Stochastic modeling of the spot price of electricity incorporating commodities and renewables as exogenous factors

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Abstract

Models for electricity spot prices are used for a variety of valuation issues, e.g. pricing of (real) options and pricing on the retail market. These issues require adequate stochastic price models. A major requirement is the reproduction of stylized facts of the spot price of electricity. Furthermore, the dependencies on other commodities need to be modeled as well. Combined models of all relevant commodities gain importance in the context of risk management of energy utilities. The complexity of options and of portfolios of energy utilities increases due to the number of products with multiple commodities included. This makes the use of combined models essential.

The spot price of electricity is set by the principle of merit order. The most important drivers for the merit order curve on the EEX market are grid load, generation of renewables and prices of coal, natural gas, oil and emission allowances. We present a model incorporating these influence factors in a functional approximation of the non-deterministic merit order curve. The stylized facts mean reversion, seasonalities, negative prices and price spikes as observable in historical spot prices are reproduced.

The input factors require models as well. The residual load model relies on distinct models for renewables including their increase and grid load. We introduce a gas price model incorporating temperature and oil price as exogenous factors. Three model alternatives are presented for the considered commodity prices. A comparison gives evidence that the cointegration approach describes the dependencies best. Finally, the electricity price model is applied to risk-adequate pricing on the retail market.

Zusammenfassung

Für eine Reihe von Bewertungen werden Strompreismodelle benötigt, beispielsweise für die Bewertung von (Real-) Optionen oder die Risikobewertung bei Kundenverträgen. Diese Anwendungen erfordern stochastische Spotpreismodelle, die die Eigenschaften der Preise sowie die Abhängigkeiten von anderen Brennstoffpreisen wiedergeben. Für das Risikomanagement von Energieversorgern sind kombinierte Preismodelle für alle Brennstoffe und CO_2 von großer Bedeutung. Durch die steigende Zahl von Handelsprodukten, die auf mehreren Brennstoffpreisen basieren, steigt die Komplexität der zu bewertenden Portfolios. Daher sind kombinierte Modelle unverzichtbar.

Der Stromspotpreis wird durch das Prinzip der "merit order" bestimmt. Dabei sind die Netzlast, die Erzeugung durch erneuerbare Energien und die Preise für Kohle, Gas, Öl und CO_2-Zertifikate die wichtigsten Einflussfaktoren auf dem EEX Markt. In dieser Arbeit wird eine funktionale Approximation der nicht-deterministischen "merit order" unter Berücksichtigung der wesentlichen Einflussfaktoren angegeben. Das resultierende Modell reproduziert mean reversion, Saisonalitäten, negative Preise und Preisspitzen der historischen Preise.

Für die Einflussfaktoren des Strompreismodells sind ebenfalls Modelle notwendig. Das Restlastmodell basiert auf Modellen für die Netzlast und die Erzeugung der erneuerbaren Energien unter Berücksichtigung der erwarteten Kapazitätserhöhungen. Im Gaspreismodell werden Temperatur und Ölpreis als exogene Größen verwendet. Für die Brennstoffpreise werden drei verschiedene Modellansätze präsentiert. Der Vergleich dieser Ansätze zeigt, dass die Abhängigkeiten im Kointegrationsansatz am besten berücksichtigt werden. Abschließend wird das Modell zur Bewertung der Risiken in Kundenverträgen angewendet.

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List of symbols

Chapter 3 - Modeling the spot price of electricity

$\mathbf{1}_A(t)$	Indicator function
$\widehat{X}_t$	Forecast of a time series X_t
B_t	Hourly power generation by biomass and minor energy sources
C_t	Daily price of coal in Euro per ton
E_t	Daily price of emission allowances in Euro per ton
g	Polynomial approximation of the merit order curve
G_t	Daily day-ahead price of natural gas in Euro per megawatt hour
L	Lag operator
L_t	Hourly grid load
l_t	Hourly residual load
N_t	Hourly power generation by nuclear energy
O_t	Daily price of crude oil in Euro per barrel
PV_t	Hourly power generation by solar energy
S_t	Hourly day-ahead price of electricity in Euro per megawatt hour
v_t	Hourly average availability of power plants
W_t	Hourly power generation by wind energy
$WN\left(0, \sigma^2\right)$	White noise process with mean 0 and variance σ^2
$X_t^{(Z)}$	Stochastic short term process belonging to the model for Z_t
$Y_t^{(Z)}$	Stochastic long term process belonging to the model for Z_t

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Chapter 4 - Modeling the spot price of natural gas

$\Lambda_{d,w}$ Normalized cumulated heating degree days on day d of winter w, also denoted as Λ_t

Ψ_t Oil price formula

$CHDD_{d,w}$ Cumulated heating degree days on day d of winter w

$HDD_{d,w}$ Heating degree days on day d of winter w, also denoted as HDD_t

T_t Daily average temperature in °C

Chapter 5 - Modeling commodity prices

$\boldsymbol{\beta}$ Cointegrating vector

$\boldsymbol{\Gamma}_i$ Coefficient matrix of the VAR(k) process in VECM representation

$\boldsymbol{\Omega}$ Covariance matrix

$\boldsymbol{\Pi}_i$ Coefficient matrix of the VAR(k) process

χ_t Short term process of the two factor model

Δ Difference operator

$\mathcal{N}_N(\mathbf{0}, \boldsymbol{\Omega})$ N-dimensional normal distribution with mean vector $\mathbf{0}$ and covariance matrix $\boldsymbol{\Omega}$

ρ Correlation of short term and long term process of the two factor model

$\widehat{a}$ Estimator of a parameter a

ξ_t Long term process of the two factor model

$A(z)$ Characteristic polynomial of the VAR(k) process

$H(r)$ Cointegration model of rank r

I_n $(n \times n)$ identity matrix

r Rank of cointegration

1. Introduction

Electricity markets started as local markets where the generation from any power plant was consumed by surrounding companies and households. Due to the increase of transmission networks and the market liberalization in the late 90's local electricity markets developed to national or international markets with competing suppliers. Instead of relying on the local supplier customers began to choose between several suppliers. Generation capacities might exceed the demand for some supplier and vice versa. In general terms, generation and consumption in a supplier's portfolio might not match anymore. On the one hand, suppliers with surplus generation try to sell their power in order to maximize their profit. On the other hand, suppliers with a lack of generation buy electricity to meet their customers' demand. Thus, competition makes suppliers start trading electricity.

A portfolio of customers requires the allocation of strongly varying quantities of electricity at any time. This flexibility might exceed the flexibility of generation assets so that the supplier needs to purchase additional flexibility from the market. In addition to the physical need for electricity, there are more reasons for trading electricity. Future contracts are used to hedge generation and demand of energy utilities. Furthermore, speculators trade future contracts.

Energy exchanges, such as the European Energy Exchange (EEX) in Leipzig, offer a variety of trading products. While electricity with hourly delivery for the following day is traded in the spot market, contracts with longer future delivery periods such as quarters or years are traded in the futures market. In addition to these standard trading products, a wide range of derivatives, partly known from financial markets, is traded in the electricity market: plain vanilla options, multi-exercise options or virtual power plants. These financial instruments increase a supplier's flexibility.

Derivatives in the electricity market need to be priced as any products in financial or energy markets. In some cases, e.g. plain vanilla options on future contracts, well-known concepts from financial markets can be adapted to derive prices. Most electricity derivatives are specific for the energy market. Therefore, an adaptation of concepts from financial markets is not always possible.

1. Introduction

For example, swing options contain a certain number of exercise rights within a certain period of time. Such complex options are not easy to price as optimal exercise strategies are required. Analytic pricing formulas do not exist but Monte Carlo methods are commonly used. Pricing of real-options like power plants or risk-adequate pricing of retail power contracts are further examples where Monte Carlo methods are applied. These examples require simulation models for the underlying electricity prices. Derivatives on electricity can be distinguished with respect to the underlying: prices of future contracts or spot prices. As many derivatives such as multi-exercise options and virtual power plants are based on the spot price, this work focuses on spot price models for electricity.

At first sight, electricity prices are the result of supply meeting demand. As large amounts of electricity generation are not storable at reasonable costs supply may not exceed demand. Thus, the price is set by the demand. This idea is the basis for many spot price models in the literature where the load is used as a fundamental factor. A variety of functional dependencies between load and spot price is proposed. Such models neglect several important influence factors:

1. Fuel prices.

2. Prices of emission allowances.

3. Generation of renewables.

Electricity cannot be generated at the same price at different times due to varying costs of generation. This is due to fluctuations of generation capacities and volatility of commodity prices. For example, coal-fired power plants constitute a considerable part of generation capacities in many markets. Thus, the price of electricity is affected by new coal-fired power plants as well as by changes of prices of coal and emission allowances. The former makes the new plant replace more expensive power plants on the merit order curve. Volatility of commodity prices has a direct influence on generation costs of each power plant. Thus, increased commodity prices directly lead to increased electricity prices. Both influences need to be covered by an adequate spot price model.

In this work a new stochastic model for the spot price of electricity is introduced. This model accounts for fluctuations of demand, commodity prices, renewables and capacities. To our knowledge there is no model in the literature combining all these influence factors in one model framework. We introduce a function of residual load and prices of coal, gas and emission allowances to explain the spot price behavior. Especially, the inclusion of commodity prices

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increases the variety of possible applications of our model. The valuation of thermal power plants relies on models covering electricity prices as well as prices of the underlying fuel and prices of emission allowances. Derivatives on electricity or any of the commodity prices mentioned above can be priced by our model. This includes load-dependent products such as full-service contracts in the retail market.

Price formation in the EEX market is the result of an auction where bids and offers of energy utilities determine the price. Within this auction the so-called merit order curve is constructed. A function of residual load and commodity prices in our model is an approximation of the merit order curve. Deviations from the curve and model errors are covered by a stationary stochastic process. Apart from the deduction of the spot price model we introduce models for the underlying input variables.

The residual load is composed of grid load, nuclear generation and generation of renewables. A grid load model including seasonalities and a long term factor for economic influences is proposed. This component induces seasonalities to the spot price model. Recently, the spot price behavior changed due to the strong increase of renewables and the decrease of nuclear capacities. Both changes and expectations about future capacities are regarded in the corresponding models.

In addition to the residual load model we present models for the prices of gas, coal, oil and emission allowances. A gas price model incorporating temperature and oil price as exogenous factors is introduced. It is shown that these fundamental components are important drivers of the gas price. The oil price is part of a combined model with prices of coal and emission allowances. Due to this model framework correlations between prices of gas, coal, oil and emission allowances are incorporated in the electricity price model.

As an alternative approach we present a cointegrated model for the four commodities considered within this work. This concept describes a stronger relationship than correlation between these variables. Evidence for this alternative is given by an out-of-sample validation using the clean dark spread. This spread is the basis for the valuation of coal-fired power plants being one of the typical applications of a multi-commodity model. Apart from valuations of further (real) options on the wholesale markets, there are applications for the model in the retail market. We present risk factors for suppliers participating in the retail market including a model framework to derive prices for typical retail power contracts. As a spot price model including the grid load is essential within this framework we can apply our new model. The application to a retail pricing example gives further evidence for the reliability of our models.

Structure of the thesis

After this introduction we give a short overview of the commodity markets that are relevant within this thesis. We restrict ourselves to the information necessary to understand the following modeling issues. In Chapter 3 our spot price models are introduced. We describe the price formation on the EEX market and derive adequate spot price models. These are categorized into the existing literature. The underlying models for the residual load are presented as well. The gas price model is presented in Chapter 4. Different modeling approaches for commodity prices are presented in Chapter 5.

After introduction of all models their features are compared in Chapter 6. This includes an out-of-sample model validation. Chapter 7 contains the application of the model to the issue of risk-adequate pricing of retail power contracts. The results of the thesis are concluded in Chapter 8.

2. Commodity markets

Modeling of commodity prices requires knowledge about the market structures and the traded products. We give a short overview of the relevant markets for our models. This includes the choice of adequate data to estimate the model parameters. For further details on commodity markets we refer to more comprehensive publications like Burger et al. (2007) and Geman (2005).

2.1. Electricity market

There are two characteristics that distinguish electricity from other commodities:

1. Electricity is not storable to a large extent. The generation exceeds the capacities of (hydro pumped) storages by far.

2. Transport of electricity is linked to transmission networks.

These facts have consequences for the market structure, price behavior and the products.

Which market is considered? Due to transmission networks the electricity market is not global. Adjacent national electricity markets start to link to one another but the capacities give natural constraints. Therefore, we focus on the presentation of the German electricity market which is relevant for our modeling. The trade of standardized contracts in Germany takes place on the European Energy Exchange (EEX) in Leipzig (see European Energy Exchange (2013)). In addition to the trade on the exchange, there is an over-the-counter (OTC) market for any kind of contracts. The standard products are traded OTC as well as unusual specifications like peakload on weekends.

Where are the producers? Various energy utilities operate nuclear, coal-fired, gas-fired and petrol-fired power plants. A wide range of renewable energy sources owned by energy utilites, industrial companies or even private households contributes to the total generation as well.

What products are traded? A major part of the electricity trading takes place in the **futures market**. Future contracts describe delivery of electricity during a certain period of time (year, quarter, month, week). A further distinction of future contracts is given by the delivery hours. So-called **baseload contracts** guarantee delivery of energy in every hour of the period. In contrast, delivery of **peakload contracts** is restricted to 8 a.m. till 8 p.m. on working days. Both contract types are used for supply (physical) or hedging (financial) purposes. These are standard products liquidly traded on the EEX.

In addition to the futures market there is a **day-ahead market**. Everyday the EEX determines the price of electricity for every hour of the following day in an auction. The power plant operators submit quotes, i.e. a list with available capacities and corresponding prices, and the potential buyers submit their bids. The EEX publishes those 24 prices that lead to a match of bids and offers in every hour. More details on the formation of prices are given in Section 3.1. In the following the prices on the day-ahead market are referred to as **spot prices**.

Who are the traders? Market participants on the exchange and OTC market can be differentiated into three groups:

- **Energy utilities** trade for financial and physical purposes.

- **Banks** participate in the markets for speculation.

- Some major **industrial companies** have market access to buy electricity for their own use.

Due to trading volumes the price is set by energy utilities and banks. Their focus is on liquidly traded products like the baseload contract for the front year. Front year describes the first yearly contract being traded. E.g. the front year contract in July 2012 would be the future contract with delivery in 2013. The corresponding terms front month and front quarter are used as well. Yearly contracts for the following years are less liquidly traded but important

for energy utilities hedging their generation of electricity.

Which data is used? Recent future prices are needed to include all available market information into our spot price models. Market participants might have knowledge about price relevant information that cannot be deduced from historical spot prices. Therefore, we include future prices as described in Section 3.5. The historical prices of the future contracts for 2013 are given in Figure 2.1.
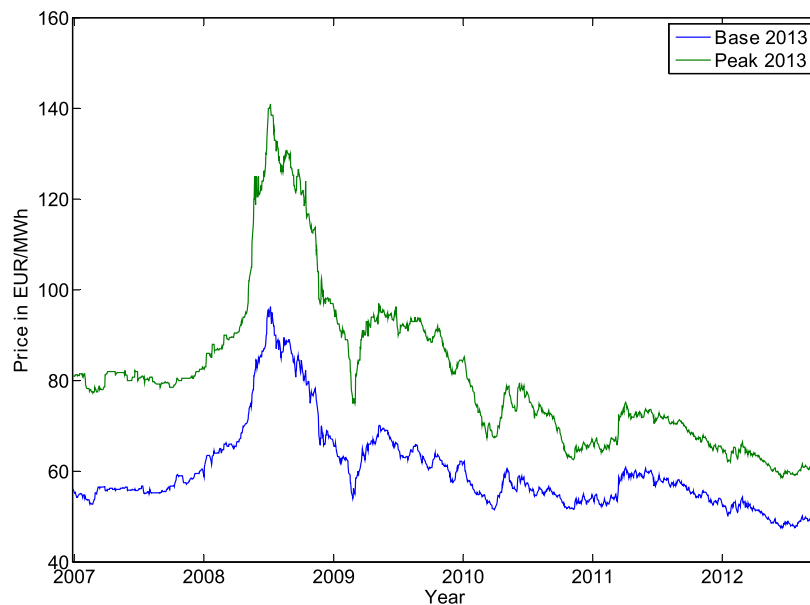
Figure 2.1.: Prices of baseload and peakload contracts for the year 2013 as quoted on the EEX from January 2007 till September 2012.

The most relevant data in this work concerns the spot price of electricity. We use the prices on the EEX for parameter estimations of the model. Amongst others, the minimum price of -500.02 EUR/MWh at 2 a.m. on 2009/10/04. is cut off to ensure that some structures can be observed in Figure 2.2. The contrast between spot prices and the prices of future contracts seen in Figure 2.1 is due to delivery periods: The spot prices are prices for delivery in a single hour of the following day. The prices of future contracts describe delivery within the year 2013. This means that any kind of seasonality within the year is removed by considering an average price for the year. As future prices are average prices for a delivery period they usually do not exhibit extreme prices

as spot prices do. Extreme spot prices occur in various situations such as a combination of low load and high infeed from renewable energy (low prices) or power plant outages (high prices). Examples of typical price patterns within a
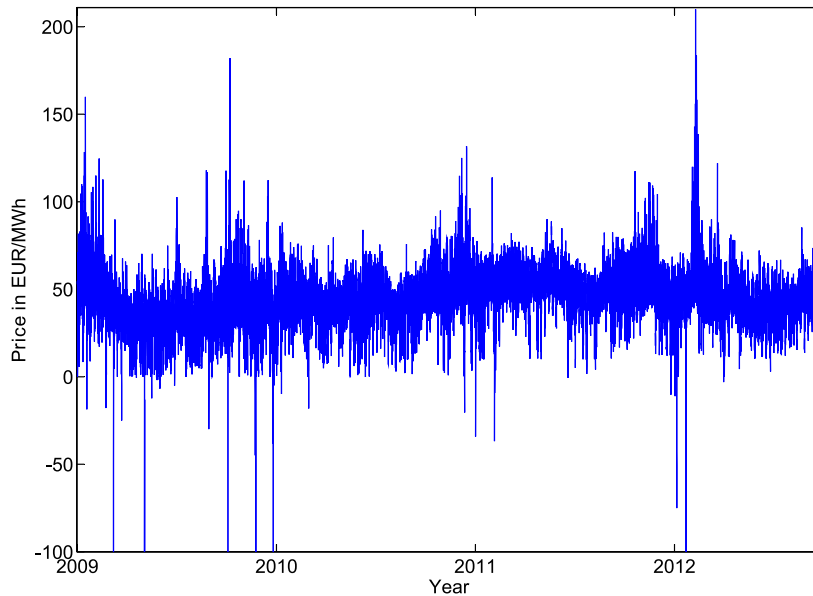
Figure 2.2.: Spot prices of electricity on the EEX from January 2009 till September 2012. Prices below -100 EUR/MWh are cut.

week are given in Figure 2.3. Prices at night and on weekends are remarkably lower than at daytime or on weekdays. Further typical price patterns such as maximal prices at noon have changed due to the increase of generation by renewables (compare Section 3.1).

