{\lambda}_t, \widehat{n}_{0,t}, \widehat{n}_{1,t}, \widehat{n}_t, \widehat{w}_t, \widehat{\mu}_t, \widehat{\psi}_t, \widehat{r}_t, \widehat{R}_t, \widehat{P}_t, \widehat{P}_{t-1}, \widehat{P}_{0,t}, \widehat{P}_{0,t-1}, \widehat{P}_{1,t}, \widehat{\Pi}_t, \widehat{M}_t$

but only 17 equations so two tautologies must be added to the model. These are

$$\widehat{P}_{0,t} = \widehat{P}_{0,t} \quad (\text{A.21})$$

$$\widehat{P}_t = \widehat{P}_t \quad (\text{A.22})$$

# Appendix B

## Price Staggering in a Monetary Stochastic Dynamic General Equilibrium Model with Labor and Capital

### B.1 Household's Equations

The efficiency condition for consumption results in

$$\begin{aligned} & -D_1u(c, n, a) \widehat{P}_{t+1} + nD_{12}u(c, n, a) \widehat{n}_{t+1} + cD_{11}u(c, n, a) \widehat{c}_{t+1} \\ = & D_1u(c, n, a) \widehat{\lambda}_t - D_1u(c, n, a) \widehat{P}_t - aD_{13}u(c, n, a) \widehat{a}_{t+1} \end{aligned} \quad (\text{B.1})$$

using  $\Omega_t$  from the derivative with respect to  $m_{t+1}$ .

For labor one gets

$$\begin{aligned} 0 = & nD_{22}u(c, n, a) \widehat{n}_t + cD_{21}u(c, n, a) \widehat{c}_t \\ & -D_2u(c, n, a) \widehat{\lambda}_t - D_2u(c, n, a) \widehat{w}_t + aD_{23}u(c, n, a) \widehat{a}_t \end{aligned} \quad (\text{B.2})$$

The cyclical behavior of money demand can be deduced from (3.42).

$$\widehat{M}_t = \widehat{c}_t + \widehat{P}_t \quad (\text{B.3})$$

The nominal interest rate follows, according to (3.16),

$$-\widehat{P}_{t+1} + \widehat{\lambda}_{t+1} = -\widehat{P}_t - \frac{R}{1+R} \widehat{R}_t + \widehat{\lambda}_t \quad (\text{B.4})$$

in the approximated form, with  $R$  (respective  $r$  for the real rate) as the steady state values. The real rate  $r_t$  was deduced via the Fisher equation (see (3.17)) so that the

approximated equation is given by

$$\widehat{\lambda}_{t+1} = -\frac{r}{1+r}\widehat{r}_t + \widehat{\lambda}_t \quad (\text{B.5})$$

Optimal investment is determined from the efficiency condition for  $i_t$ :

$$0 = -\widehat{\lambda}_t + \widehat{\theta}_t + \frac{\phi''}{\phi'} \frac{i}{k} \widehat{i}_t - \frac{\phi''}{\phi'} \frac{i}{k} \widehat{i}_{t-1} \quad (\text{B.6})$$

The first order condition for capital implies:

$$\beta z \widehat{\lambda}_{t+1} + \beta z \widehat{z}_{t+1} + \beta(1-\delta)\widehat{\theta}_{t+1} - \beta \frac{\phi''}{\phi'} \frac{i}{k} \widehat{i}_{t+1} = -\beta \frac{\phi''}{\phi'} \frac{i}{k} \widehat{i}_t + \widehat{\theta}_t \quad (\text{B.7})$$

Capital evolves over time according to

$$\widehat{k}_t = (1-\delta)\widehat{k}_{t-1} + \delta \widehat{i}_t \quad (\text{B.8})$$

## B.2 Finished Goods Firm's Equations

Since the focus is on a symmetric equilibrium the only equation that remains for the finished goods firm is the price index. In case of the Taylor model it is given by

$$0 = \frac{1}{2}\widehat{P}_{0,t} + \frac{1}{2}\widehat{P}_{0,t-1} - \widehat{P}_t \quad (\text{B.9})$$

In order to avoid too many variables  $\widehat{P}_{1,t}$  is dropped and replaced by  $\widehat{P}_{0,t-1}$ .