2.2. Coal market

The market for black coal is not a local or regional but a global market. This is due to the resources and the ports all over the world. As lignite plays a minor role within this work the term coal always refers to black coal.

Which market is considered?   The German electricity prices are influenced by prices of coal in Germany. As the German coal production decreased dur-
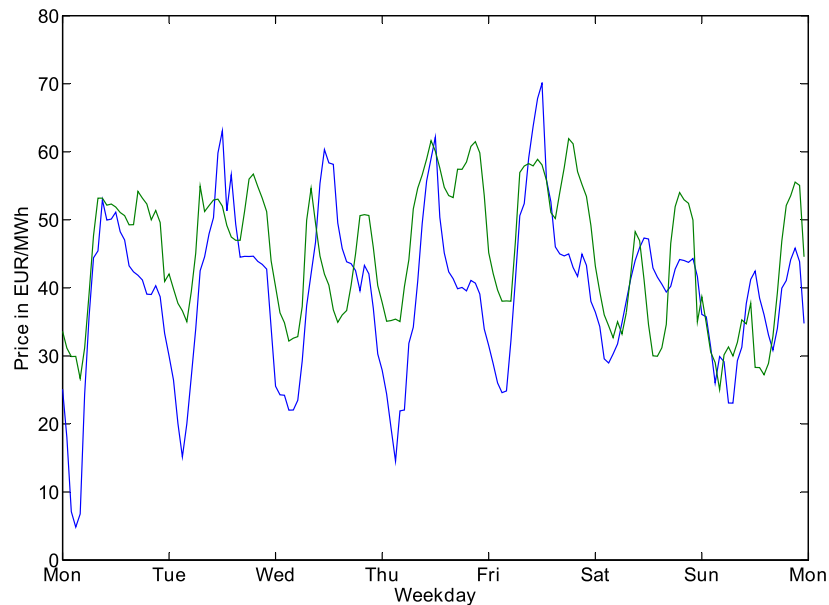
Figure 2.3.: Spot prices of electricity on the EEX from 2009/08/17 till 2009/08/23 (blue line) and from 2012/08/13 till 2012/08/19 (green line).

ing the last decades the coal prices in Germany are set by coal imported from abroad. Coal prices on the EEX and coal prices on the IntercontinentalExchange (ICE), London, (see IntercontinentalExchange (2013)) both refer to imported coal. The prices on the ICE are more reliable due to higher trading volumes. Both exchanges offer various standardized products with future delivery. A spot market meaning immediate delivery does not exist due to the transportation time of coal.

Where are the producers?   The major producers are China, United States, India, Russia, Australia, South Africa, Indonesia, Poland, Kazakhstan, Colombia and Ukraine. Due to their own consumption and freight charges not all of these countries export coal to any other country in the world. The most important coal producers for Germany are Colombia, Russia (and the countries of the former Soviet Union), United States and South Africa.

What products are traded?   As coal is a heterogenous commodity there are different qualities. These differences result from varying carbon, energy,

sulphur and ash content. These contents influence caloric value and emissions. As a consequence the price of coal is dependent on the quality.

Another influence factor for the price is given by the Incoterms (international commerce terms). These terms specify the allocation of costs concerning the transport:

- **Free on board (FOB)**: The seller delivers to the port of shipment and pays the loading costs. As soon as the goods are on the ship the buyer bears all other costs.

- **Cost, insurance and freight (CIF)**: All costs of transportation and insurance are included in the selling price.

Varying quality, transportation costs and Incoterms lead to distinct prices for coal. Therefore, physical coal trading is done by OTC deals. The API#2 index is the average price of a certain coal quality being delivered CIF ARA (Amsterdam, Rotterdam, Antwerp) within the next 90 days. Although it is not guaranteed that all coal deals are captured by the index it is the most important benchmark for physical coal trades in Central Europe. Further price indexes for different regions, different qualities and different Incoterm specifications exist but are not relevant in this work.

In addition to the physical trade there is an extensive trade of financial coal contracts on the ICE. Contracts for months, quarters, seasons (summer and winter) and years are traded. The API#2 index describing delivery within the next 90 days is the closest to spot. But for reasons of liquidity we use prices of the front month contract traded on the ICE instead of the API#2 index prices.

Who are the traders? There are four major groups of market participants in the exchanges and OTC that influence the price:

- **Energy utilities** trade for financial and physical purposes.

- **Banks** participate in the markets for speculation.

- **Metallurgical industrial companies** need coal for their production processes.

- **Coal producers** sell their production.

Which data is used? All coal prices are quoted in US dollars. As we analyze the coal price effect on the electricity price quoted in Euro coal prices in Euro per ton are used. Forward exchange rates are used for the conversion. The price data consisting of five different coal futures traded on the ICE is shown in Figure 2.4. For the electricity price model the front month prices of coal are used as an input variable. All futures are required to set up the coal price model in Section 5.3.
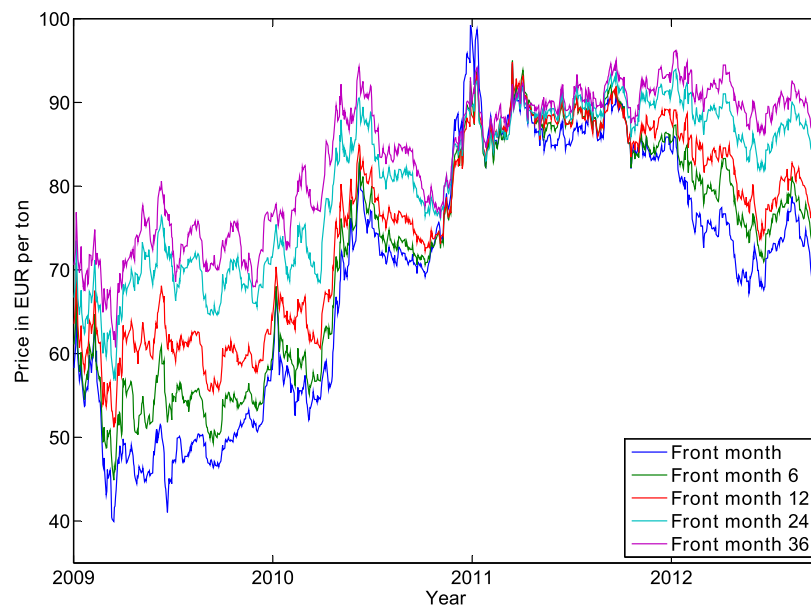
Figure 2.4.: Prices of coal futures with different maturities from January 2009 till September 2012.

2.3. Gas market

The trade of natural gas developed later than the trade of coal but due to the increasing importance of gas as a fuel for heating purposes the market is rapidly growing.

Which market is considered? The major way of transportation is a gas pipeline. Therefore, the market for gas is a regional one. The German gas

market was split into several market areas. After some steps of consolidation the two market areas Gaspool and NetConnect Germany (NCG) are left. The Dutch market is covered by the trading hub Title Transfer Facility (TTF). The Dutch gas market started earlier than the German and leads to a longer history of price data. The high pipeline capacities between both countries eliminate arbitrage possibilities. Both markets consist of various standardized trading products in the spot and future market. In Germany the EEX allows for trading in both market areas. The European Energy Derivatives Exchange (ENDEX), Amsterdam, is the exchange for TTF (see European Energy Derivatives Exchange (2013)).

Where are the producers?   The main sources of natural gas in the Western European gas markets are Russia, Norway and the Netherlands. Especially the German gas market relies on imported gas from these countries. The increasing number of liquefied natural gas (LNG) transports to Central Europe increase the available quantities as well as the variety of sources. LNG transports, e.g. from Qatar, arrive at British or Dutch LNG terminals and influence the German market as well.

What products are traded?   Spot and forward products are liquidly traded in both markets. Buying a forward contract on gas means delivery of a certain quantity during the entire delivery period. Standardized delivery periods on exchanges are months, quarters, years and seasons (October till March, April till September). The so-called spot market is a day-ahead market. The "spot" price quoted on one day is a price for delivery of gas on the next business day. In this context the day starts at 6 a.m. and ends at 6 a.m. on the following day. Another specialty concerns the weekends: A weekend price is quoted in addition to the day-ahead price. The weekend price is the price for delivery of gas on Saturday and Sunday. As a consequence the day-ahead price quoted on Friday describes the price for delivery on Monday.

Who are the traders?   Only two groups trade in the spot market for gas:

- **Energy utilities** supply their customers and operate gas-fired power plants and gas storages.

- **S**ome major **industrial companies** have access to the market to buy gas for their own use.

Which data is used? The spot price of natural gas is used to explain the spot price of electricity. As the ENDEX provides a longer consistent history of spot prices we choose this price data. As argued above the price differences are negligible. Using day-ahead prices for weekdays and weekend prices for Saturday and Sunday a daily time series of spot prices can be constructed. This data is shown in Figure 2.5. Extreme spot prices can occur in analogy to power spot prices. For example, technical problems with gas storages in winter immediately increase the price.
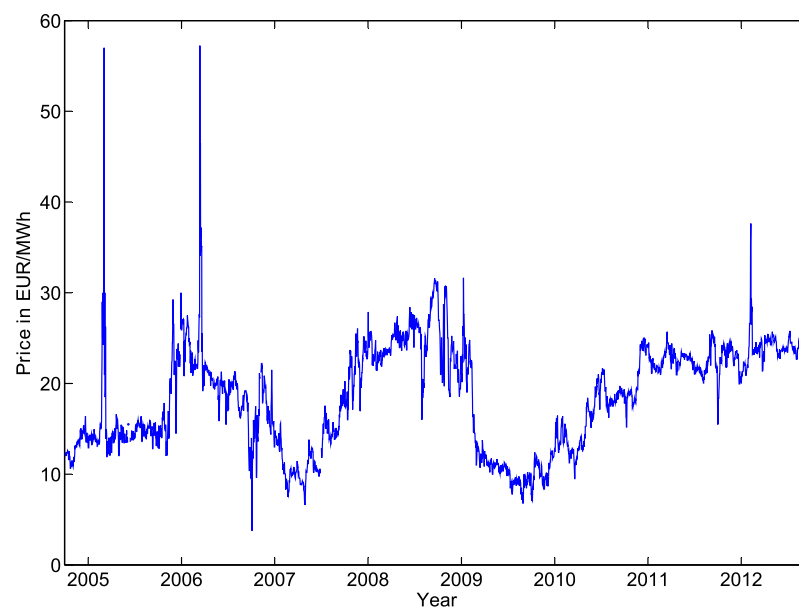
Figure 2.5.: Spot prices of gas on TTF from October 2004 till September 2012.

Prices of future contracts are needed for the combined commodity price model in Section 5.4. The time series of front month prices for the TTF market area is given in Figure 2.6. This data contains the decreasing prices as a consequence of the crisis 2008/2009.

2.4. Oil market

Throughout this work oil is the abbreviation for crude oil. Reasons why gasoil or fueloil are not used are given in the particular chapters.
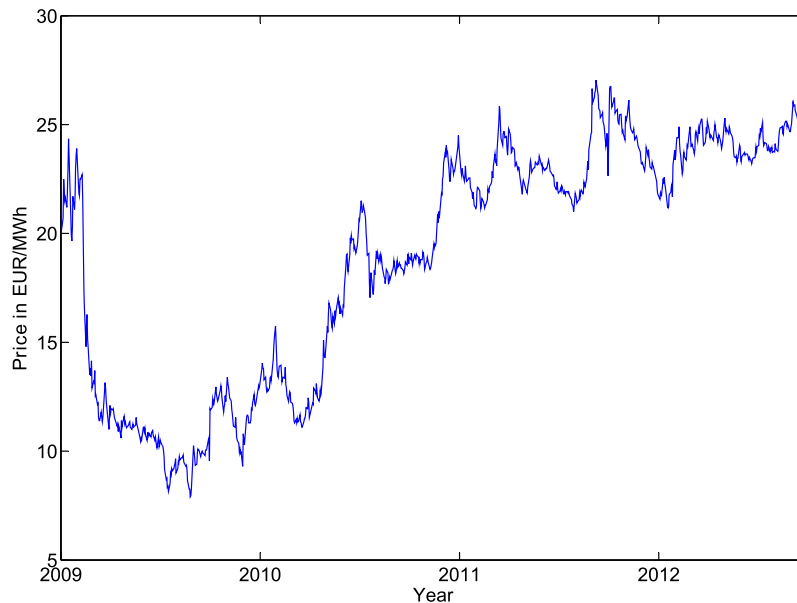
Figure 2.6.: Prices of front month contracts of gas on TTF from January 2009 till September 2012.

Which market is considered?   The oil market is not a regional but a global market. The crude oil grade Brent is the most relevant for the European market. Liquid trading of future contracts takes place on the ICE. Physical trades are OTC and include an oil price as well as costs for transportation. As these costs differ there is no standardized trade of physical oil shipments on an exchange.

Where are the producers?   Major oil reserves can be found in the Middle East, the former Soviet Union and North America. Countries without any considerable oil reserves, like Germany, rely on supply from these regions (or other oil producers not mentioned here).

What products are traded?   Similar to coal, oil has different qualities depending on the source. The most important characteristics are viscosity and sulphur content. In order to achieve some standardization for trading purposes some oil qualities became benchmarks. The relevant price index for oil in Germany is the Brent index. This index describes the North Sea oil market with crude oil having 0.3% sulphur content and 38° API viscosity. Although the

oil reserves in the Brent area declined the price index is still used in Europe. The exact price of an oil shipment is determined by the transport costs, the Incoterms and the oil price itself.

Monthly financial future contracts are liquidly traded on the ICE and OTC. Price quotes for the upcoming 36 front month are available. Physical oil trades are OTC deals. Their prices are closely related to the exchange prices for the corresponding period of time. The energy news service Platts (see Platts (2013)) assesses prices for physical Brent shipments loading FOB 10-25 days forward. These prices are known as dated Brent. Spot prices as known from other markets do not exist due to the transportation by ship.

Who are the traders?   The trading volumes in the financial oil market exceed the physically traded volumes by far. Financial trades take place for reasons of speculation or hedging of physical demand.

- **Banks and funds** trade for speculation.

- **Energy utilities** need to supply their power plants with oil and hedge these prices. Speculation is another purpose of them.

- The highest physical demand have **refineries**.

- The **chemical industry** needs oil as the basis for many processes.

Which data is used?   As prices of financial future contracts for the front month and prices of dated Brent nearly refer to the same period of time we choose the more liquid prices of future contracts for our model calibration. Another reason is the easier availability as prices quoted on the ICE are public information.

As we use historical data for the parameter estimation in our model we need to make sure that currencies of electricity, gas and oil prices are consistent. To exclude any future currency effects Brent prices are converted from Dollar per Barrel into Euro per Barrel by means of forward exchange rates. Though the spot price model for electricity relies only on the front month contract we need prices of Brent futures with five different delivery months (see Figure 2.7) for the commodity price model.
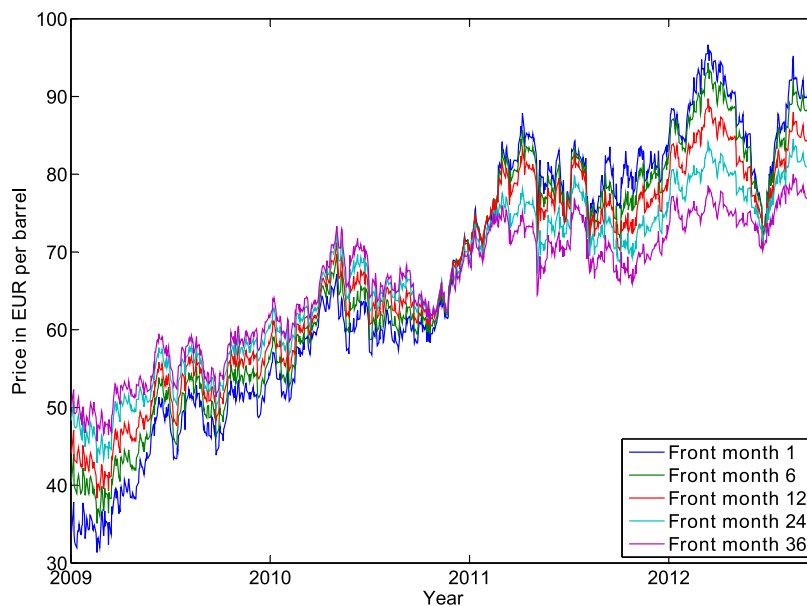
Figure 2.7.: Prices of Brent futures with different maturities from January 2009 till September 2012.

2.5. Emissions market

In order to reduce emissions the European Union decided that companies in industries with high carbon dioxide emissions need allowances for their emissions. Therefore, the European Union started the European Union Emission Trading System (EU ETS) in 2005. The EU ETS consists of trading periods. The first period from 2005 to 2007 and the second period from 2008 to 2012 started with allocation of allowances to companies that cause emissions. As these allocations did not exactly match the realized demand for allowances the companies started trading. By the end of April allowances for the emissions of the year before need to be submitted. In the third period from 2013 to 2020 the allocation changes and companies need to buy all allowances. Nevertheless, a sufficient number of allowances can be achieved by means of trading. A lack of allowances causes high penalties. For more detailed information including the official documents see European Commission (2013).

Which market is considered? Emission allowances are traded on the EEX and ICE. Due to its size and the relevance for Europe the trade on the ICE

is more liquid than on the EEX. In addition, an extensive OTC trade takes place. There is no arbitrage between prices on the ICE and prices on the EEX or OTC.

Where are the producers? Allowances are allocated to certain companies owning plants with high emissions. These companies can sell surplus allowances on the market.

What products are traded? One contract for the upcoming months March, June, September and December is quoted on the ICE. Contracts with delivery in December are quoted for several years. In contrast to commodities such as electricity or gas, allowances can be delivered at one point of time. The product specifications on the ICE determine the day of delivery as the last Monday of the month being a business day. If there are less than four business days following the last Monday, the expiry will be on the penultimate Monday. Spot trade of allowances with immediate delivery is possible in case of emission allowances but it only takes place OTC.

Who are the traders? The market participants on the market for emission allowances have different trading purposes.

- All **industrial companies** emitting CO_2 sell surplus allocated emission allowances or buy additional allowances.

- **Energy utilities** hedge their future emissions due to planned electricity generation by their power plants.

- **Banks and financial institutions** speculate on prices of emission allowances.

Which data is used? The number of allocated allowances in period one exceeded the demand. The companies realized this situation at the end of the period and sold their surplus of allowances. As a consequence the price declined close to zero. At the beginning of period two, the price raised to the usual level. This price effect is due to the regulations on the emissions market. It is not a result of the typical price behavior. Therefore, we exclude period one from our data.

2. Commodity markets

Although prices for delivery in various months are quoted, the trade focusses on the December contract of the current year. Therefore, we use the price of the December contract for the current year as the basis for our model calibration. The March contract is important as a last chance to get allowances for the previous year. Nevertheless, due to speculation the trading volumes of the December contracts are higher.

In contrast to the other commodities mentioned above we do not need prices close to spot in case of emission allowances. The allowances need to match the emissions by the end of a year. Therefore, there is no difference between buying allowances on the spot market, in December of the same year or in March of the following year. Thus we use the most liquid prices for model calibration, i.e. the prices of the December contract on the ICE. The price data used within this work is shown in Figure 2.8. Several future contracts are used to calibrate the combined commodity price model presented in Section 5.3.
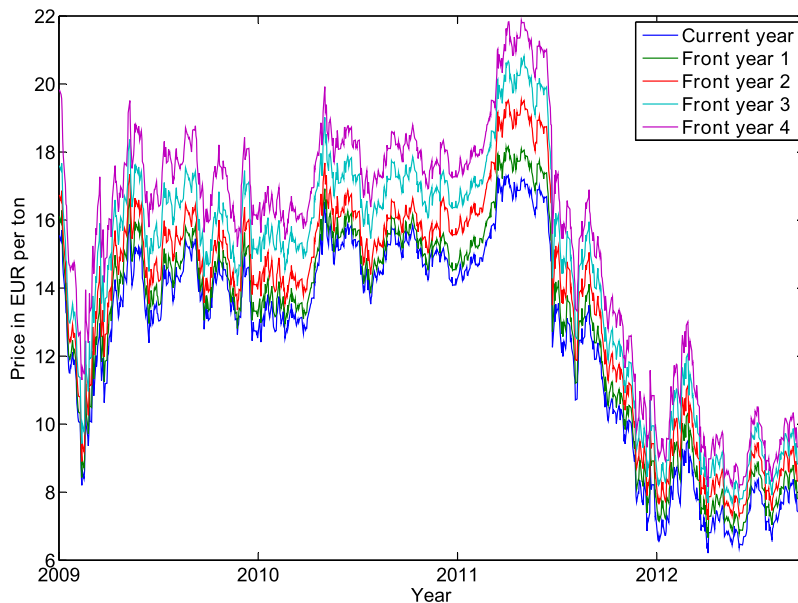
Figure 2.8.: Prices of emission allowances with different maturities from January 2009 till September 2012.

3. Modeling the spot price of electricity

There are various factors with a considerable influence on the spot price of electricity. These factors and their effect on the price behavior are presented in this chapter. Requirements for adequate spot price models can be derived from this list of features. The wide range of existing literature on price models is distinguished with respect to their focus on some of the price features. Finally, we introduce a new model meeting the requirements. As the spot price model depends on load and renewables as input factor we present the corresponding models. A short overview of the model, its alternative and the underlying load model can be found in Section 6.1.

3.1. The merit order curve

The generation of electricity in Germany relies on various energy sources (see Figure 3.1). Traditionally the most important sources were lignite, coal, gas
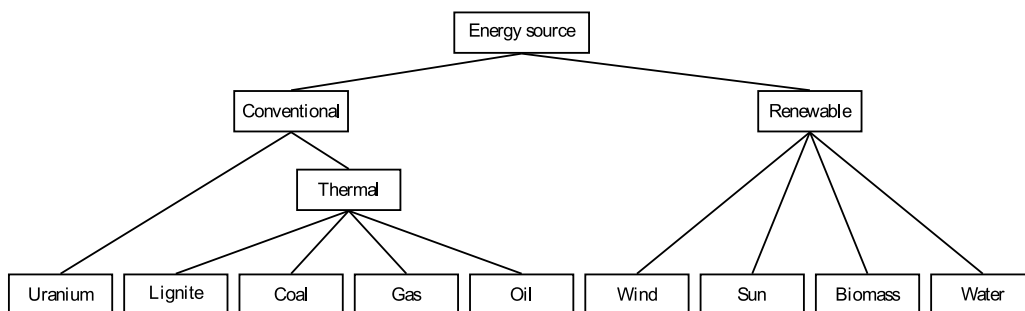
Figure 3.1.: Classification of major energy sources.

and uranium. Within the last years the generation portfolio experienced large changes. Capacities of renewable energy sources, especially wind and photovoltaic (PV) power plants, extremely increased. In contrast the capacities of

nuclear power plants were reduced by the German government as a consequence of the nuclear disaster in Fukushima, Japan, in March 2011. The generation volumes distinguished by energy sources in 2009 and 2011 are given in Figure 3.2 (see Statistisches Bundesamt (2013)). The obvious decrease of electricity generation by uranium is covered by an increase in biomass (major part of "Others"), wind and PV generation.
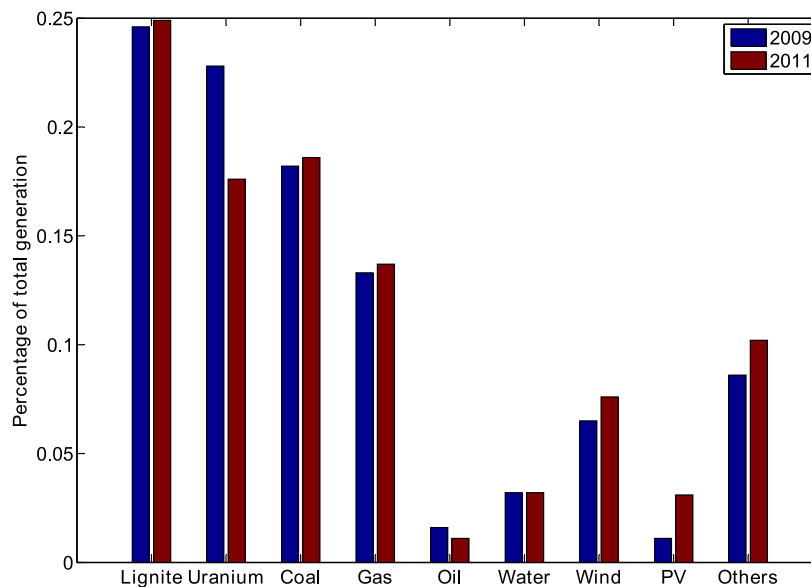
Figure 3.2.: Comparison of percentages of generated electricity by energy sources in Germany for 2009 and 2011. Total generation: 592.4 TWh in 2009 and 614.5 TWh in 2011.

Pricing on the EEX

The total demand for electricity needs to be met by generated electricity from various energy sources and imported quantities. The relation between demand and generation plus import determines the price. The price is determined in a standardized process on the EEX. All market participants submit bids and offers for arbitrary quantities. The bids and offers usually include a maximum buying price and a minimal selling price. For example, the use of various buying price limits might ensure that lower quantities are bought at higher prices. This might be reasonable if a market participant has some expensive

generation capacities to match the own demand. These capacities will only be used above a certain price on the exchange. All orders in the auction are restricted to the price range from -3000 EUR/MWh to 3000 EUR/MWh. The auction takes place every day at 12:00 p.m. for every hour of the following day. 24 order sets need to be submitted by the market participants.

Assuming a market participant with a coal-fired power plant tries to sell its generation on the spot market. Due to the auction structure the result of the auction might be that the plant runs only one hour on that day. But this would not be profitable as there are high starting costs for such power plants. To avoid such problems block orders are allowed in the auction as well. The market participants may place orders for blocks of several adjacent hours instead of single hours. These orders are either completely accepted or completely rejected. This situation allows for the profitable marketing of power plants in the spot market.

The exchange constructs a so-called **merit order curve** from all incoming offers for every hour. The offers are sorted ascending with respect to the prices. The corresponding quantities are cumulated. As the offers are based on power plants with costs that strongly depend on the fuel price the sorted curve consists of several blocks. These blocks can usually be dedicated to the different fuel types. An overlapping of adjacent blocks is possible. New plants of an expensive fuel with a high efficiency might sometimes be cheaper than old plants of a cheaper fuel type with a low efficiency. Therefore, not all plants of one fuel type produce with the same variable costs. Hence, the offers will be different as well. Variable costs resulting from fuel prices are increased by further variable costs and fixed costs of the power plant.

Variable costs of a power plant do not only depend on fuel costs but also on (high) costs for starting the plant. E.g. coal plants need to be preheated which is usually done by burning petrol. In order to avoid these starting costs power plants sometimes produce and sell electricity below fuel costs. For these reasons, the merit order curve is not constructed by sorting the fuel prices and adding up the capacities of the corresponding fuel types. The typical order of power plants (depending on prices) would be nuclear, lignite, coal, gas (as a general term for gas used in gas turbines and combined cycle gas turbines) and oil. Two example of the merit order curve on the EEX are given in Figure 3.3. Small price movements might change place of single power plants within the merit order curve. Extreme price movements might even change places of all power plants of different fuels. For this reason the merit order curve is not constant, hence not easy to identify.

Once the merit order curve for a certain point of time is determined the bids
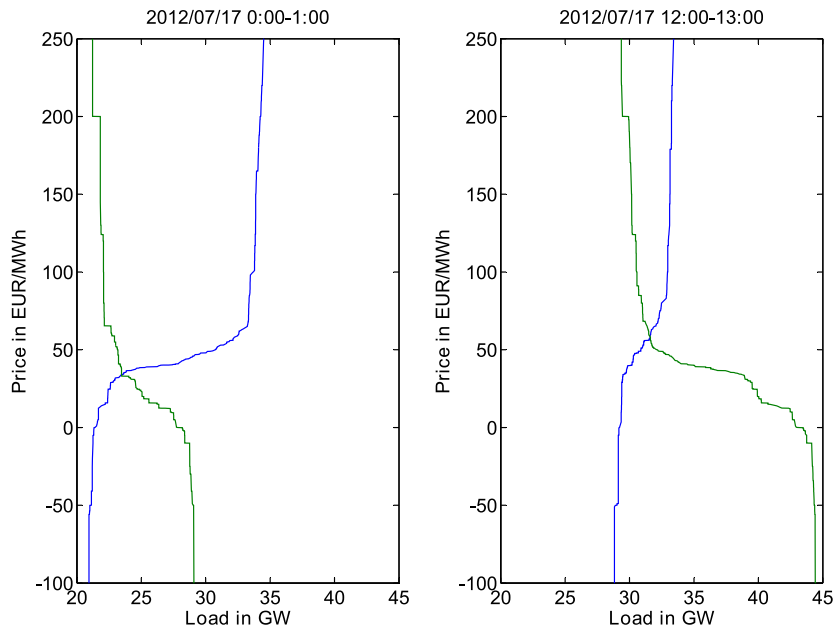
Figure 3.3.: Two examples of the price formation via merit order curve on the EEX on Tuesday 2012/07/17. The bids (green line) and offers (blue line) cut at $[-100, 250]$ cover the total price range $[-3000, 3000]$.

of the market participants are considered. The spot price is determined as the price where bids and offers match (compare Figure 3.3). This price is published by the exchange. As explained in Section 2.1 a market for forward contracts with physical delivery exists. These contracts reduce the demand on the spot market as well as the supply. Therefore, the price level is not affected by the trading volumes in the forward market. The spot price is still set by the costs of producing an additional unit of electricity.

Influence factors on the merit order curve

Apart from changes due to fuel price movements there are changes in the merit order curve due to renewable energy. Electricity from wind, PV, biomass, run-of-river power plants and seasonal storages is (almost) free of variable costs. For regulatory reasons these energy sources have higher priority than conventional energy sources. Renewable energy moves the rest of the merit order curve to the right and takes the place of uranium. Though the installed

capacity is known the generation of renewable energy is unknown. Therefore, we cannot determine how far the merit order curve is moved to the right.

Another type of power plants contributes to the variability of the merit order curve: pumped storage power stations. As long as pumped storages are not used like seasonal storages the assumption of zero costs for pumped storages is not appropriate. At least the costs of pumping the water up again must be covered by the generated electricity. As the use of pumped storages is highly dependent on the load and the prices for pumping we cannot assign a certain place within the merit order to pumped storages.

Due to market coupling with adjacent countries the spot price on the German market depends on exports and imports from abroad. Currently Germany, France, Netherlands, Belgium, Austria and Luxembourg have a simultaneous day-ahead auction. This leads to the optimal use of border capacities as the combined spot price auction on all markets results in trade of surplus generation capacities between these countries. With increasing imports and exports the differences between electricity prices in these countries decrease. As a consequence not even the knowledge about the variable costs of all German power plants would be sufficient to describe the merit order curve and the resulting spot price exactly. Historical import and export data might be available but forecasts for the future are not possible without consideration of all markets. Therefore, market coupling effects are neglected within this work.

Now that we understand the construction of the merit order curve we start considerations with respect to modeling issues. From a fundamental point of view the spot price is the solution of an optimization problem. The market participants optimize their outcome of the auction with respect to the restrictions given by the government, their power plants and their demand. Assumed that commodity prices, generation capacities, variable, fixed and starting costs of all power plants are known we can determine the optimal constellation of operating power plants by means of an optimization algorithm. The most expensive operating power plant determines the spot price. Ignoring imports and exports the daily optimization problem can be stated as

$$\min_{q_{it}} \sum_{t=1}^{24} \sum_{i=1}^{M} q_{it} \cdot f\left(P_t, t, i\right)$$

with respect to

$$\sum_{i=1}^{M} q_{it} = D_t, \qquad t = 1, ..., 24$$

where q_{it} denotes the generation of power plant i at time t, $f\left(P_t, t, i\right)$ describes the costs of power plant i at time t depending on the fuel price P_t, M is the

number of power plants and D_t the demand at time t. Solving this problem results in the quantities q_{it} that need to be generated by the power plants. Then, the spot price is determined as

$$S_t = \max_{\{i:q_{it}>0\}} f\left(P_t, t, i\right), \qquad t = 1, \ldots, 24.$$

Apart from the lack of knowledge about the costs of power plants this modeling approach is not tractable for computational reasons. For the future, commodity prices need to be simulated in this situation. Hence, the optimization algorithm needs to run once for every point in time per path. Typical applications of a simulation model require at least 1000 scenarios for one year. In case of electricity this means $1000 \cdot 8760$ optimizations, i.e. this approach takes too much computation time. Burger et al. (2007) give a more detailed overview of optimization approaches.

As the first idea of exact determination of the merit order curve by the optimization approach is not tractable we need to focus on approximations of the merit order curve. Either we neglect some fundamental influence factors without loosing to much precision or we go for a mathematical approximation which neglects fundamental influence factors. Any model needs to reproduce the characteristics of electricity prices. Hence, we need to know about these. Afterwards we can determine which approach might be able to incorporate the characteristics.

Stylized facts of electricity spot prices

A closer look on the behavior of spot prices in the past reveals some stylized facts. Some of these can be explained by the structure of demand.

- The (average) demand for electricity is higher by day than by night. This leads to a **daily seasonality** of spot prices. Before the increase of renewable energy sources the maximum demand and price was around noon. Due to the increase of PV this price maximum disappeared and the highest prices can be identified around 8 a.m. and 6 p.m (compare Figure 2.3).

- Many industrial companies do not operate on weekends. Companies with shift operation might even have less demand on Friday afternoon as the last shift might stop early and the machines are shut down. The corresponding situation is found on Monday morning. As industrial companies represent a major part of the total demand in Germany the described

weekly seasonality is reflected in the total load and with that in the price.

- In many countries like the USA the electricity demand in summer is higher than in winter due to air conditioning. As air conditioning is not that popular in Germany the load differences between summer and winter are due to the use of electricity for heating systems and light. Hence, the load in winter is higher than in summer. The prices reflect this **yearly seasonality** (compare Figure 2.2).

- The daily seasonality causes price fluctuations on an intra-day basis. These fluctuations typically remain in a certain range. For example, maximum prices are usually below 100 EUR/MWh. As soon as power plants are unavailable, due to either planned maintenance or outage, unexpected high load can cause extreme prices, so-called **price spikes**. An extremely high load might be caused by extremely low temperatures. Even a normal load level can cause spikes if the electricity generation by wind and PV is close to zero. The reason for the high spot prices can be found in the merit order curve. A high load, low renewable electricity generation and unavailability of power plants lead to the right end of the curve where it has a steep slope.

- Spot prices of electricity exhibit **mean reversion**. As soon as the price increases over a longer period new power plants will be constructed, transmission capacities to adjacent countries are increased and load will decrease as companies try to save energy to reduce costs. These measures lead the price back to its mean. The other way around, in case of low spot prices exports will increase, less efficient power plants are shut down and the load increases. Expected long term load changes are captured by new power plants. Hence, the spot price will always fluctuate around its mean.

- The occurrence of **negative prices** is the most recent development in the German spot market of electricity. Due to the increase of renewable energy supply exceeds the demand from time to time. A result of this mismatch are negative prices, i.e. one gets paid to take some of the surplus electricity. Common times for this situation are windy public holidays or windy nights on weekends.

These characteristics need to be included by an adequate spot price model. In the following section we give an overview of the literature on spot price models for electricity.

3.2. Literature on electricity price models

As there is a wide range of literature on price models for electricity we give an overview of some models. This allows for a classification of the model described in Section 3.3.

Electricity prices in various markets have a number of attributes in common: Seasonalities, mean reversion and price spikes (compare Section 3.1). These stylized facts are described by Pilipovic (1998), Eydeland and Wolyniec (2003), Weron (2006) and Burger et al. (2007). These authors give comprehensive overviews concerning risk management and pricing issues on energy markets. Schwartz (1997) and Baker et al. (1998) focus on the mean reversion of commodity prices.

Early well-known price models were published by Schwartz and Smith (2000) and Lucia and Schwartz (2002). Lucia and Schwartz (2002) introduce one and two factor models for (log) prices in the Scandinavian power market. These factors are responsible for short and long term variations of spot prices. Schwartz and Smith (2000) follow a similar approach: In their model a Geometric Brownian motion determines the level of price and a mean reverting process covers short term deviations from this level.