Under Calvo pricing the price level is given by (3.39) so that the Taylor approximation reads:

$$0 = \frac{1}{1-\varphi}\widehat{P}_t - \frac{\varphi}{1-\varphi}\widehat{P}_{t-1} - \widehat{P}_{0,t} \quad (\text{B.10})$$

## B.3 Intermediate Goods Firm's Equations

### B.3.1 The Producing Unit

The optimum conditions of the cost minimization problem determine the real wage and the rental rate of capital (see (3.24) and (3.25)), with the  $j$ 's dropped of course.

$$0 = (\alpha - 1)\widehat{n}_t + (1 - \alpha)\widehat{k}_{t-1} + \widehat{\psi}_t + \widehat{a}_t - \widehat{w}_t \quad (\text{B.11})$$

$$0 = \alpha\widehat{n}_t - \alpha\widehat{k}_{t-1} + \widehat{\psi}_t + \widehat{a}_t - \widehat{z}_t \quad (\text{B.12})$$

The production function is given by the Cobb-Douglas-functions of the intermediate goods firms and valid in aggregate variables.

$$0 = -\widehat{y}_t + \alpha\widehat{n}_t + (1 - \alpha)\widehat{k}_{t-1} + \widehat{a}_t \quad (\text{B.13})$$

### B.3.2 The Pricing Unit under Taylor Staggering

The condition for optimal two period pricing is given in (3.32). Its Taylor approximation can be written as

$$\begin{aligned} & \beta [\epsilon\psi - (\epsilon - 1)] \widehat{\lambda}_{t+1} + \beta [\epsilon^2\psi - (\epsilon - 1)^2] \widehat{P}_{t+1} + \beta [\epsilon\psi - (\epsilon - 1)] \widehat{y}_{t+1} \\ & + \beta\epsilon\psi\widehat{\psi}_{t+1} = (\epsilon - 1)(1 + \beta) \widehat{P}_{0,t} + [(\epsilon - 1) - \epsilon\psi] \widehat{\lambda}_t \\ & + [(\epsilon - 1)^2 - \epsilon^2\psi] \widehat{P}_t + [(\epsilon - 1) - \epsilon\psi] \widehat{y}_t - \epsilon\psi\widehat{\psi}_t \end{aligned} \quad (\text{B.14})$$

### B.3.3 The Pricing Unit under Calvo Staggering

As stated in the main text the approximation of (3.38) yields the New Keynesian Phillips curve and is given by

$$\widehat{\pi}_t = (1 - \varphi)(1 - \beta\varphi)\varphi^{-1}\widehat{\psi}_t + \beta E_t \widehat{\pi}_{t+1} \quad (\text{B.15})$$

## B.4 Market Clearing Conditions and Other Equations

The Taylor expansion of the aggregate market clearing condition is given by

$$0 = -\widehat{y}_t + \frac{c}{y}\widehat{c}_t + \frac{i}{y}\widehat{i}_t \quad (\text{B.16})$$

The markup  $\mu_t$  is determined by the ratio of price over nominal marginal cost ( $\mu = P/(P\psi)$ ) and as there is no steady state inflation it follows that  $\mu_t = 1/\psi_t$ . So the Taylor approximation can be written as

$$0 = \widehat{\mu}_t + \widehat{\psi}_t \quad (\text{B.17})$$

## B.5 The Monetary Authority and Further Equations

To close the model one needs to assume some exogenous process for money supply. Here it will be assumed that money  $\widehat{M}_t$  follows an AR(2)-process (see the discussion in the main text). This implies that the growth rate of  $\widehat{M}_t$  follows an AR(1)-process. In order to model this properly one has to add the equation

$$0 = \widehat{M}_t - \widehat{g}_{M_t} \quad (\text{B.18})$$

where  $\widehat{g}_{M_t}$  is the exogenous stochastic process that will have the same characteristics as  $\widehat{M}_t$ .

As it is interesting to study the implications for the inflation rate  $\Pi$  this equation is further added to the system:

$$0 = -\widehat{\Pi}_t + \widehat{P}_t - \widehat{P}_{t-1} \quad (\text{B.19})$$

In the model with Taylor staggering there are now 20 variables

$\widehat{c}_t, \widehat{i}_t, \widehat{y}_t, \widehat{\lambda}_t, \widehat{\theta}_t, \widehat{k}_t, \widehat{k}_{t-1}, \widehat{n}_t, \widehat{w}_t, \widehat{z}_t, \widehat{\mu}_t, \widehat{\psi}_t, \widehat{r}_t, \widehat{R}_t, \widehat{P}_t, \widehat{P}_{t-1}, \widehat{P}_{0,t}, \widehat{P}_{0,t-1}, \widehat{\Pi}_t, \widehat{M}_t$

but only 17 equations so three tautologies must be added to the model. These are

$$\widehat{P}_{0,t} = \widehat{P}_{0,t} \quad (\text{B.20})$$

$$\widehat{P}_t = \widehat{P}_t \quad (\text{B.21})$$

$$\widehat{k}_t = \widehat{k}_t \quad (\text{B.22})$$

In the Calvo pricing model there are only 19 variables since  $\widehat{P}_{0,t-1}$  does not show up. So only two tautologies must be added to the model. These are given by

$$\widehat{P}_t = \widehat{P}_t \quad (\text{B.23})$$

$$\widehat{k}_t = \widehat{k}_t \quad (\text{B.24})$$

# Appendix C

## Habit Persistence and Price Staggering in a Monetary Stochastic Dynamic General Equilibrium Model with Labor and Capital