The seasonal behavior of electricity prices is the result of seasonal variations of electricity demand. Therefore, various models describing the spot price as a functional of the load are proposed. Gubina et al. (2000) describe the price as an exponential function of the load. A more sophisticated early approach is presented by Barlow (2002). This diffusion model is based on a supply function derived by means of a Box-Cox-transformation. Kanamura and Ohashi (2007) present a structural model to approximate the merit order curve. The idea of Burger et al. (2004) is the starting point for the model in this work (compare Section 3.3). They describe the log spot price by means of cubic splines of the grid load adjusted by average availability of power plants. These approaches do not consider the influence of varying commodity prices. These factors are included by Coulon and Howison (2009). They give an analytic expression for the bid stack function considering fuel prices, demand and capacity in two US markets. A different approach is chosen by Emery and Liu (2002): They analyze the cointegration between electricity and natural gas futures prices.

A new spot price model is introduced by Andersson et al. (2013). They introduce a fuel-adjusted heat-rate model. This model explains the price behavior in normal load situations by means of the coal price. The typical price behavior in times of high load (strong increase of the price) and in times of low

load (possibly negative prices) is covered by a dual exponential function. This function is similar to the hyperbolic sine function but provides more parameters to achieve a better fit. Fundamental factors besides load and coal prices are neglected in this approach.

Beside this class of fundamental models there are various jump-diffusion models. The deterministic functions covering the seasonalities within these models range from a constant term up to a complex function including daily, weekly and yearly seasonalities. These models have some more common features: A stochastic process covering the usual price behavior and a jump diffusion component capturing price spikes. Deng (1999), Escribano et al. (2002), Cartea and Figueroa (2005) and Geman and Roncoroni (2006) show well-known examples of this class of models. A combination of jump diffusion and fundamental influences is proposed by Elliott et al. (2003). They use a Markov chain to describe the number of operating large generators. Hirsch (2009) gives a comparison of a regime-switching model, a jump-diffusion model and a normal inverse Gaussian process with respect to the valuation of swing options.

Another common way of modeling price spikes is using so-called regime switching models. These models explain price spikes by hidden Markov chains. In case of Benth et al. (2010) the Markov chain represents the state of economy. The model parameters like rate and level of mean reversion depend on the state of the Markov chain. The regime switching model by Huisman and Mahieu (2003) contains three regimes: One regime for "normal" price behavior, one regime for upward spikes and another regime for price movements from spike level down to "normal" price level. There are transition probabilities for the transitions between all regimes. E.g. the probability to move from the normal regime to the downward regime is zero. This ensures upward spikes.

Several authors mention the occurrence of negative prices but only a few regard them in modeling issues. Schneider (2011) proposes the use of the area hyperbolic sine transformation instead of the typical log transformation. This trigonometric function mirrors the behavior of the logarithm to the negative range. Close to zero it is almost linear. Therefore, this transformation keeps most of the characteristics of the log transformation but adds the possibility to cover negative prices. Fichtner et al. (2012) model the log price by a regime switching model with a base, an upper jump and a lower jump regime. In the lower jump regime some jumps are turned negative with a given probability and their height is determined by a mixture of a log-normal and an exponential distribution. Another model for negative prices is proposed by Fanone et al. (2013). They use an arithmetic model including a Lï¿½$\frac{1}{2}$vy-process for upward and downward jumps.

A recent more comprehensive survey of literature on electricity price models is given by Carmona and Coulon (2012).

3.3. The new spot price model

Each of the spot price models in the literature overview given above covers only some of the stylized facts described in Section 3.1 but none of them covers all characteristics. The first approaches contain stochastic factors and seasonalities. Further developments include price spikes as well as the load as a fundamental factor. Recently, models including negative prices are published. Some of the models incorporate single fuels as influence factors but we need to cover all commodity prices relevant for the market. In case of the German market we need to consider prices of coal, oil, gas and emission allowances.

As mentioned in Section 3.1 the inclusion of commodity prices prevents the use of the optimization approach for simulation purposes. Therefore, we approximate the merit order curve by a mathematical function that depends on the various commodity prices and the load as input variables. This idea is similar to the existing **SMaPS** model by Burger et al. (2004). In the following we give a short overview of the SMaPS model and indicate the need for improvements.

In Burger et al. (2004) the logarithm of the hourly spot price is described by an estimation of the merit order curve, f, a stochastic short term process $\left(X_t^{(S)}\right)$ and a stochastic long term process $\left(Y_t^{(S)}\right)$:

$$\log S_t = f\left(\frac{L_t}{v_t}, t\right) + X_t^{(S)} + Y_t^{(S)}. \tag{3.1}$$

The function f is a cubic spline fitted to historic price and load data. Burger et al. (2004) have shown that a time dependency of the price load curve improves the fit (see Figure 3.4).

Furthermore, the load L_t is divided by the average relative availability of power plants $v_t \in [0, 1]$ to improve the fit. $\left(X_t^{(S)}\right)$ was chosen as a SARIMA process with a season of 24 hours (see Definition 3.1). The uncertainty of price development on a long term horizon is represented by the process $\left(Y_t^{(S)}\right)$ following a random walk with drift. Its parameters are derived from prices of future contracts.
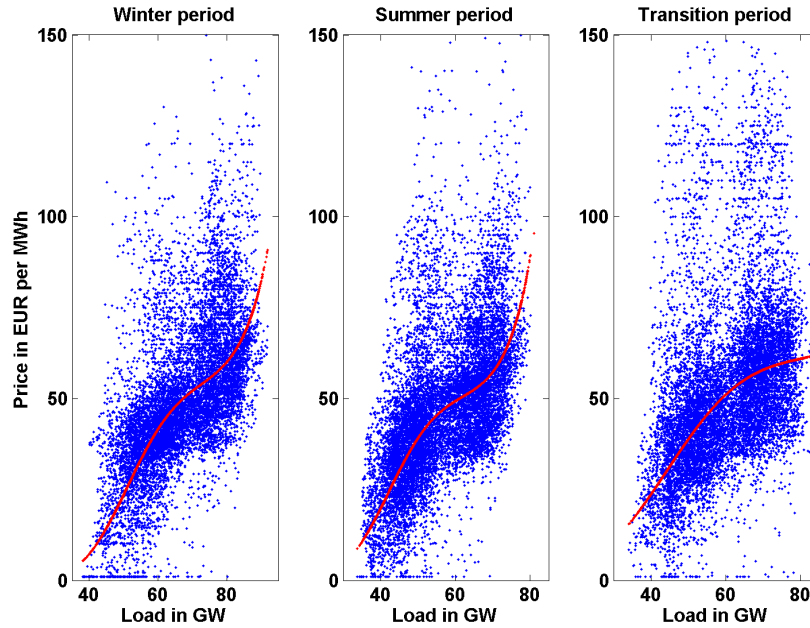
Figure 3.4.: Fit of the cubic spline (red) to empirical price and grid load data (blue) in different periods.

As the spot price usually follows the merit order curve the basic idea of the SMaPS model is still valid. But there are a few shortcomings that have occurred since development of the model.

- Price spikes cannot be captured by the normal distribution of the innovations of a SARIMA process.

- A model completely calibrated on logarithms of positive prices cannot reproduce negative prices.

- Renewable energy sources, especially wind and solar power, change price behavior. Especially, maximum prices within a day changed from noon to morning and late afternoon.

- Fuel prices and their relations to each other as major influence factors on the electricity price are neglected.

- Emission allowances need to be purchased for CO_2 emissions and increase the costs of coal, gas and petrol fired power plants.

3. Modeling the spot price of electricity

Some of these problems can be solved by small adaptations of the model. A heavy tailed distribution out of the class of generalized hyperbolic distributions (see Appendix A.3.5) can be used for the innovations of the SARIMA process. The area hyperbolic sine transformation as proposed by Schneider (2011) can be used to reproduce negative prices. But the incorporation of commodity prices requires major changes. Therefore, we propose a new model.

The SMaPS model approximates the merit order curve by a function of one fundamental factor, the load adjusted by average availability of power plants, although the merit order curve depends on many factors as described in Section 3.1. We include further variables in our approximation. This leads to an improvement of the approximation but is still less exact than the optimization approach including all restrictions. Our model for the spot price process, (S_t), can be written as

$$S_t = g\left(l_t, G_t, C_t, O_t, E_t\right) + X_t^{(S)} \tag{3.2}$$

with a load component (l_t), gas price (G_t), coal price (C_t), oil price (O_t), price of emission allowances (E_t) and a SARIMA process $\left(X_t^{(S)}\right)$. The daily commodity prices in this model are assumed constant within the day to match the hourly electricity prices. As our approximation g does not exactly behave like the real merit order curve we describe the deviations by a stochastic process. These residuals contain the errors of our approximation. Furthermore, market participants that include fix costs in their offers do not behave exactly like the merit order curve would expect it based on variable costs. Such deviations are covered by the short term process as well. In the following we describe both model components more detailed. The choice of the function g and the stochastic process $\left(X_t^{(S)}\right)$ are deduced.

3.3.1. The polynomial approximation

The function g in Equation (3.2) describes the relationship between different commodity prices and the load with respect to their effect on the spot price of electricity. It is a more complex approach to estimate the merit order curve than the SMaPS model used. Looking at Figure 3.5 suggests a simple modeling approach like

$$g\left(l_t, G_t, C_t, O_t, E_t\right) = a_0 + a_1 l_t + a_2 G_t + a_3 C_t + a_4 O_t + a_5 E_t \tag{3.3}$$

The spot price seems to grow at least linear in each of the variables. Estimation of the coefficients $a_0, \ldots, a_5$ can be done via ordinary least squares regression and the model is set up.
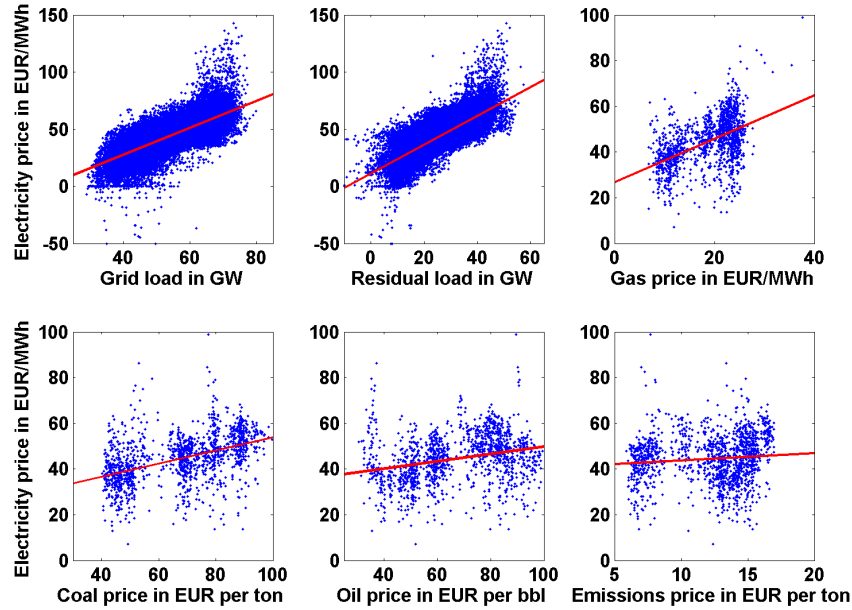
Figure 3.5.: Dependencies between grid load (top left), residual load (top center), commodity prices and the price of electricity. Least squares fit of a linear function (red). Daily average spot price in case of commodity prices.

The approach of Equation (3.3) does not represent one of the main ideas of the merit order curve: The cheapest power plants will be used first. If the gas price is constant and the coal price increases, the spot price will follow this price movement. But only to a certain point. Neglecting capacity constraints electricity will only be generated by coal power plants up to a certain coal price. Beyond this price it is cheaper to run gas power plants. As a consequence, further coal price increases would not have an impact on the electricity price. In analogy to this fuel switch from coal to gas, energy provider will switch between any other fuels as soon as the switch is profitable.

We neglected the capacities of power plants in this consideration. Taking capacities into account increases the complexity of this fuel switch: If the constellation of commodity prices requires a switch from coal to gas and the load is high the gas power plants might not be able to supply enough electricity so that some coal power plants will run as well. A total fuel switch can only be done if the capacities of both fuels match the load. Otherwise a mixture of both fuels is used. Obviously, the impact on the price is strongly related to the load level.

3. Modeling the spot price of electricity

So far, emission allowances were not mentioned in this consideration. All kinds of thermal power plants emit certain quantities of CO_2 per MWh. As these quantities differ among the fuel types the effect of emission allowances on the price of electricity depends on the type of power plants running. Only if the total load is covered by nuclear generation and renewable energies the price of emission allowances can be neglected. As these situations play a minor role we need to include prices of emission allowances depending on the load level in our model.

The previous arguments suggest the inclusion of commodity prices taking into account the load level. This requirement is not met by the approach in Equation (3.3). A linear combination of commodity prices and load is not sufficient as a spot price model. The description of the merit order curve and the illustration in Figure 3.3 suggest a nonlinear behavior of the curve which should be covered by the approximating function as well.

Although the choice of any complex function might considerably increase the goodness-of-fit to historical data this is not advisable either. The computational tractability needs to be considered. On the one hand, the function needs to be evaluated for any point of time in the future for every realization of the commodity price model. This requires the function to be easy to evaluate. On the other hand, the parameters of the function need to be estimated. The method used for parameter estimation based on historical data needs to provide reliable results within reasonable time. Not any function $g : \mathbb{R}^5 \to \mathbb{R}$ (or even $g : \mathbb{R}^6 \to \mathbb{R}$, if g depends on the time) fulfills both conditions.

By choosing an adequate function from the class of polynomials we ensure that parameters can easily be estimated via ordinary least squares regression. The goodness-of-fit can be increased by choosing polynomials of high degrees as this increases the number of explanatory variables in the regression. The problem about polynomials of high degrees becomes apparent in simulations. As we use stochastic models for each variable a degree of n would lead to $\mathrm{Var}(X^n)$ where X is the distribution of the variable. $\mathrm{Var}(X^n) < \infty$ might not hold. Hence, adequate scenarios cannot be generated by the model. Therefore, we choose from the class of polynomials of degree ≤ 2. This restriction scales the variance problem down to the question of existing moments of order four but it maintains the opportunity to model combined influences.

The loss of accuracy by choosing a polynomial of degree ≤ 2 cannot be neglected. But we show that the goodness-of-fit of other spot price models is still exceeded (compare Section 6.2). Furthermore, the structure of the model allows for all features of electricity prices to be reproduced in simulations.

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Influence of renewables

We did not take into account renewable energy sources so far. Renewables can be distinguished with respect to their electricity generation. Run-of-river and biomass power plants contribute a steady, slightly seasonal generation of electricity to the market. Generation from wind and solar power plants is subject to considerable fluctuations. Both kinds of renewable power plants lead to a decrease of the spot price as they are free of variable costs. As the fluctuations of water levels are comparably low the generation by run-of-river power plants has small fluctuations and leads to a permanent decrease of the spot price. The same is applicable to biomass power plants. Due to the increase of fuel costs the number of biomass (and other renewable energy sources) power plants increases. Nevertheless, once these power plants are built they provide a steady generation.

The installed capacities of wind and solar power plants are remarkably higher than the installed capacities of other renewables. But due to the dependency on weather conditions their generation might even be zero. There is no reliable long term forecast or at least "good" estimation of power generation by wind and solar power plants. As a consequence the impact on the spot price is not as steady as by run-of-river power plants.

Once the power plants are built the generation by renewable energy sources is free of variable costs. In addition to this economic advantage, renewable energy sources are supported by most governments. This means that their generation has highest priority. Renewable power plants are not turned off at any time. The generation by wind and PV power plants is placed in the markets by transmission system operators. In Germany they have to sell the energy at any price. Thus, they offer certain quantities even at a price of -3000EUR/MWh though paying a fixed price to producers. This ensures that the first part of consumer demand is satisfied by renewable energy sources and the rest needs to be satisfied by conventional power plants. As a consequence of this the usual principle of merit order takes effect after the renewable energy sources have supplied a part of the demand. The commodity prices set the electricity price for the grid load less the generation by renewable energy sources.

Nuclear power plants contribute a major part of the power generation in Germany. These plants cannot easily be turned off or reduced in their generation for financial, technical and legal reasons. Therefore, these plants run nonstop with the exception of maintenance outages. As a consequence of the nuclear disaster in Fukushima, Japan, in March 2011 the German government forced the shutdown of several German nuclear power plants. This means that the

cheapest conventional source of energy is partly replaced by more expensive thermal power plants. In contrast to this increasing effect on the price level there is an increase of flexibility. The generation of thermal power plants used instead can be adjusted to react on changing infeed from renewables. This flexibility increases the smoothness of prices. Nuclear power plants generate electricity at almost constant prices. The fluctuations of uranium prices are negligible and do not contribute to major electricity price movements. Therefore, we can consider the grid load less generation from renewable and nuclear energy sources as the load that sets the price. We define **residual load** as

$$l_t = L_t - PV_t - W_t - B_t - N_t \tag{3.4}$$

where (L_t) is the grid load, (PV_t) is the generation by solar power plants, (W_t) is the generation by wind, (B_t) is the generation by biomass, run-of-river and minor renewable energy sources and (N_t) is the generation by nuclear power plants. This definition ensures that the effect of renewables on the price is considered as well as the shut down of nuclear power plants.

This approach is in line with recent literature: Thoenes (2011) refers to grid load less wind power generation as residual load. This variable and commodity prices are used to explain the historic spot price behavior. The model is not specified for simulation purposes. Wagner (2012) proposes the use of residual demand (total demand minus generation from renewables) to improve model fit of load based electricity price models. Even the fit of a linear function of the load to the price is improved by the use of the residual load as it can be seen in Figure 3.5.

In the explanations above all sources of energy contributing to the generation of electricity in Germany are mentioned except of lignite. Lignite is produced in Germany and immediately consumed for generation of electricity as transportation of lignite is too expensive. Due to the comparably low energy content of lignite huge quantities are required. Transportation of such quantities would exceed the profitability of lignite-fired power plants. Therefore, there is no market for lignite. The costs of production can be regarded as constant and the capacities are almost constant. Some new power plants are built but these do not change the total capacity of generation by lignite too much. Therefore, the electricity generation with nearly constant capacities and production costs will not make a major contribution to the variation of the spot price of electricity. Lignite influences the total level of the spot price but not the hourly or daily variation. Thus, the price effect of lignite generation is constant and can be captured by the constant in our price model. If major capacity changes of lignite lead to a different behavior of the electricity price the residual load as defined above might be adapted by the lignite capacities as well. Such a step should work in analogy to the consideration of nuclear capacities.

Consideration of capacities for gas or coal power plants is more complex. These power plants operate only in certain hours. Changes of capacity will only affect these hours. As explained in the description of the merit order curve there is no clear indication of these hours. Therefore, we assume constant capacities of coal and gas within our data and for the time period of simulation.

A more sophisticated approach for our model in Equation (3.2) is given by

$$
\begin{aligned}
g\left(l_t, G_t, C_t, O_t, E_t, t\right) = \ & a_0 + a_1 \mathbf{1}_{Sat}(t) + a_2 \mathbf{1}_{Sun}(t) \\
& + a_3 l_t + a_4 G_t \mathbf{1}_A(t) + a_5 C_t + a_6 O_t + a_7 E_t \\
& + a_8 l_t G_t + a_9 l_t C_t + a_{10} l_t O_t + a_{11} l_t E_t.
\end{aligned}
$$

Apart from statistical significance there are fundamental and heuristic reasons for the choice of these factors:

- a_0: A constant term for the average price level.

- $\mathbf{1}_{Sat}$ and $\mathbf{1}_{Sun}$: Indicator variables equal to one on Saturdays or Sundays. Some power plants run on weekends to avoid starting costs although their variable costs are higher than the (expected) spot price. The coefficients for these indicator variables take account for this situation and decrease the price level on weekends. The goodness-of-fit is significantly increased by the use of two indicator variables instead of one indicator describing the weekend.

- l_t, G_t, C_t, O_t and E_t: Residual load as well as commodity prices have a linear influence on the spot price. The higher the commodity prices or the residual load, the higher the spot price (compare Figure 3.5). These terms neglect the non-linear relationship between the variables described above.

- $\mathbf{1}_A$: A describes the hours from 7 a.m. to midnight as the statistical significance of the gas price is only given in these hours. Gas fired power plants are used in times of high load only due to the high variable costs in comparison to coal fired power plants. Therefore, there is no influence on the spot price at night. This represents the situation in our data and neglects a possible future fuel switch from coal to gas.

- $l_t G_t, l_t C_t, l_t O_t$ and $l_t E_t$: The influence of the various commodity prices is strongly related to the level of the residual load. Therefore, we include these combinations of residual load and commodity prices.

3. Modeling the spot price of electricity

The model components are chosen symmetrically. Each commodity occurs twice in the model: alone and in combination with the residual load. All fuel types are included due to the portfolio of generation assets in Germany. Nevertheless, it turns out that negligence of the oil price component has a minor effect on the goodness-of-fit. The fluctuation around the linear fit in Figure 3.5 suggests that there is no strong dependency on the oil price. The statistical significance of the component might be due to the correlation to other commodities. In the model without oil prices the other components take over the role of oil. This reduces the complexity of the function but keeps the goodness-of-fit measured by the coefficient of determination at a level of $R^2 = 0.69$.

Apart from such statistical reasons there is a fundamental justification for the negligence of oil prices in the spot price model. As the generation by oil-fired power plants is below two percent of the total generation in the German market (compare Figure 3.2) it is considered a minor source of electricity. The model approach above changes to

$$
\begin{aligned}
g\left(l_t, G_t, C_t, E_t, t\right) \;=\;& a_0 + a_1 \mathbf{1}_{Sat}(t) + a_2 \mathbf{1}_{Sun}(t) + a_3 l_t + a_4 G_t \mathbf{1}_A(t) \\
&+ a_5 C_t + a_6 E_t + a_7 l_t G_t + a_8 l_t C_t + a_9 l_t E_t. \qquad (3.5)
\end{aligned}
$$

The formulation of model (3.5) as

$$
\begin{aligned}
g\left(l_t, G_t, C_t, E_t, t\right) \;=\;& a_0 + a_1 \mathbf{1}_{Sat}(t) + a_2 \mathbf{1}_{Sun}(t) + a_4 G_t \mathbf{1}_A(t) + a_5 C_t \\
&+ a_6 E_t + l_t \left(a_3 + a_7 G_t + a_8 C_t + a_9 E_t\right).
\end{aligned}
$$

allows for another heuristic explanation of the model structure. The term $l_t \left(a_3 + a_7 G_t + a_8 C_t + a_9 E_t\right)$ describes residual load times average costs of generation. These average costs consist of a weighted average of commodity prices representing the mixture of generation assets responsible for electricity generation in the market. The terms $a_4 G_t \mathbf{1}_A(t)$, $a_5 C_t$ and $a_6 E_t$ cover deviations from this average due to price movements of single commodities.

Time dependent model

The goodness-of-fit of the model (3.5) measured by $R^2 = 0.69$ can be increased by a simple modification. Instead of using one set of parameters for the data set it is possible to use 24 sets of parameters. One set for each hour:

$$
\begin{aligned}
g\left(l_t, G_t, C_t, E_t, t\right) \;=\;& a_0(t) + a_1(t)\mathbf{1}_{Sat}(t) + a_2(t)\mathbf{1}_{Sun}(t) \\
&+ a_3(t)l_t + a_4(t)G_t \mathbf{1}_A(t) + a_5(t)C_t + a_6(t)E_t \\
&+ a_7(t)l_t G_t + a_8(t)l_t C_t + a_9(t)l_t E_t.
\end{aligned}
$$

Technically, the data set is divided into 24 data sets and the regression is done 24 times. The errors of the regression are aggregated to one time series which is the basis for the estimation of the stochastic process. On the one hand, the R^2 increases to 0.74. On the other hand, the total number of parameters is increased by factor 24. As the goodness-of-fit always increases as the number of parameters increases we need to judge whether the huge number of parameters is justified by the increased R^2. An obvious criterion would be the adjusted coefficient of determination

$$\bar{R}^2 = R^2 - (1 - R^2)\frac{p}{T - p - 1}$$

with the number of parameters p and the sample size T. As in our case $T > 30000, p < 300$ holds the adjustment term will not change the decision taken according to R^2. A likelihood based information criterion considering the number of parameters like the Bayesian information criterion (BIC) is not applicable due to the missing likelihood function in our situation. Thus, we try to reduce the number of parameters used in the models without further consideration of concepts of model selection.

As a first step it turns out, that the indicator variables for Saturday and Sunday can be aggregated to one weekend indicator variable. There is no fundamental reasons for having two different indicator variables for Saturday and Sunday in the initial model but the R^2 significantly increases. Having only one indicator variable is what we expect and it is supported by the 24 models. In this representation the indicator variable $\mathbf{1}_A(t)$ is neglected as the parameter $a_3(t)$ might be zero, if it is not significant in certain hours.

$$\begin{aligned} g\left(l_t, G_t, C_t, E_t, t\right) = \ & a_0(t) + a_1(t)\mathbf{1}_{We}(t) \\ & + a_2(t)l_t + a_3(t)G_t + a_4(t)C_t + a_5(t)E_t \\ & + a_6(t)l_tG_t + a_7(t)l_tC_t + a_8(t)l_tE_t \end{aligned} \qquad (3.6)$$

The fluctuations of the ten parameters in the 24 models should be considered. If these fluctuate randomly from one model to the next, it is neither possible to call the 24 models reliable nor the initial model. Under the assumption that the parameters in adjacent hours have a similar level we can try to model adjacent hours together. Instead of having 24 models a smaller number of models implying less parameters is needed.

In Figure 3.6 the parameters of the 24 models are compared. As the coefficients of terms including the residual load as a factor are very small we compare the parameters multiplied by the historical mean of the corresponding variables. Terms containing the same commodity are aggregated. The results as given in
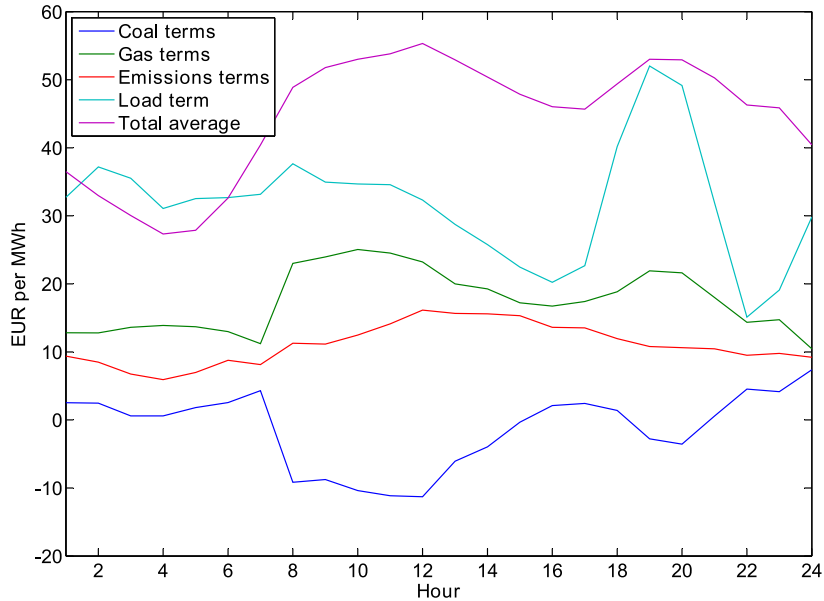
Figure 3.6.: Parameters of 24 models multiplied by average price and load levels of the history. Aggregated over commodities.

Figure 3.6 meet the expectations. In times of increased residual load the gas terms have a higher influence (8 a.m. - 8 p.m.). When the coal terms decrease (7 a.m.) the gas terms take over the role of the fuel that sets the spot price. The emissions terms show a similar behavior in all models. These terms lead to an almost constant increase of the price level. The load term is responsible for the increased price level between 6 p.m. and 8 p.m. Altogether these terms and the constant term lead to an appropriate average price in every hour (the weekend indicator variable is neglected).

As the estimation results for the 24 models do not show "random" fluctuations it is reasonable to choose a time dependent modeling approach. This approach needs to be adapted due to the high number of parameters. For this purpose we try to identify blocks having similar parameters. The most obvious block is formed by the first seven hours. There are no remarkable changes in the parameters during these hours. Especially, coal and gas terms show a mirrored behavior with several slopes. If we take hours where the slope changes its direction as boundaries for further blocks, this choice results in the blocks 0-7, 7-12, 12-18 and 18-24. Applying the model to these four blocks leads to an R^2 of 0.71. The goodness-of-fit does not change if we take 0-6 as the first block. This block is a commonly traded OTC contract which is another indicator for

the similar behavior of prices in these hours.
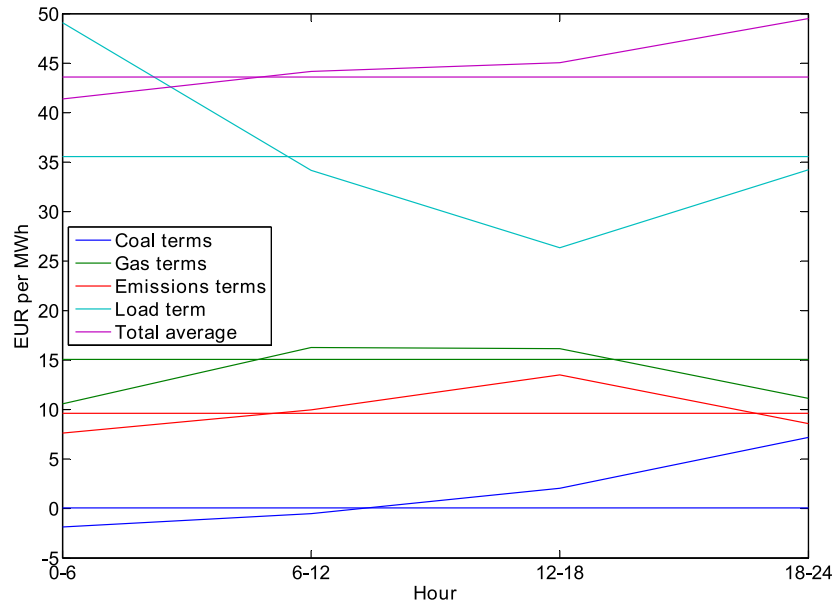
Figure 3.7.: Parameters of models for four blocks of six hours multiplied by average price and load levels of the history (solid lines). Parameters of the initial model for comparison (dotted). Aggregated over commodities.

The parameters of the four models (see Figure 3.7) show similar behavior to the parameters of 24 models discussed above. As expected, parameters of the initial model are in between those of the four models (and the 24 models as well).

We presented two alternatives to the initial model (3.5). These alternatives were discussed with respect to the goodness-of-fit and the number of parameters. A final decision about the "best" model cannot be made. The alternative using 24 parameter sets is neglected due to the high number of parameters. Further discussion can be found in Chapter 6. In the following we refer to the initial model unless otherwise stated. Due to the similarity of the alternatives and the initial model the choice of the stochastic process is not affected by the decision between these models. Though the estimated parameters differ the process is adequate in all three cases.

Residuals

Using the model components given in the function g the normal price behavior can be described. In times of major power plant outages this function is not able to reproduce the extreme spot price. Thus, extreme prices, so-called **outliers**, are excluded from the data before parameter estimation of g. Any data being further than four times the standard deviation away from the mean is declared to be an outlier. This simple approach is sufficient to increase the goodness-of-fit of the model.

After fitting g to the data without outliers

$$S_t - g\left(l_t, G_t, C_t, E_t, t\right)$$

is the basis for the estimation of $\left(X_t^{(S)}\right)$. This means, that outliers are due to stochastic events in our model. The stochastic process is not only supposed to cover outliers but any kind of abnormal price behavior as a consequence of planned or unplanned power plant outages, imports and exports, strategic decisions of energy utilities and approximation errors due to the choice of g. Therefore, we fit a stochastic process (see Section 3.3.2) to cover the price range in our simulation model.

A comparison of Equation (3.2) and Equation (3.1) reveals a missing long term factor in the new model. We discuss that long term uncertainty of electricity prices is a result of long term uncertainty of commodity prices (see Section 6.2). Therefore, we do not need a long term process for the electricity price but rather include adequate terms in each commodity price model.

3.3.2. The stochastic process

The difference between (S_t) and g contains all unexplained influences on the spot price. Although there might be fundamental reasons for every deviation from g we declare them to be random. This randomness can be covered by an adequate stochastic process which is specified below.

Although the residual time series does not have any deterministic structure a high autocorrelation can be identified. Within the autocorrelation function (ACF) a periodicity can be identified. All lags being multiples of 24 have a slightly higher autocorrelation than other lags (see Figure 3.8). As the time series is stationary we apply a **seasonal autoregressive integrated moving average (SARIMA)** process as defined by Brockwell and Davis (1987,

Chapter 9.6). The class of SARIMA processes is an extension of the ARMA processes described in Appendix A.1.1.

Definition 3.1 (SARIMA process)**.** *If p, d, q, P, D, Q, s are non-negative integers, then (X_t) is a SARIMA $(p, d, q) \times (P, D, Q)_s$ process with period s if the differenced process*

$$Y_t = (1 - L)^d (1 - L^s)^D X_t$$

is a causal ARMA process

$$\phi(L) \Phi(L^s) Y_t = \theta(L) \Theta(L^s) Z_t, \quad Z_t \sim WN\left(0, \sigma^2\right), \tag{3.7}$$

where

$$
\begin{aligned}
\phi(z) &= 1 - \phi_1 z - \ldots - \phi_p z^p, \\
\Phi(z) &= 1 - \Phi_1 z - \ldots - \Phi_P z^P, \\
\theta(z) &= 1 + \theta_1 z + \ldots + \theta_q z^q, \\
\Theta(z) &= 1 + \Theta_1 z + \ldots + \Theta_Q z^Q, \qquad z \in \mathbb{C}.
\end{aligned}
$$

Applying a SARIMA process to a time series means determination of the model orders p, d, q, P, D, Q, s and estimation of $p + q + P + Q$ coefficients of the polynomials $\phi, \theta, \Phi, \Theta$. According to the model choice in case of ARMA processes the orders of a SARIMA process can be determined by an information criterion. But due to computational complexity one is interested in very small numbers of parameters. Therefore, we try to find adequate orders by analyzing ACF and partial autocorrelation function (PACF).

First, we can apply e.g. the Augmented Dickey-Fuller-test (see Appendix A.1.3) to state that the residuals of the spot price process are stationary. Hence, we do not take differences of the time series: $d = D = 0$. Obviously, ACF and PACF exhibit a period with lag 24. This is not only a mathematical finding but it can be supported by a heuristic argument: A model error in one hour is correlated to the model error in the same hour on the day before. Therefore, we set $s = 24$.

The first s lags of ACF and PACF can be used to choose p and q as it is done in the model choice for ARMA models. Dividing the whole time series into 24 individual time series, one for each hour of the day, and analyzing 24 ACFs and PACFs gives an idea about adequate choices for P and Q. This gives us four orders which are separately determined. To make sure that the result is an adequate model for the time series we check ACF and PACF of the innovations. The innovations should be white noise without any autocorrelations (see Figure 3.8).
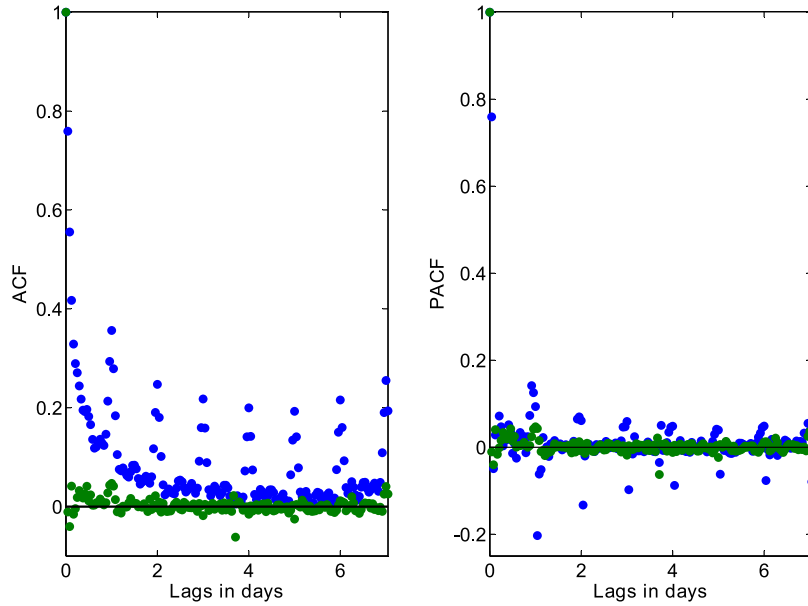
Figure 3.8.: Autocorrelation function (left) and partial autocorrelation function (right) of price residuals (blue) and innovations of the SARIMA process (green).