### C.1 Household's Equations

The efficiency condition for consumption results in

$$\begin{aligned} & (1 - \sigma) \beta b c^{\sigma b - b - \sigma} \widehat{c}_{t+1} \\ = & [-\sigma - \beta b (\sigma b - b - 1)] c^{\sigma b - b - \sigma} \widehat{c}_t + b (\sigma - 1) c^{\sigma b - b - \sigma} \widehat{c}_{t-1} \\ & - (1 - \beta b) c^{\sigma b - b - \sigma} \widehat{\lambda}_t \end{aligned} \quad (\text{C.1})$$

A hat ( $\widehat{\phantom{x}}$ ) represents the relative deviation of the respective variable from its steady state ( $\widehat{c}_t = (c_t - c) / c$ ).

The cyclical behavior of labor is determined by

$$\begin{aligned} 0 = & -n\gamma\sigma(1-n)^{-\sigma-1}\widehat{n}_t \\ & +\gamma(1-n)^{-\sigma}\widehat{\lambda}_t + \gamma(1-n)^{-\sigma}\widehat{w}_t \end{aligned} \quad (\text{C.2})$$

The efficiency condition for money determines the respective demand function. So

one gets

$$\begin{aligned}
& \beta(1-\beta b)c^{\sigma b-b-\sigma}\widehat{P}_{t+1}-\beta(1-\beta b)c^{\sigma b-b-\sigma}\widehat{\lambda}_{t+1} \\
= & -\sigma m^{-\sigma}\widehat{M}_t \\
& -(1-\beta b)c^{\sigma b-b-\sigma}\widehat{\lambda}_t \\
& +[\beta(1-\beta b)c^{\sigma b-b-\sigma}+\sigma m^{-\sigma}]\widehat{P}_t
\end{aligned} \tag{C.3}$$

The nominal interest rate follows, according to (4.18),

$$-\widehat{P}_{t+1}+\widehat{\lambda}_{t+1}=-\widehat{P}_t-\frac{R}{1+R}\widehat{R}_t+\widehat{\lambda}_t \tag{C.4}$$

in the approximated form, with  $R$  (respective  $r$  for the real rate) as the steady state values. The real rate  $r_t$  was deduced via the Fisher equation (see (4.19)) so that the approximated equation is given by

$$\widehat{\lambda}_{t+1}=-\frac{r}{1+r}\widehat{r}_t+\widehat{\lambda}_t \tag{C.5}$$

Optimal investment is determined from the efficiency condition for  $i_t$ :

$$0=-\widehat{\lambda}_t+\widehat{\theta}_t+\frac{\phi''}{\phi'}\frac{i}{k}\widehat{i}_t-\frac{\phi''}{\phi'}\frac{i}{k}\widehat{k}_{t-1} \tag{C.6}$$

The first order condition for capital implies:

$$\beta z\widehat{\lambda}_{t+1}+\beta z\widehat{z}_{t+1}+\beta(1-\delta)\widehat{\theta}_{t+1}-\beta\frac{\phi''}{\phi'}\frac{i}{k}\frac{i}{k}\widehat{i}_{t+1}=-\beta\frac{\phi''}{\phi'}\frac{i}{k}\frac{i}{k}\widehat{k}_t+\widehat{\theta}_t \tag{C.7}$$

Capital evolves over time according to

$$\widehat{k}_t=(1-\delta)\widehat{k}_{t-1}+\delta\widehat{i}_t \tag{C.8}$$

## C.2 Finished Goods Firm's Equations

Since the focus is on a symmetric equilibrium the only equation that remains for the finished goods firm is the price index.

$$0=\frac{1}{2}\widehat{P}_{0,t}+\frac{1}{2}\widehat{P}_{0,t-1}-\widehat{P}_t \tag{C.9}$$

In order to avoid too many variables  $\widehat{P}_{1,t}$  is dropped and replaced by  $\widehat{P}_{0,t-1}$ .



### C.3 Intermediate Goods Firm's Equations

The optimum conditions of the cost minimization problem determine the real wage and the rental rate of capital (see (4.27) and (4.28)), with the  $j$ 's dropped of course.

$$0 = (\alpha - 1)\hat{n}_t + (1 - \alpha)\hat{k}_{t-1} + \hat{\psi}_t + \hat{a}_t - \hat{w}_t \quad (\text{C.10})$$

$$0 = \alpha\hat{n}_t - \alpha\hat{k}_{t-1} + \hat{\psi}_t + \hat{a}_t - \hat{z}_t \quad (\text{C.11})$$

The production function is given by the Cobb-Douglas-functions of the intermediate goods firms and valid in aggregate variables.

$$0 = -\hat{y}_t + \alpha\hat{n}_t + (1 - \alpha)\hat{k}_{t-1} + \hat{a}_t \quad (\text{C.12})$$

The condition for optimal two period pricing is given in (4.34). Its Taylor approximation can be written as

$$\begin{aligned} & \beta [\epsilon\psi - (\epsilon - 1)]\hat{\lambda}_{t+1} + \beta [\epsilon^2\psi - (\epsilon - 1)^2]\hat{P}_{t+1} + \beta [\epsilon\psi - (\epsilon - 1)]\hat{y}_{t+1} \\ & + \beta\epsilon\psi\hat{\psi}_{t+1} = (\epsilon - 1)(1 + \beta)\hat{P}_{0,t} + [(\epsilon - 1) - \epsilon\psi]\hat{\lambda}_t \\ & + [(\epsilon - 1)^2 - \epsilon^2\psi]\hat{P}_t + [(\epsilon - 1) - \epsilon\psi]\hat{y}_t - \epsilon\psi\hat{\psi}_t \end{aligned} \quad (\text{C.13})$$

### C.4 Market Clearing Conditions and Other Equations

The Taylor expansion of the aggregate market clearing condition is given by

$$0 = -\hat{y}_t + \frac{c}{y}\hat{c}_t + \frac{i}{y}\hat{i}_t \quad (\text{C.14})$$

The markup  $\mu_t$  is determined by the ratio of price over nominal marginal cost ( $\mu = P/(P\psi)$ ) and as there is no steady state inflation it follows that  $\mu_t = 1/\psi_t$ . So the Taylor approximation can be written as

$$0 = \hat{\mu}_t + \hat{\psi}_t \quad (\text{C.15})$$

### C.5 The Monetary Authority and Further Equations

To close the model one needs to assume some exogenous process for money supply. Here it will be assumed that money  $\widehat{M}_t$  follows an AR(2)-process (see the discussion

in the main text). This implies that the growth rate of  $\widehat{M}_t$  follows an AR(1)-process. In order to model this properly one has to add the equation

$$0 = \widehat{M}_t - \widehat{g}_{M_t} \quad (\text{C.16})$$

where  $\widehat{g}_{M_t}$  is the exogenous stochastic process that will have the same characteristics as  $\widehat{M}_t$ .

As it is interesting to study the implications for the inflation rate  $\Pi$  this equation is further added to the system:

$$0 = -\widehat{\Pi}_t + \widehat{P}_t - \widehat{P}_{t-1} \quad (\text{C.17})$$

There are now 21 variables

$\widehat{c}_t, \widehat{c}_{t-1}, \widehat{i}_t, \widehat{y}_t, \widehat{\lambda}_t, \widehat{\theta}_t, \widehat{k}_t, \widehat{k}_{t-1}, \widehat{n}_t, \widehat{w}_t, \widehat{z}_t, \widehat{\mu}_t, \widehat{\psi}_t, \widehat{r}_t, \widehat{R}_t, \widehat{P}_t, \widehat{P}_{t-1}, \widehat{P}_{0,t}, \widehat{P}_{0,t-1}, \widehat{\Pi}_t, \widehat{M}_t$

but only 17 equations so four tautologies must be added to the model. These are

$$\widehat{P}_{0,t} = \widehat{P}_{0,t} \quad (\text{C.18})$$

$$\widehat{P}_t = \widehat{P}_t \quad (\text{C.19})$$

$$\widehat{k}_t = \widehat{k}_t \quad (\text{C.20})$$

$$\widehat{c}_t = \widehat{c}_t \quad (\text{C.21})$$

# Appendix D

## Wage Staggering and Sticky Prices in a Monetary Stochastic Dynamic General Equilibrium Model with Labor and Capital

### D.1 Household's Equations

The Taylor approximation for the consumption decision is given by

$$\begin{aligned} 0 = & -mD_{12}u(c, m, n, a)\widehat{P}_t \\ & +cD_{11}u(c, m, n, a)\widehat{c}_t - D_1u(c, m, n, a)\widehat{\lambda}_t \\ & +mD_{12}u(c, m, n, a)\widehat{M}_t + aD_{14}u(c, m, n, a)\widehat{a}_t \end{aligned} \quad (\text{D.1})$$

A hat ( $\widehat{\cdot}$ ) represents the relative deviation of the respective variable from its steady state ( $\widehat{a}_t = (a_t - a)/a$ ).  $D_i u(\cdot)$  denotes the first partial derivative of the  $u$ -function with respect to the  $i$ -th argument. Similarly  $D_{ij} u(\cdot)$  denotes the partial derivative of  $D_i u(\cdot)$  with respect to the  $j$ -th argument, all evaluated at the steady state. It should be noted that  $D_{3j} u(\cdot) = D_{j3} u(\cdot)$  for  $j = c, m$  will be equal to zero because of the separability assumption in the utility function.