According to this analysis a SARIMA $(1, 0, 0) \times (1, 0, 1)_{24}$ process is appropriate to describe the features of the residual time series. The innovations of this process, (Z_t), are uncorrelated but not normally distributed. The histogram of the empirical data of (Z_t) (see Figure 3.9) reveals the typical shape of a Student's t-distribution (compare Appendix A.3.1). A result of Hannan (1973) allows to choose different distributions than the normal distribution for the innovations of an ARMA process as long as mean and variance of the distribution exist (see Appendix A.1.2).

A Student's t-distribution with scale parameter can be fitted to the data via MLE. The variance of a Student's t-distribution for any degree of freedom $\nu > 2$ is given as $\nu/(\nu - 2)$. As we obtain $\nu = 2.7$ the conditions for the application of this distribution to the innovations of the SARIMA process are met.

Our spot price model consisting of a polynomial with several input variables and a stochastic short term process is specified. In the following we present models for the input variables. We start with two modeling approaches for the residual load.
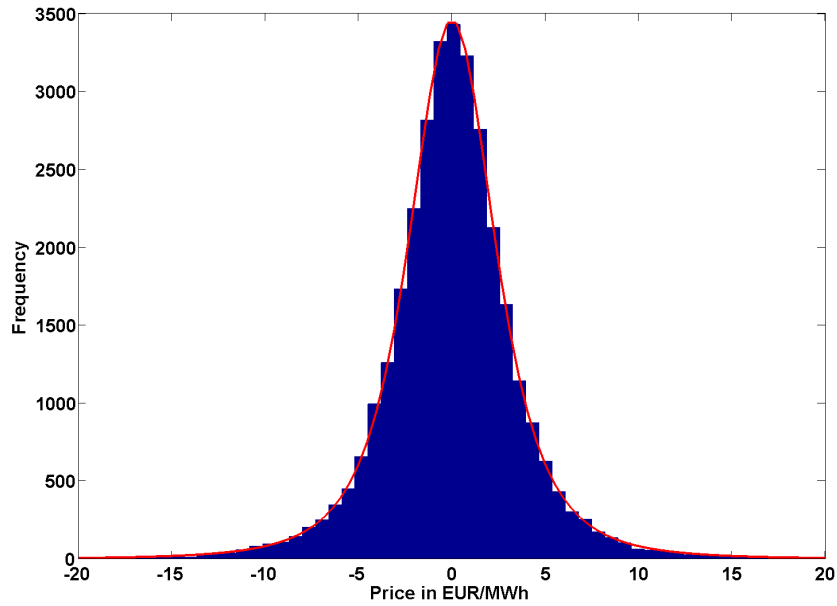
Figure 3.9.: Fit of a Student's t-distribution (red line) to the histogram of the empirical innovations of the SARIMA process of the spot price (blue).

3.4. Modeling the load

The spot price models presented in the previous chapter have the residual load (see Figure 3.10)

$$l_t = L_t - PV_t - W_t - B_t - N_t$$

as an input variable. In this chapter we introduce two stochastic modeling approaches.

1. Distinct models for the grid load L_t, solar generation PV_t, wind generation W_t, miscellaneous generation B_t and nuclear generation N_t are used. The residual load scenarios are composed of simulations of these distinct models according to the formula above.

2. One model for the residual load l_t is set up. It includes variations from all sources of uncertainty included in the distinct variables.

From a theoretical point of view both model structures are thinkable. Therefore, we will present both approaches in the following. A discussion including

practical reasons as well as mathematical arguments reveals advantages and disadvantages of both approaches. We will give this discussion partly within this chapter. The remaining part follows in Chapter 6.
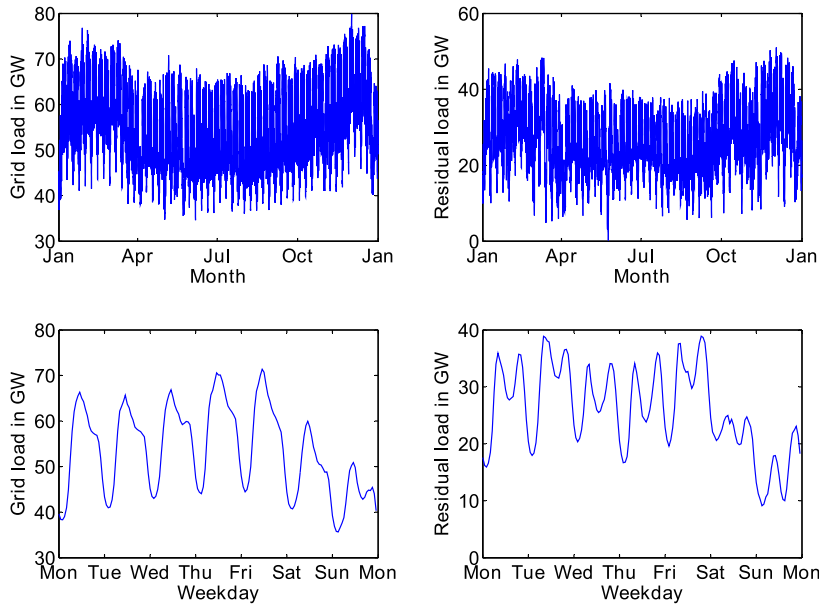
Figure 3.10.: German grid load (left) and residual load (right) in 2010 (top) and one week in April 2012 (bottom).

The data basis for both approaches is given by grid load data provided by European Network of Transmission System Operators for Electricity (2013). The historical generation of renewables is given by the Transparency Platform of the EEX (see European Energy Exchange (2013)).

3.4.1. Modeling the grid load and renewable energy generation

The first approach requires models for all five components of the residual load. We refrain from a model for the nuclear generation. For reasons of flexibility the future generation by nuclear power plants is considered constant. This approach neglects maintenance outages and outages due to technical problems. This error is as small as the benefit from a more detailed generation forecast for the nuclear generation. Nevertheless, the model structure gives the chance

for later inclusion of a different forecast. The remaining four variables are modeled in the following.

Modeling the grid load

Modeling the load is rather about identifying adequate functions to cover various seasonalities than about finding a complex stochastic process. The grid load exhibits the seasonalities explained in Section 3.1. Forecasting the seasonal behavior of the grid load by means of deterministic functions is easy in comparison to other time series requiring complex model structures and exogenous variables.

The load of a large grid is the sum of a large number of consumer loads. To a certain extent, fluctuations of single consumers cancel each other out so that the total is quite stable and predictable. Nevertheless, deviations from the forecast, $\widehat{L}_t$, occur due to weather influences or unpredicted events. This means that weather dependent variables might even improve the model fit. The weather dependent deviations and forecast errors are included in a short term process $\left(X_t^{(L)}\right)$. In the long run load changes due to fundamental changes in the economy, as seen in the crisis 2008/2009, can be included by a long term process. Altogether, we model the grid load as

$$L_t = \widehat{L}_t + X_t^{(L)} + Y_t^{(L)}. \tag{3.8}$$

The long term process $\left(Y_t^{(L)}\right)$ is supposed to cover load changes for economic reasons. This kind of changes is characterized by long lasting changes of the load level. An increase of load during one week is not considered as a fundamental change of economy but rather changes lasting a few months or years. Therefore, we do not use hourly data to identify the long term fluctuations. Long term load data with a monthly granularity is available with a longer history than hourly data. Using this data, fluctuations due to economy can be identified.

In order to ensure that short term and long term volatility do not influence each other we aggregate monthly data to quarterly data. This data still contains yearly seasonality and a slight long term trend. After removal of these statistically significant deterministic components we obtain a residual time series containing all economic influences (and potential errors of the removal of seasonalities and trend). These residuals are stationary as we expect it in advance: There are economic fluctuations but after a crisis economy usually

tends to revert to its mean. These fluctuations around the mean can be modeled by an AR(1) process (see Figure 3.11). Although we remove a linear trend from the historical data we ignore it in our simulation. There is no clear trend observable after 2005. Therefore, we consider the linear growth as negligible for the future.
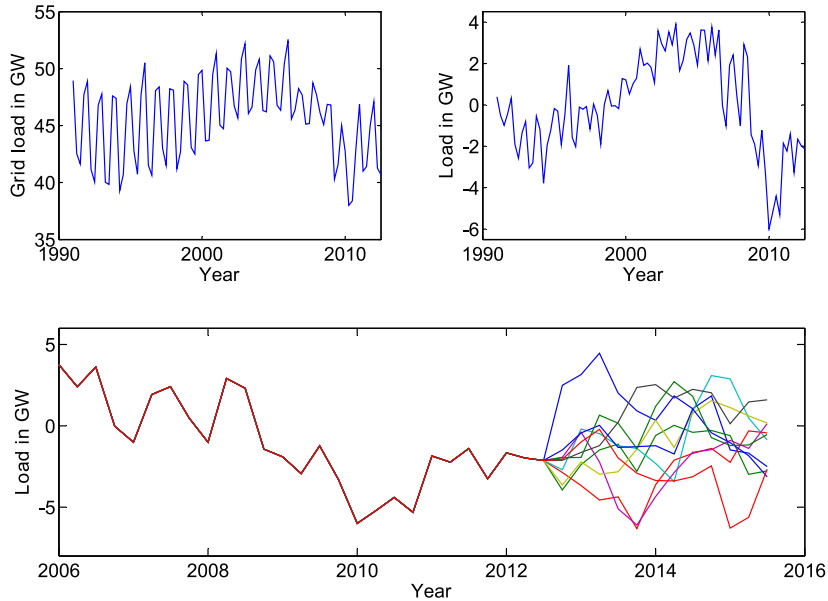
Figure 3.11.: German quarterly load data from January 1991 to September 2012 (top left); Quarterly load data without trend and season (top right); Historical data and 10 simulated paths for 3 years (bottom).

We use the long term process to model the level of load. The structure on top of this level is given by seasonalities. The easiest way to cover all daily and weekly seasonalities is to use a system of similar days. Each similar day is represented by an indicator variable. Working days, Saturdays, Sundays and public holidays are the most popular choices for similar days. One has to decide whether all working days are represented by one indicator variable or by up to five indicator variables. Such a system covers the weekly seasonality. As we work on hourly data each similar day is not only represented by one indicator variable but rather by 24 to specify all 24 hours of the similar day. If differences between the load structure in different periods of the year exist, one set of similar day for each period can be chosen. Further possibilities to incorporate the yearly seasonality are the use of 12 indicator variables on monthly basis or

a combination of sine and cosine to have a continuous function.

Depending on the choice of similar days, the number of periods within a year and the type of function for the yearly seasonality the system of variables might be quite large. As we want to fit $\widehat{L}_t$ by linear regression a big system of regressor variables can lead to computational problems (close to singular matrices). As the load is comparatively easy to predict even a rather small system of regressors captures the most relevant characteristics. We use sine and cosine for the yearly seasonality, similar days for the weekly seasonality and 24 indicator variables per similar day for the daily seasonality. For the comparison of different models we choose the R^2 as a measure of goodness-of-fit. Holidays, single days between holidays and weekends, working days and weekends in summer, winter and spring/autumn give us a system of similar days leading to $R^2 = 0.89$. The same system applied to the grid load model without a long term process results in $R^2 = 0.92$. Seven similar days in four periods already produce a rather high goodness-of-fit. Further improvements can be achieved by more similar days or periods and the inclusion of weather dependent exogenous variables. The model improvement by inclusion of further similar days decreases. For our purposes this simple modeling approach is sufficient. A detailed description of load modeling techniques with further references is given by Weron (2006). Alfares and Nazeeruddin (2002) provide a survey and classification of load models.

As it can be seen above the inclusion of the long term component reduces the goodness-of-fit in the period of time considered in this work. As the component has a fundamental justification we keep it in the model. The representation of long term fluctuations is an essential part of the grid load model. According to the state of the economy the demand for electricity fluctuates. Such fluctuations on a long term horizon need to be covered by a grid load model.

The presentation within this chapter slightly differs from Chapter 7. In Chapter 7 the long term component is based on monthly data instead of quarterly data. Nevertheless, the purpose behind both ways is the same. Especially, during the crisis 2008 the economic load fluctuations could be explained by the long term component and the model fit was improved. The use of the component decreases as the data used in this work starts in 2009. But still, load fluctuations have a major contribution to the level of the spot prices and the riskiness of retail power contracts.

Estimation of long term process and deterministic function $\widehat{L}_t$ leads to the residual time series. This part of the model contains all unpredictable fluctuations due to short term load changes (see Figure 3.12).
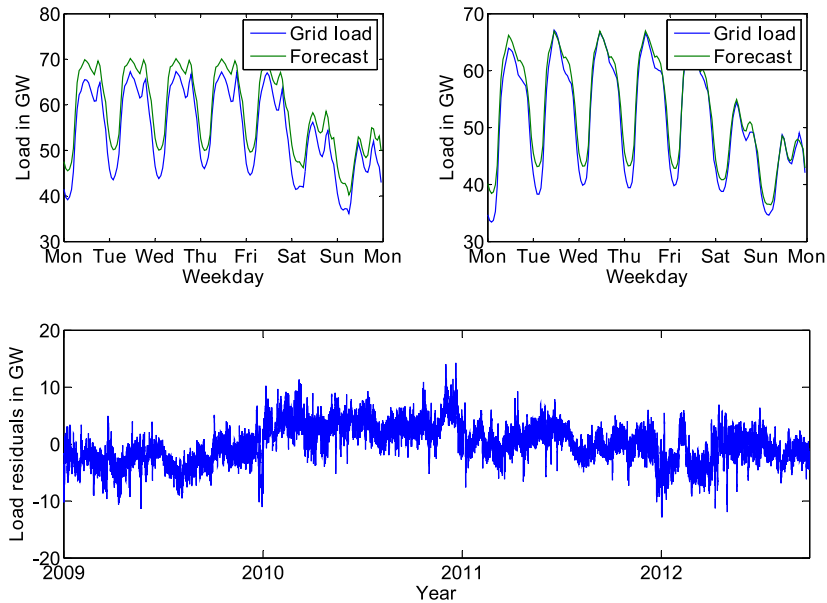
Figure 3.12.: German grid load and forecast from 2012/3/12 till 2012/3/18
(top left) and from 2012/8/27 till 2012/9/2 (top right). The
empirical load short term process (bottom).

The short term effects usually do not occur in single hours but in blocks of
hours. Therefore, we need to analyze the autocorrelations within the time
series. This reveals strong autocorrelations. On the one hand, lags of 1-3
hours are significant. On the other hand, lags of (multiples of) 24 hours show
an increased autocorrelation (see Figure 3.13). The former can be explained
by the fact that events leading to short term load changes usually last more
than one hour and influence a few adjacent hours as well. The latter is a result
of the model specification: The daily structure of load changes within a year
e.g. due to the decreasing need for electric lighting in summer. We include
various periods for our similar day system to cover these changes but they are
somehow continuous and not stepwise as it is suggested by the similar day
system. Therefore, a model error in any hour on one day is likely to occur in
the same hour on the days before as well.

As the ADF-test indicates stationarity of the residual time series we choose
a seasonal ARMA process with period 24. The model order of this process
is determined as described above in case of the residual price time series (see
Section 3.3.2). The result is a SARIMA $(2, 0, 0) \times (1, 0, 1)_{24}$ process. After
determination of the model order an adequate distribution for the innovations
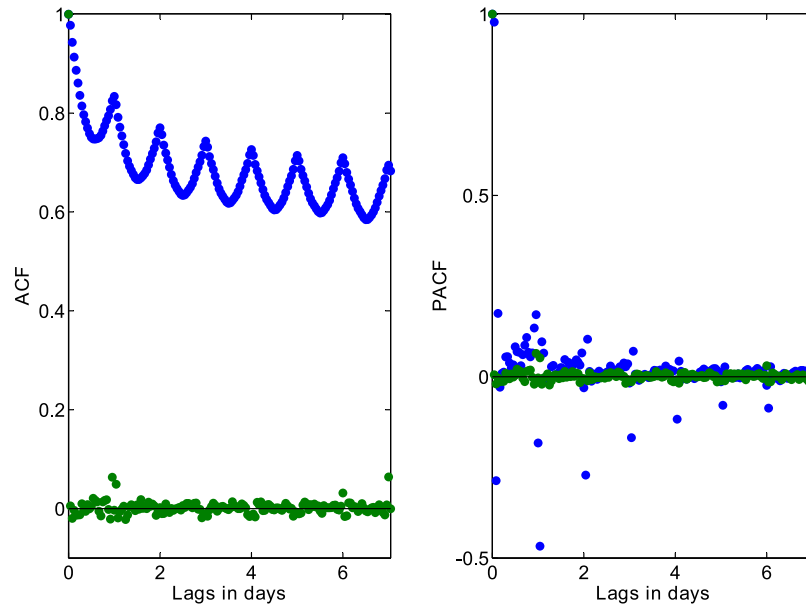
Figure 3.13.: ACF (left) and PACF (right) of residual time series (blue) and
innovations of SARIMA process (green) for the grid load.

of the process has to be fitted. We use a Student's t-distribution with scale
parameter to account for the increased variance in comparison to the standard
t-distribution. The parameter of the t-distribution (degrees of freedom) allows
for a good fit to the empirical distribution, even in the tails (see Figure 3.14).
The grid load is not likely to show any extreme values. Extreme values occur-
ring in the residual time series or in the innovations of the process are due to
forecast errors. The original load data does not exhibit any extreme values or
outliers (see Figure 3.10). Therefore, a truncated version of the t-distribution
leads to more realistic simulation results. We choose the range of the empirical
distribution as the support of the t-distribution.

Altogether these model components form a reliable model to reproduce the
characteristics of the grid load (see Figure 3.15).