The efficiency condition for the optimal nominal wage is determined by

$$\begin{aligned}
& -\beta c \frac{D_{11}u(c, m, n, a)}{D_1u(c, m, n, a)} \widehat{c}_{t+1} - \beta m \frac{D_{12}u(c, m, n, a)}{D_1u(c, m, n, a)} \widehat{M}_{t+1} \\
& + \beta n \frac{D_{33}u(c, m, n, a)}{D_3u(c, m, n, a)} \widehat{n}_{t+1} + \beta \left( m \frac{D_{12}u(c, m, n, a)}{D_1u(c, m, n, a)} + 1 \right) \widehat{P}_{t+1} \\
& + \epsilon_w n \frac{D_{33}u(c, m, n, a)}{D_3u(c, m, n, a)} \widehat{W}_{t+1} \\
& + \beta a \left( \frac{D_{34}u(c, m, n, a)}{D_3u(c, m, n, a)} - \frac{D_{14}u(c, m, n, a)}{D_1u(c, m, n, a)} \right) \widehat{a}_{t+1} \\
= & (1 + \beta) \left( 1 + \epsilon_w n \frac{D_{33}u(c, m, n, a)}{D_3u(c, m, n, a)} \right) \widehat{W}_{0,t} \\
& + c \frac{D_{11}u(c, m, n, a)}{D_1u(c, m, n, a)} \widehat{c}_t + m \frac{D_{12}u(c, m, n, a)}{D_1u(c, m, n, a)} \widehat{M}_t \\
& - n \frac{D_{33}u(c, m, n, a)}{D_3u(c, m, n, a)} \widehat{n}_t - \left( m \frac{D_{12}u(c, m, n, a)}{D_1u(c, m, n, a)} + 1 \right) \widehat{P}_t \\
& - \epsilon_w n \frac{D_{33}u(c, m, n, a)}{D_3u(c, m, n, a)} \widehat{W}_t + a \left( \frac{D_{14}u(c, m, n, a)}{D_1u(c, m, n, a)} - \frac{D_{34}u(c, m, n, a)}{D_3u(c, m, n, a)} \right) \widehat{a}_t
\end{aligned} \tag{D.2}$$

The efficiency condition for money determines the respective demand function. So one gets

$$\begin{aligned}
& \beta D_1u(c, m, n, a) \widehat{P}_{t+1} - \beta D_1u(c, m, n, a) \widehat{\lambda}_{t+1} \\
= & c D_{21}u(c, m, n, a) \widehat{c}_t + m D_{22}u(c, m, n, a) \widehat{M}_t \\
& - D_1u(c, m, n, a) \widehat{\lambda}_t \\
& + [\beta D_1u(c, m, n, a) - m D_{22}u(c, m, n, a)] \widehat{P}_t \\
& + a D_{24}u(c, m, n, a) \widehat{a}_t
\end{aligned} \tag{D.3}$$

The nominal interest rate follows, according to (5.20),

$$-\widehat{P}_{t+1} + \widehat{\lambda}_{t+1} = -\widehat{P}_t - \frac{R}{1+R} \widehat{R}_t + \widehat{\lambda}_t \tag{D.4}$$

in the approximated form, with  $R$  (respective  $r$  for the real rate) as the steady state values. The real rate  $r_t$  was deduced via the Fisher equation (see (5.21)) so that the approximated equation is given by

$$\widehat{\lambda}_{t+1} = -\frac{r}{1+r} \widehat{r}_t + \widehat{\lambda}_t \tag{D.5}$$

Optimal investment is determined from the efficiency condition for  $i_t$ :

$$0 = -\widehat{\lambda}_t + \widehat{\theta}_t + \frac{\phi''}{\phi'} \frac{i}{k} \widehat{i}_t - \frac{\phi''}{\phi'} \frac{i}{k} \widehat{k}_{t-1} \tag{D.6}$$

The first order condition for capital implies:

$$\beta z \widehat{\lambda}_{t+1} + \beta z \widehat{z}_{t+1} + \beta (1 - \delta) \widehat{\theta}_{t+1} - \beta \frac{\phi''}{\phi'} \frac{i}{k} \frac{i}{k} \widehat{i}_{t+1} = -\beta \frac{\phi''}{\phi'} \frac{i}{k} \frac{i}{k} \widehat{k}_t + \widehat{\theta}_t \quad (\text{D.7})$$

Capital evolves over time according to

$$\widehat{k}_t = (1 - \delta) \widehat{k}_{t-1} + \delta \widehat{i}_t \quad (\text{D.8})$$

## D.2 The Labor Market Intermediary's Equation

Since the focus is on a symmetric equilibrium the only equation that remains for the labor market intermediary is the wage index.

$$0 = \frac{1}{2} \widehat{W}_{0,t} + \frac{1}{2} \widehat{W}_{0,t-1} - \widehat{W}_t \quad (\text{D.9})$$

In order to avoid too many variables  $\widehat{W}_{1,t}$  is dropped and replaced by  $\widehat{W}_{0,t-1}$ .

## D.3 Intermediate Goods Firm's Equations

The optimum conditions of profit maximization problem determine the real wage and the rental rate of capital (see (5.38) and (5.39)).

$$0 = (\alpha - 1) \widehat{n}_t + (1 - \alpha) \widehat{k}_{t-1} + \widehat{\xi}_t - \widehat{\lambda}_t + \widehat{a}_t - \widehat{w}_t \quad (\text{D.10})$$

$$0 = \alpha \widehat{n}_t - \alpha \widehat{k}_{t-1} + \widehat{\xi}_t - \widehat{\lambda}_t + \widehat{a}_t - \widehat{z}_t \quad (\text{D.11})$$

The production function is given by the Cobb-Douglas-functions of the intermediate goods firms and valid in aggregate variables.

$$0 = -\widehat{y}_t + \alpha \widehat{n}_t + (1 - \alpha) \widehat{k}_{t-1} + \widehat{a}_t \quad (\text{D.12})$$

The Taylor approximation for optimal price setting (5.41) is given by

$$-\beta \mu \phi_p \widehat{P}_{t+1} = \mu (1 - \epsilon_p) \widehat{\lambda}_t - (\mu \phi_p + \beta \mu \phi_p) \widehat{P}_t + \epsilon_p \widehat{\xi}_t + \mu \phi_p \widehat{P}_{t-1} \quad (\text{D.13})$$

## D.4 Market Clearing Conditions and Other Equations

The Taylor expansion of the aggregate market clearing condition is given by<sup>1</sup>

$$0 = -\widehat{y}_t + \frac{c}{y}\widehat{c}_t + \frac{i}{y}\widehat{i}_t \quad (\text{D.14})$$

The markup  $\mu_t$  is determined by the ratio of price over nominal marginal cost and as there is no steady state inflation it follows that  $\mu_t = 1/\psi_t$ . So the Taylor approximation can be written as

$$0 = \widehat{\mu}_t + \widehat{\psi}_t \quad (\text{D.15})$$

Real marginal cost are linked to the Lagrange multipliers by

$$0 = \widehat{\psi}_t + \widehat{\xi}_t - \widehat{\lambda}_t \quad (\text{D.16})$$

The real wage equation is represented by

$$0 = -\widehat{w}_t + \widehat{W}_t - \widehat{P}_t \quad (\text{D.17})$$

## D.5 The Monetary Authority and Further Equations

To close the model one needs to assume some exogenous process for the money supply. Here it will be assumed that the growth rate of  $\widehat{M}_t$  follows an AR(1)-process. This means that the level of money will follow an AR(2)-process (see the discussion in the main text). In order to model this properly one has to add the equation

$$0 = \widehat{M}_t - \widehat{g}_{M_t} \quad (\text{D.18})$$

where  $\widehat{g}_{M_t}$  is the exogenous stochastic process that will have the same characteristics as  $\widehat{M}_t$ .

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<sup>1</sup>The adjustment cost term does not appear in this equation because the steady state inflation rate is zero in this model. This is fundamentally different for a positive inflation rate, see Gerke (2003), p. 175. It also confirms the result of Ascari (2003b) that a positive inflation rate not only changes the steady state (long run properties of the model) but also the dynamics (short run properties).

As it is interesting to study the implications for the inflation rate  $\Pi$  this equation is further added to the system:

$$0 = -\widehat{\Pi}_t + \widehat{P}_t - \widehat{P}_{t-1} \quad (\text{D.19})$$

There are now 22 variables

$$\begin{aligned} & \widehat{c}_t, \widehat{i}_t, \widehat{y}_t, \widehat{\lambda}_t, \widehat{\theta}_t, \widehat{k}_t, \widehat{k}_{t-1}, \widehat{n}_t, \widehat{w}_t, \widehat{z}_t, \widehat{\mu}_t, \\ & \widehat{\psi}_t, \widehat{r}_t, \widehat{R}_t, \widehat{P}_t, \widehat{P}_{t-1}, \widehat{W}_{0,t}, \widehat{W}_{0,t-1}, \widehat{W}_t, \widehat{\Pi}_t, \widehat{M}_t, \widehat{\xi}_t \end{aligned}$$

but only 19 equations so three tautologies must be added to the model. These are

$$\widehat{W}_{0,t} = \widehat{W}_{0,t} \quad (\text{D.20})$$

$$\widehat{P}_t = \widehat{P}_t \quad (\text{D.21})$$

$$\widehat{k}_t = \widehat{k}_t \quad (\text{D.22})$$

# Appendix E

## Optimal Monetary Policy in a Monetary Stochastic Dynamic General Equilibrium Model with Price Staggering

### E.1 The Real Variables

The linearized equations for the firms' labor inputs are given by

$$0 = -\widehat{\Omega}_t + \widehat{\rho}_{0,t} + \widehat{a}_t \quad (\text{E.1})$$

$$0 = -\widehat{\Omega}_t + \widehat{\rho}_{1,t} + \widehat{a}_t \quad (\text{E.2})$$