Modeling wind and PV generation

The generation of wind and PV power plants is characterized by strong weather
dependent fluctuations. Reliable forecast models of weather conditions are
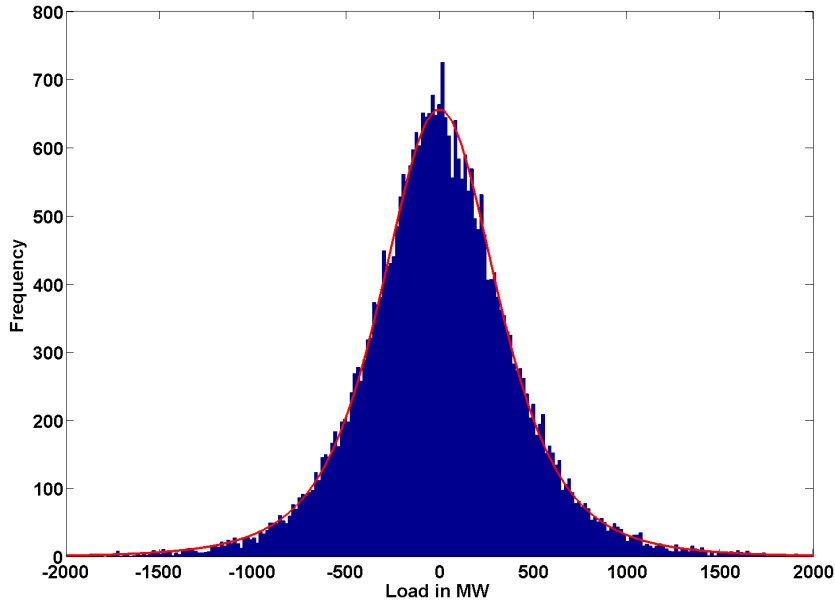
Figure 3.14.: Fit of a truncated Student's t-distribution (red line) to the histogram of the empirical innovations of the SARIMA process of the load (blue). The figure is restricted to $[-2000, 2000]$.

available for a few days only. Hence, a simulation model for a few years cannot easily be constructed by a forecast plus a stochastic process as it is done in case of the grid load (see Section 3.4.1).

The major drivers for generation of wind and PV power plants are weather situation and installed capacities. As the installation of renewable capacities is supported by most governments there is a strong increase that is not expected to stop within the next few years. Thus, it is not easy to identify a stochastic process describing weather dependent generation based on given installed capacities. Therefore, we use a pragmatical method to obtain realistic simulations.

The basic concept of our model is historical simulation. Historical data contains realistic fluctuations which is an essential condition for any model. Nevertheless, there are two problems concerning the use of historical data.

1. There is only a short history of PV and wind generation data (less than five years, compare Figure 3.16).
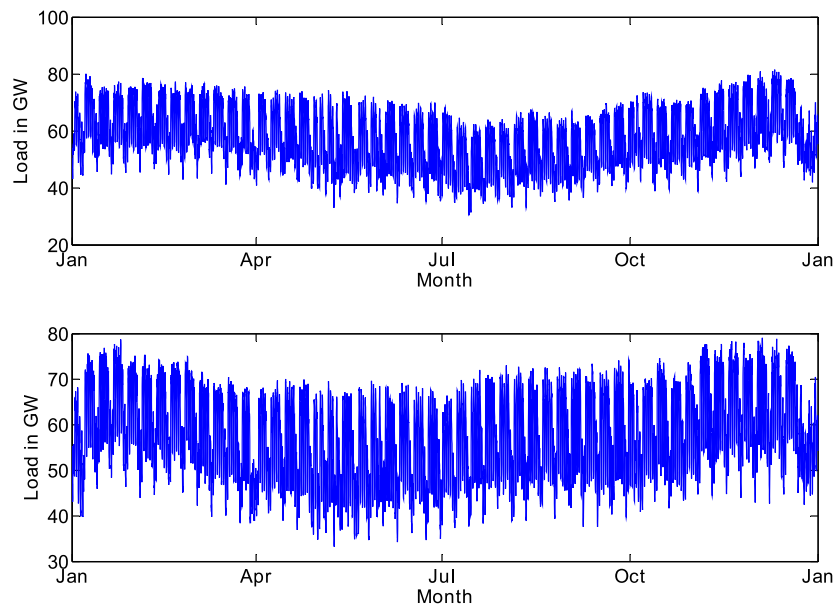
Figure 3.15.: Two scenarios of the grid load for 2013.

2. Historical capacities are not valid for the future.

In order to extend the pool of data that we can draw from we generate further data. Weather data from the last 20 years is used to derive wind and PV generation data under the assumption of current capacities. With this extension a historical simulation generates sufficient variations.

Though future capacities are not exactly known there are forecasts. Thus, we can scale the historical scenarios to expected future capacities. As the forecast is subject to uncertainty we include fluctuations of the forecast in our model. In combination with the historical simulation this ensures sufficient stochastic behavior of the resulting scenarios which are subtracted from grid load scenarios.

More sophisticated models can be applied instead of this simple approach as soon as a longer history of data and a consistent generation capacity are available.
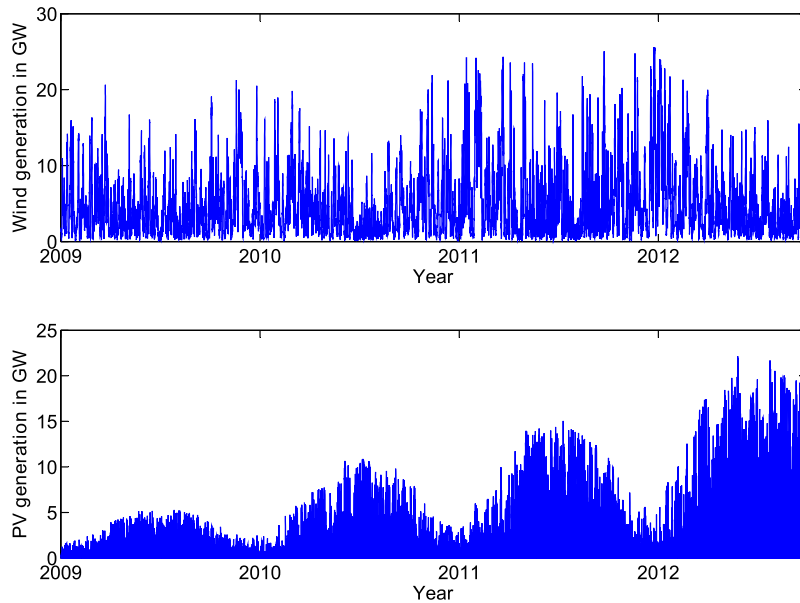
Figure 3.16.: Historical wind (top) and PV (bottom) generation.

Modeling miscellaneous renewable generation

Miscellaneous renewable generation describes sources such as run-of-river and biomass power plants. Further sources of energy with minor quantities are included as well. The increase of capacities is small due to the availability of the corresponding fuels. Most profitable locations for run-of-river power plants are already used. This reduces the speed of extension. Therefore, it is acceptable to assume the capacities as constant for the next few years.

Neglecting the increase of biomass power plants we can assume constant capacities of miscellaneous renewable generation. Apart from seasonal variations by run-of-river power plants there is almost no fluctuation in miscellaneous generation. Thus, we assume the generation of the last year in the historical data to be continued for the years to be simulated. This is analog to the approach for nuclear capacities.

Using the historical simulation models for renewable generation and the model for the grid load allows for calculation of the residual load. These scenarios contain estimations of the increase of renewable capacities as well as load fluctuations due to various short term influences (see Figure 3.17).
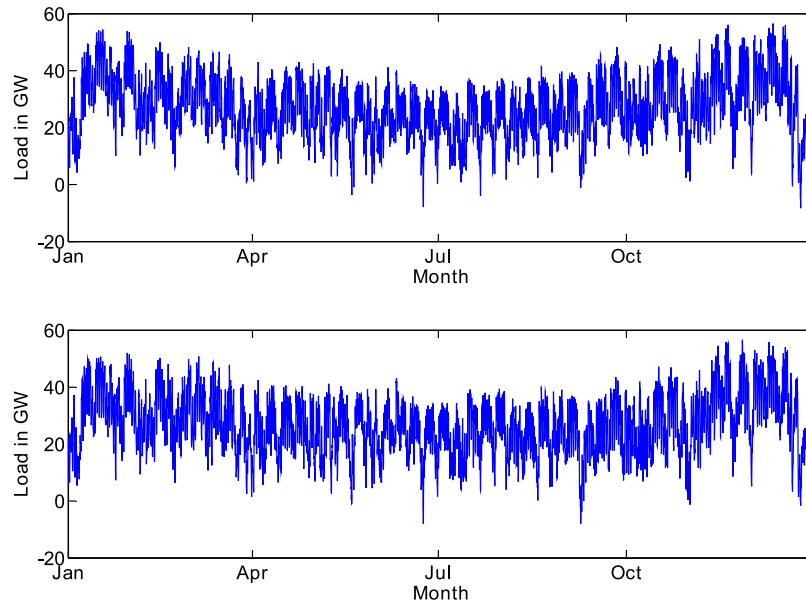
Figure 3.17.: Two residual load scenarios for 2013. The residual load is composed by scenarios from the grid load model and the models for renewables.

The model for miscellaneous renewable generation is the last part needed for the first modeling approach for the residual load. Distinct models for each residual load component were presented. In the following the second approach with only one model is introduced.

3.4.2. Modeling the residual load

The residual load as defined in Equation (3.4) contains the grid load as a major variable. Therefore, grid load and (l_t) contain the same seasonalities. In case of residual load, these seasonalities are "disturbed" by renewable generation. If the grid load model is applied to the residual load the goodness-of-fit of the similar day system described in Section 3.4.1 decreases. This is due to the disturbances by renewables. They reduce the predictability.

The mismatch of the grid load model results in increased errors. Hence, the variance of the stochastic process increases. But this is what we expect: Solar and wind power are unpredictable and lead to an increased volatility in the

model. Thus, the stochastic short term process in the model for (l_t) contains grid load volatility as well as solar and wind power generation volatility.

These considerations justify the use of the same similar day system as in case of the grid load to obtain the forecast $\widehat{l}_t$ in

$$l_t = \widehat{l}_t + X_t^{(l)}. \tag{3.9}$$

Thus, we use the similar day system and a SARIMA-process $\left(X_t^{(l)}\right)$ to describe the residual load. The goodness-of-fit measured by R^2 reduces to 0.72 in comparison to 0.92 for the grid load.

The scenarios resulting from the grid load model applied to the residual load can be seen in Figure 3.18. Due to the structure of the model, the increase of renewable capacities is not reflected in the model. The obvious loss of the

Figure 3.18.: Two realizations of the grid load model applied to the residual load for 2013.

clear yearly seasonality can be explained by the decreased model fit. The lower load level in summer is not identified by the model due to the renewable energy sources. Therefore, the scenarios cannot reproduce this feature of the historical input data.

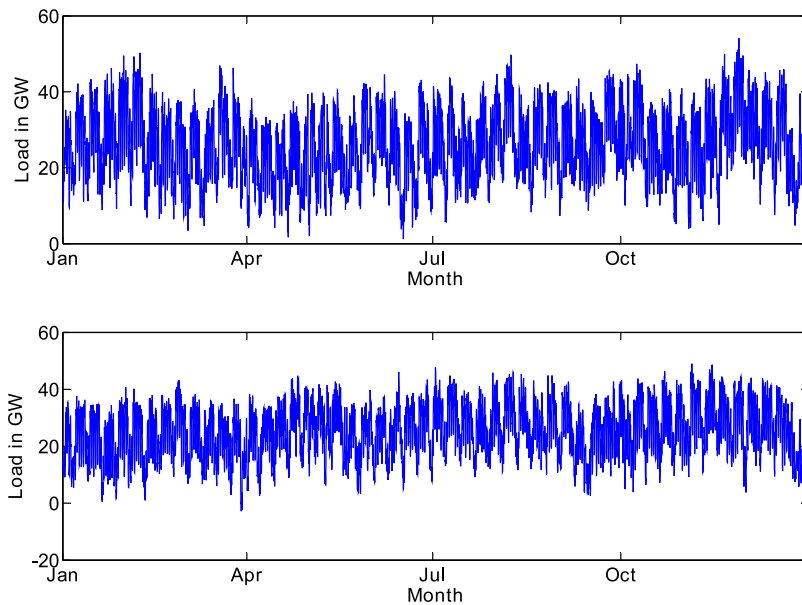The missing increase of renewable capacities is the major drawback of this model. The simplicity of having only one model instead of several models as in Section 3.4.1 cannot compensate this drawback. Another drawback occurs in Chapter 7.

In the following we present a method to include current information from the futures market in our price models.

3.5. Inclusion of futures market information

An adequate price model is supposed to cover the (stochastic and deterministic) price behavior of the commodity in the past as well as the expected price level for the future. As models are usually set up on historical prices some kind of average of those prices or the latest available price is used as the expected future price level. This expectation from the past might be a good estimator of the "true" price level, i.e. the level that will realize.

Any expectation resulting from a model can only contain the information that is in the model. Therefore, information about future changes in the market structure, political interventions and other fundamental changes might not be included in this expectation. All information available to market participants is included in the prices of future and forward contracts. These prices can be seen as the market's expectation about the future price level.

As prices of future contracts are available for all considered commodities and electricity we introduce the incorporation of these prices in a general way before the individual models are described. After estimation of all model parameters sets of scenarios can be generated from a model. The mean of these scenarios in every hour (on every day) is shifted to the price of the future contract for the corresponding period of time, i.e. we need to determine b_t in

$$\frac{1}{n} \sum_{i=1}^{n} x_t^{(i)} + b_t = F(t) \quad \forall \, t \tag{3.10}$$

with $x_t^{(i)}$ the i-th realization of a random variable X_t, n the number of scenarios and $F(t)$ the price of future contracts for time t. $F(t)$ is either a stepwise function consisting of the different prices of future contracts for different times of delivery or a continuous function containing the expected price structure within each delivery period. In the latter case the average price in each delivery period corresponds to the price of the price of the future contract for this

delivery period (compare Section 7.2.1). By setting

$$\widetilde{x}_t^{(i)} = x_t^{(i)} + b_t \quad i = 1, \ldots, n, \forall\, t$$

the futures market information is incorporated into the scenarios from our model.

To be consistent with the model structure we only apply the shift to additive models. In case of multiplicative models, i.e. models for log prices, the summand b_t in Equation (3.10) turns into a scaling factor. In the following all scenarios from the proposed models are either scaled or shifted to the prices of the corresponding future contracts until further notice.

4. Modeling the spot price of natural gas

For inclusion in the electricity price model a spot price model for gas is needed. In the following we present such a model including temperature and oil price as exogenous factors. The chapter is based on the work of Hirsch et al. (2013).

4.1. Introduction

During the last years trading of natural gas has become more important. The traded quantities OTC and on energy exchanges have strongly increased and new products have been developed. For example, swing options increase the flexibility of suppliers and they are used as an instrument for risk management purposes. Important facilities for the security of supply are gas storages. The storages are filled in times of low prices and emptied in times of high prices which usually coincides with low and high consumption of natural gas. Although the liquidity is comparably low, an OTC market for gas storages exists. This means, physical or virtual gas storage contracts are traded.

These two examples of complex options illustrate the need of reliable pricing methods. Both options rely on non-trivial trading strategies where exercise decisions are taken under uncertainty. Therefore, analytic pricing formulas cannot be expected. The identification of an optimal trading strategy under uncertainty is a typical problem of stochastic dynamic programming, where even numerical solutions are difficult to obtain due to the curse of dimensionality. Therefore, simulation based approximation algorithms have been successfully applied in this area. Longstaff and Schwartz (2001) introduced the least square Monte Carlo method for the valuation of American options. Meinshausen and Hambly (2004) extended the idea to Swing options, and Boogert and de Jong (2008) applied it to the valuation of gas storages. Their least-squares Monte Carlo algorithm requires a stochastic price model for daily spot prices generating adequate gas price scenarios. We prefer this approach

to methods using scenario trees or finite differences as it is independent of the underlying price process.

The literature on stochastic gas price models is dominated by purely stochastic approaches. The one and two factor models by Schwartz (1997) and Schwartz and Smith (2000) are general approaches applicable to many commodities, such as oil and gas. Cortazar and Schwartz (2003) present a three factor model for the term structure of oil prices. These models can be applied to gas prices as well. The various factors represent short and long term influences on the price.

Extensions of these factor models are given by Jaillet et al. (2004) and Xu (2004). Especially the inclusion of deterministic functions to cover seasonalities within gas prices is considered. Cartea and Williams (2008) introduce a two factor model including a function for the seasonality. Their focus is on the market price of risk. An important application of gas price models is the valuation of gas storage facilities. Within this context, Chen and Forsyth (2010) and Boogert and de Jong (2011) propose gas price models. Chen and Forsyth (2010) analyze regime-switching approaches incorporating mean-reverting processes and random walks. The class of factor models is extended by Boogert and de Jong (2011). The three factors in their model represent short and long term fluctuations as well as the behavior of the winter-summer spread.

In contrast to these models Stoll and Wiebauer (2010) propose a fundamental model with temperature as an exogenous factor. They use the temperature component as an approximation of the filling level of gas storages which have a remarkable influence on the price. In this chapter we will extend the model of Stoll and Wiebauer (2010) by introducing another exogenous factor to their model: the oil price. There are at least two reasons why we believe this to be useful. The main reason is that an oil price component can be considered as a proxy that covers the dependence of the gas price on the state of the world economy in the near future. In contrast to other indicators such as the gross domestic product the oil price is available on a daily basis at least for the 36 front months. Furthermore, the prices for gas imports in countries such as Germany are oil price indexed. This fundamental link between gas and oil prices is covered by our oil component as well.

The rest of the chapter is organized as follows. In Section 4.2 we present the model by Stoll and Wiebauer (2010) including a short description of their model for the temperature component. In their model the influence of the temperature is described by so-called normalized cumulated heating degree days. In Section 4.3 we describe our new oil price component and discuss the need for an oil price component in the model. The choice of the component

is explained as well. After introducing the stochastic processes that we use to model oil prices and temperature, we finally fit the model to data from the TTF market in Section 4.4. It turns out that for the innovations of the process a heavy tailed distribution like the normal inverse Gaussian (NIG) is more appropriate than the classical normal distribution. We finish with a short conclusion in Section 4.5.

4.2. A review of the model by Stoll and Wiebauer (2010)

Modeling the price of natural gas in Central Europe requires knowledge about the structure of supply and demand. On the supply side there are only a few sources in Central Europe while most of the natural gas is imported from Norway and Russia (compare Section 2.3). On the demand side there are mainly three groups of gas consumers: Households, industrial companies and gas fired power plants. While households only use gas for heating purposes at low temperatures, industrial companies use gas as heating and process gas. Households and industrial companies are responsible for the major part of total gas demand.

These two groups of consumers cause seasonalities in the gas price:

- Weekly seasonality: Many industrial companies do not need gas on weekends. Their operation is restricted to working days.

- Yearly seasonality: Heating gas is needed in winter when temperatures are low.

An adequate gas price model has to incorporate these seasonalities as well as stochastic deviations of these.

Stoll and Wiebauer (2010) propose a model meeting these requirements and incorporating another major influence factor: the temperature. To a certain extent the temperature dependency is already covered by the deterministic yearly seasonality. This component describes the direct influence of temperature: The lower the temperature, the higher the price. But the temperature influence is more complex than this. A day with average temperature of zero degrees Celsius at the end of a long cold winter has a different impact on the price than a daily average of zero at the end of a "warm" winter. Similarly, a

4. Modeling the spot price of natural gas

cold day at the end of a winter has a different impact on the price than a cold day at the beginning of the winter.