A hat ( $\widehat{\phantom{x}}$ ) represents the relative deviation of the respective variable from its steady state ( $\widehat{a}_t = (a_t - a) / a$ ). For the consumption levels one gets

$$\begin{aligned} 0 = & \phi n D_{13} x(c_0, c, n, a) \widehat{n}_t + [\phi c_0 D_{11} x(c_0, c, n, a) + \Lambda c_0 D_{11} c(c_0, c_1)] \widehat{c}_{0,t} \\ & + \Lambda c_1 D_{12} c(c_0, c_1) \widehat{c}_{1,t} + \phi c D_{12} x(c_0, c, n, a) \widehat{c}_t + \Lambda D_1 c(c_0, c_1) \widehat{\Lambda}_t \\ & - \rho_0 \widehat{\rho}_{0,t} + \phi D_1 x(c_0, c, n, a) \widehat{\phi}_t + \phi a D_{14} x(c_0, c, n, a) \widehat{a}_t \end{aligned} \quad (\text{E.3})$$

$$\begin{aligned} 0 = & \phi n D_{13} x(c_1, c, n, a) \widehat{n}_t + \Lambda c_0 D_{21} c(c_0, c_1) \widehat{c}_{0,t} \\ & + [\phi c_1 D_{11} x(c_1, c, n, a) + \Lambda c_1 D_{22} c(c_0, c_1)] \widehat{c}_{1,t} + \phi c D_{12} x(c_1, c, n, a) \widehat{c}_t \\ & + \Lambda D_2 c(c_0, c_1) \widehat{\Lambda}_t - \rho_1 \widehat{\rho}_{1,t} + \phi D_1 x(c_1, c, n, a) \widehat{\phi}_{t-1} \\ & + \phi a D_{14} x(c_1, c, n, a) \widehat{a}_t \end{aligned} \quad (\text{E.4})$$

It should be noted that the equality of  $c_0$ ,  $c_1$  and  $c$  is not yet considered here in order to make clear the different derivatives of the  $x$ -function with respect to  $c_0$ ,  $c_1$  and



$c$ .<sup>1</sup> The condition for aggregate consumption results in

$$\begin{aligned}
0 = & [nD_{12}u(c, n, a) + \phi nD_{23}x(c_0, c, n, a) + \phi nD_{23}x(c_1, c, n, a)] \widehat{n}_t \\
& + \phi c_0 D_{21}x(c_0, c, n, a) \widehat{c}_{0,t} + \phi c_1 D_{21}x(c_1, c, n, a) \widehat{c}_{1,t} \\
& + [cD_{11}u(c, n, a) + \phi cD_{22}x(c_0, c, n, a) + \phi cD_{22}x(c_1, c, n, a)] \widehat{c}_t \\
& - \Lambda \widehat{\Lambda}_t + \phi D_2x(c_0, c, n, a) \widehat{\phi}_t + \phi D_2x(c_1, c, n, a) \widehat{\phi}_{t-1} \\
& + [aD_{13}u(c, n, a) + \phi aD_{24}x(c_0, c, n, a) + \phi aD_{24}x(c_1, c, n, a)] \widehat{a}_t
\end{aligned} \tag{E.5}$$

whereas for aggregate labor the linearized equation is

$$\begin{aligned}
0 = & [nD_{22}u(c, n, a) + \phi nD_{33}x(c_0, c, n, a) + \phi nD_{33}x(c_1, c, n, a)] \widehat{n}_t \\
& + \phi c_0 D_{31}x(c_0, c, n, a) \widehat{c}_{0,t} + \phi c_1 D_{31}x(c_1, c, n, a) \widehat{c}_{1,t} \\
& + [cD_{21}u(c, n, a) + \phi cD_{32}x(c_0, c, n, a) + \phi cD_{32}x(c_1, c, n, a)] \widehat{c}_t \\
& + \Omega \widehat{\Omega}_t + \phi D_3x(c_0, c, n, a) \widehat{\phi}_t + \phi D_3x(c_1, c, n, a) \widehat{\phi}_{t-1} \\
& + [aD_{23}u(c, n, a) + \phi aD_{34}x(c_0, c, n, a) + \phi aD_{34}x(c_1, c, n, a)] \widehat{a}_t
\end{aligned} \tag{E.6}$$

The production functions (resource constraints) must obey

$$0 = \widehat{n}_{0,t} - \widehat{c}_{0,t} + \widehat{a}_t \tag{E.7}$$

$$0 = \widehat{n}_{1,t} - \widehat{c}_{1,t} + \widehat{a}_t \tag{E.8}$$

and for the consumption aggregator one arrives at

$$0 = \frac{1}{2} \widehat{c}_{0,t} + \frac{1}{2} \widehat{c}_{1,t} - \widehat{c}_t \tag{E.9}$$

A crucial condition for the dynamic responses is the linearized implementation constraint:

$$\begin{aligned}
& \beta n D_3x(c_1, c, n, a) \widehat{n}_{t+1} + \beta c_1 D_1x(c_1, c, n, a) \widehat{c}_{1,t+1} + \beta c D_2x(c_1, c, n, a) \widehat{c}_{t+1} \\
& = -n D_3x(c_0, c, n, a) \widehat{n}_t - c_0 D_1x(c_0, c, n, a) \widehat{c}_{0,t} - c D_2x(c_0, c, n, a) \widehat{c}_t \\
& - a D_4x(c_0, c, n, a) \widehat{a}_t - \beta a D_4x(c_1, c, n, a) \widehat{a}_{t+1}
\end{aligned} \tag{E.10}$$

The labor resource constraint results in

$$0 = \frac{1}{2} \widehat{n}_{0,t} + \frac{1}{2} \widehat{n}_{1,t} - \widehat{n}_t \tag{E.11}$$

So far the system contains eleven equations and twelve variables. These are

$$\widehat{n}_{0,t}, \widehat{n}_{1,t}, \widehat{c}_{0,t}, \widehat{c}_{1,t}, \widehat{c}_t, \widehat{n}_t, \widehat{\rho}_{0,t}, \widehat{\rho}_{1,t}, \widehat{\Omega}_t, \widehat{\Lambda}_t, \widehat{\phi}_t, \widehat{\phi}_{t-1}$$

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<sup>1</sup> $D_i x(\cdot)$  denotes the first partial derivative of the  $x$ -function with respect to the  $i$ -th argument. Similarly  $D_{ij} x(\cdot)$  denotes the partial derivative of  $D_i x(\cdot)$  with respect to the  $j$ -th argument.

To close the system one has to add one extra equation. This is the tautology  $\phi_t = \phi_t$  which is given in linearized form by

$$\widehat{\phi}_t = \widehat{\phi}_t \quad (\text{E.12})$$

## E.2 The Nominal Variables

As is known from the main text the nominal interest rate  $R_t$  is determined from the efficiency condition for bond holdings in the household's optimization plan (6.10). The real rate  $r_t$  was deduced via the Fisher equation (see (6.14)) so that the approximated equation is given by

$$\widehat{\lambda}_{t+1} = -\frac{r}{1+r}\widehat{r}_t + \widehat{\lambda}_t \quad (\text{E.13})$$

where  $\lambda_t$  is equal to the marginal utility of consumption  $\partial u(c_t, n_t, a_t)/\partial c_t$ . This implies for the Taylor approximation

$$\begin{aligned} 0 = & nD_{12}u(c, n, a)\widehat{n}_t + cD_{11}u(c, n, a)\widehat{c}_t - D_1u(c, n, a)\widehat{\lambda}_t \\ & + aD_{13}u(c, n, a)\widehat{a}_t \end{aligned} \quad (\text{E.14})$$

The nominal interest rate follows, according to (6.13),

$$-\widehat{P}_{t+1} + \widehat{\lambda}_{t+1} = -\widehat{P}_t - \frac{R}{1+R}\widehat{R}_t + \widehat{\lambda}_t \quad (\text{E.15})$$

in the approximated form, with  $R$  (respective  $r$  for the real rate) as the steady state values. As discussed earlier firms are unable to change their prices for two periods so  $P_{0,t-1} = P_{1,t}$ . The Taylor approximation for this condition is given by

$$0 = -\widehat{P}_{0,t-1} + \widehat{P}_{1,t} \quad (\text{E.16})$$

Using the demand function (6.18) allows to determine the relation between  $P_{0,t}$ ,  $P_{1,t}$  and consumption.

$$\widehat{P}_{0,t} = -\frac{1}{\epsilon}\widehat{c}_{0,t} + \frac{1}{\epsilon}\widehat{c}_{1,t} + \widehat{P}_{1,t} \quad (\text{E.17})$$

The price level is uniquely determined since  $P_{1,t}$  is predetermined and  $P_{0,t}$  is given by (E.17). Using (6.21) one gets

$$0 = \frac{1}{2}\widehat{P}_{0,t} + \frac{1}{2}\widehat{P}_{1,t} - \widehat{P}_t \quad (\text{E.18})$$

The cyclical behavior of money demand can be deduced from (6.12).

$$0 = \widehat{c}_t + \widehat{P}_t - \widehat{M}_t \quad (\text{E.19})$$

Since there are now eight new variables  $(\hat{\lambda}_t, \hat{r}_t, \hat{R}_t, \hat{P}_t, \hat{P}_{0,t}, \hat{P}_{0,t-1}, \hat{P}_{1,t}, \hat{M}_t)$  but only seven equations another tautology must be added to the model. This is

$$\hat{P}_{0,t} = \hat{P}_{0,t} \tag{E.20}$$

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