The different impacts are due to gas storages which are essential to cover the demand in winter. The total demand for gas is higher than the capacities of the gas pipelines from Norway and Russia. Therefore, gas provider use gas storages. These storages are filled during summer (at low prices) and emptied in winter months. At the end of a long and cold winter most gas storages will be rather empty. Therefore, additional cold days will lead to comparatively higher prices than in a normal winter.

The filling level of all gas storages in the market would be the adequate variable to model the gas price. However, these data are not available as they are private information. Therefore, we need a proxy variable for it. As the filling levels of gas storages are strongly related to the demand for gas which in turn depends on the temperature, an adequate variable can be derived from the temperature.

Stoll and Wiebauer (2010) use **normalized cumulated heating degree days** to cover the influence of temperature on the gas price. They define a temperature of $15\,°\mathrm{C}$ as the limit of heating. Any temperature below $15\,°\mathrm{C}$ makes households and companies switch on their heating systems. Heating degree days are measured by $HDD_t = \max{(15 - T_t, 0)}$ where T_t is the average temperature of day t. As mentioned above the impact on the price depends on the number of cold days observed so far in the winter. In this context we refer to winter as the 1st of October and the 181 following days till end of March. We will write $HDD_{d,w}$ for HDD_t, if t is day number d of winter w. Cumulation of heating degree days over a winter leads to a number indicating how cold the winter was so far. Then we can define the cumulated heating degree days on the day d in winter w as

$$CHDD_{d,w} = \sum_{k=1}^{d} HDD_{k,w} \text{ for } 1 \leq d \leq 182.$$

The impact of cumulated heating degree days on the price depends on the comparison to a normal winter. This information is included in normalized cumulated heating degree days

$$\Lambda_{d,w} = CHDD_{d,w} - \frac{1}{w-1} \sum_{\ell=1}^{w-1} CHDD_{d,\ell} \text{ for } 1 \leq d \leq 182.$$

We will use Λ_t instead of $\Lambda_{d,w}$ for simplicity, if t is a day in a winter. The definition of Λ_t for a summer day is described by a linear return to zero during

summer. This reflects the fact that we use Λ_t as a proxy variable for filling levels of gas storages. Assuming a constant filling rate during summer we thus get the linear part of normalized cumulated heating degree days (see Figure 4.1). Positive values of Λ_t describe winters colder than the average. (Λ_t) is
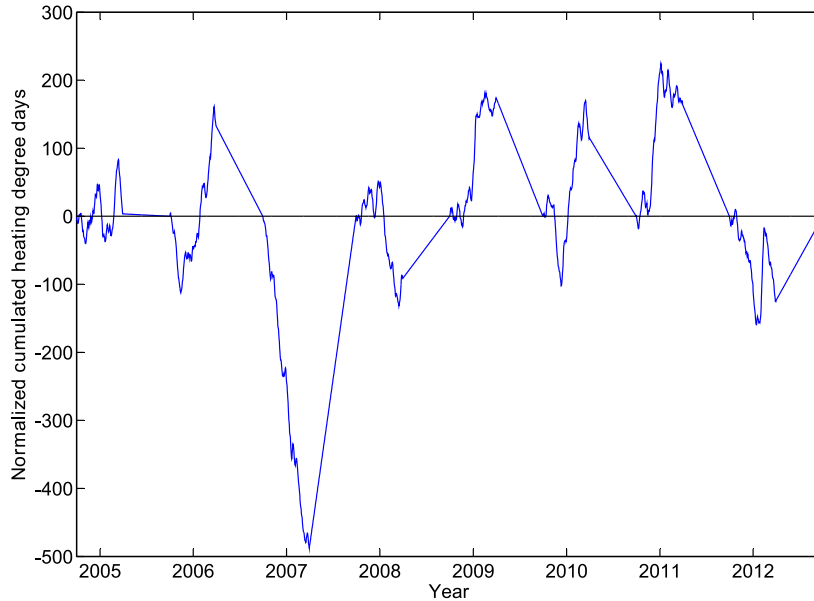
Figure 4.1.: Normalized cumulated heating degree days in Eindhoven, Netherlands, for 2003-2012.

included into the gas price model by a regression approach. As the seasonal components and the normalized cumulated heating degree days are linear with respect to the parameters we can use ordinary least squares regression for parameter estimation. The complete model can be written as

$$G_t = \widehat{G}_t + \alpha \cdot \Lambda_t + X_t^{(G)} + Y_t^{(G)} \tag{4.1}$$

with the day-ahead price of gas G_t, the deterministic seasonality $\widehat{G}_t$, the normalized cumulated heating degree days Λ_t, an ARMA process $X_t^{(G)}$ and a Geometric Brownian motion $Y_t^{(G)}$. For model calibration day-ahead gas prices from TTF and daily average temperatures from Eindhoven, Netherlands, are used. The fit to historical prices before the crisis can be seen in Figure 4.2. Outliers have been removed (see Section 4.4 for details on treatment of outliers).
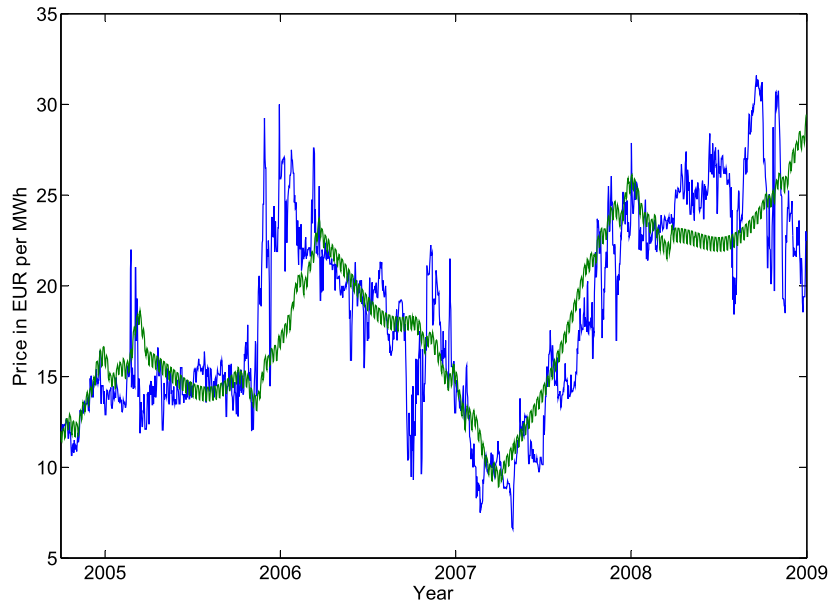
Figure 4.2.: The model in Equation (4.1) fitted to TTF prices from 2003-2009.

4.3. The oil price dependence of gas prices

The model described in Equation (4.1) is capable to cover all influences on the gas price related to changes in temperature. But changes of the economic situation are not covered by that model. This was clearly observable in the economic crisis 2008/2009 (see Figure 4.5). During that crisis the demand for gas by industrial companies in Central Europe was falling by more than 10 percent. As a consequence the gas price rapidly decreased more than 10 Euro per MWh.

The oil price showed a very similar behavior in that period. Economic changes are the main driver for remarkable changes of the oil price level. Short term price movements caused by speculators or other effects cause deviations from the price level that represents the state of the world economy. Therefore, the gas price is not influenced by daily changes of the oil price but rather by long term changes of the oil price level. Such an influence can be modeled by means of a moving average of past oil prices. The averaging procedure removes short term price movements if the averaging period is chosen sufficiently long. The result is a time series containing only the long term trends of the oil price.

Using such an oil price component in a gas price model explains the gas price movements due to changes of the economic situation.

Another important argument for the use of this oil price component is based on Central European gas markets. Countries such as Germany import gas via long term oil price indexed supply contracts. The import price of gas in these contracts usually is an average of past oil prices. The pricing in import contracts is done via **oil price formulas**. These formulas consist of three parameters:

1. The number of averaging months. The gas price is the average of past oil prices within a certain number of months.

2. The time lag. Possibly there is a time lag between the months the average is taken of and the months the price is valid for.

3. The number of validity months. The price is valid for a certain number of months.

An example of a 3-1-3 formula is given in Figure 4.3.
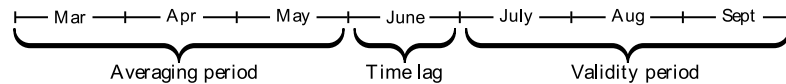
Figure 4.3.: In a 3-1-3 formula the price is determined by the average price of 3 months (March to May). This price is valid for July to September. The next day of price fixing is the 1st of October.

The formulas used in the import contracts are not known to all market participants. Theoretically any choice of three natural numbers is possible. But from other products, like oil price indexed swing options, we know that some formulas are more popular than others. Examples of common formulas are 3-1-1, 3-1-3, 6-1-1, 6-1-3 and 6-3-3. Therefore, we assume that these formulas are relevant for import contracts as well.

As there are many different import contracts with possibly different oil price formulas we cannot ensure that one of the mentioned formulas is responsible for the price behavior on the market. The mixture of different formulas might

affect the price in the same way as one of the common formulas or a similar one.

Evaluation of the oil price formula leads to price jumps every time the price is fixed. The impact on the gas price will be more smooth, however. The new price determined on a fixing day is the result of averaging a number of past oil prices. The closer to the fixing day the more prices for the averaging are known. Therefore, market participants have reliable estimations of the new import price. If the new price will be higher it is cheaper to buy gas in advance and store it. This increases the day-ahead price prior to the fixing day and leads to a smooth transition from the old to the new price level in the day-ahead market.

This behavior of market participants leads to some smoothness of the price. In order to include this fact in a model a smoothed oil price formula can be used. A sophisticated smoothing approach for forward price curves is introduced by Benth et al. (2007). They assume some smoothness conditions in the knots between different price intervals. It is shown that splines of order four meet all these requirements and make sure that the result is a smooth curve. As oil price formulas are step functions like forward price curves this approach is applicable to our situation.

If the number of validity months is equal to one it is possible to use a moving average instead of a (smoothed) step function to simplify matters (see Figure 4.4). This alternative is much less complex than the approach with smoothing by splines, and delivers comparable results. Therefore, the simpler method is applied in case of formulas with one validity month.

In the next section we will compare various oil price formulas regarding their ability to explain the price behavior in the gas market.

4.4. Model calibration with temperature and oil price

After justification of the oil price component as a fundamental factor for our model we need to choose a way to include it in our existing model. Therefore, we compare different oil price formulas in the regression model in order to find the one explaining the gas price best.

For the choice of the best oil price formula we use the coefficient of deter-
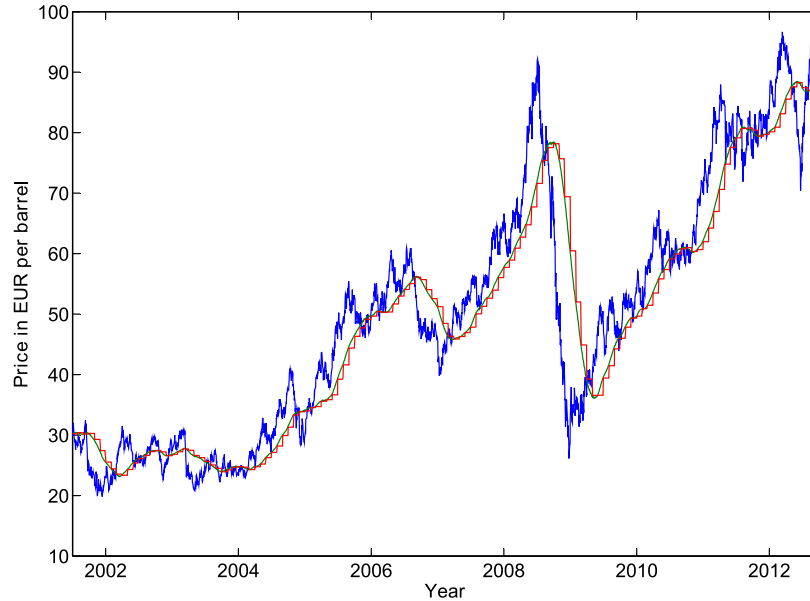
Figure 4.4.: The price of oil (blue), the 6-0-1 oil price formula (red) and the moving average of 180 days (green).

mination R^2 as the measure of goodness-of-fit. We choose the reasonable oil price formula leading to the highest value of R^2. Reasonable in this context means, that we restrict our analysis to formulas that are equal or similar to the ones known from other oil price indexed products (compare Section 4.3). The result of this comparison is a 6-0-1 formula (see Figure 4.6). Although this is not a common formula there is an explanation for it: The gas price decreased approximately six months later than the oil price in the crisis. This major price movement needs to be covered by the oil price component. As explained above we replace the step function by a moving average. Taking the moving average of 180 days is a good approximation of the 6-0-1 formula. All in all, the oil price component increases the R^2 from 0.35 to 0.83 (see Figure 4.5). Even if the new model is applied to data before the crisis the oil price component is significant. In that period the increase of R^2 is smaller but still improves the model.

These comparisons give evidence that both considerations in the previous section are valid. The included oil price component can be seen as the smoothed version of a certain oil price formula. At the same time it can be considered as a variable describing economic influences indicated by trends and level of the oil price.
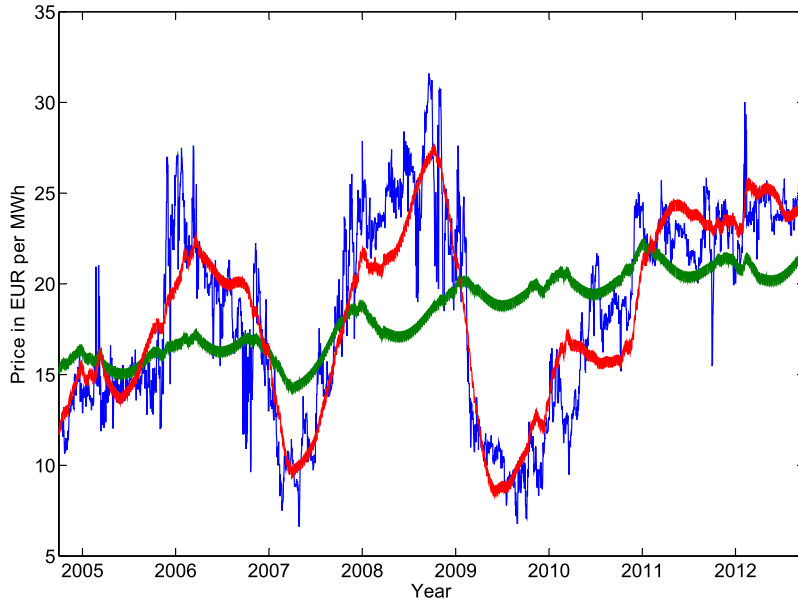
Figure 4.5.: The model of Stoll and Wiebauer (2010) (green line) and our model (red line) fitted to historical gas prices (blue line).

Therefore, we model the gas price by the new model

$$G_t = \widehat{G}_t + \alpha_1 \Lambda_t + \alpha_2 \Psi_t + X_t^{(G)} \tag{4.2}$$

with Ψ_t being the oil price formula.

For parameter estimation of our model we use day-ahead gas prices from TTF. Trading on TTF has a longer history of high trading volumes than the neighboring markets. Data from 2003-2012 is used for this model. Temperature data from Eindhoven, Netherlands, is available from 1969-2012. For the estimation of the oil price component we use prices of Brent traded on the IntercontinentalExchange (ICE). The data is available with a longer history than gas prices. We use data from 2002-2012. Using this data we can estimate all parameters applying ordinary least squares regression after some outliers are removed from the gas price data, (G_t).

Due to e.g. technical problems or a fire at a major gas storage the gas price deviated from its normal price level which was determined by temperature and oil price formulas. Thus, we exclude the prices on these occasions by an outlier treatment proposed by Weron (2006). Values outside a range around a running median are declared to be outliers. The range is defined as three times

the standard deviation. The identified outliers are excluded in the regression. We do not remove them from our model, however, as they are still included in the estimation of the parameters of the remaining stochastic process.
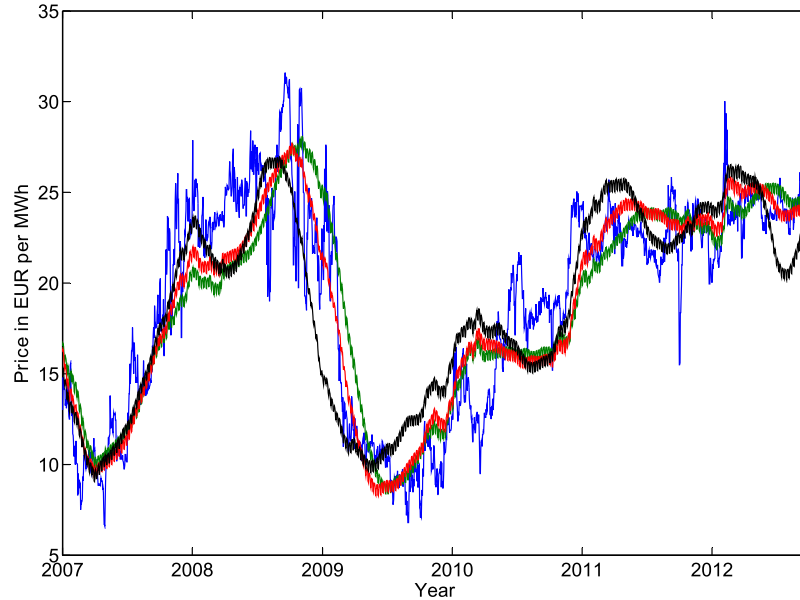
Figure 4.6.: Comparison of different oil price components in the model (4.2): 6-0-1 formula (red line), 6-1-1 formula (green line) and 3-0-1 formula (black line) fitted to the historical prices (blue line).

Altogether these model components give fundamental explanations for the historical day-ahead price behavior. Short term deviations are included by a stochastic process (see Subsection 4.4.3). Long term uncertainty due to the uncertain development of the oil price is included by the oil price process. Therefore, our model is able to generate reasonable scenarios for the future (see Figure 4.7). We will specify the stochastic models for the exogenous factors (Ψ_t) and (Λ_t) as well as the stochastic process $\left(X_t^{(G)}\right)$ in the following.

4.4.1. Oil price model

Oil prices show a different behavior than gas prices. This influences the choice of an adequate model. The most obvious fact is the absence of any seasonalities or deterministic components. Therefore, we model the oil price without a deterministic function or fundamental component. Another major difference
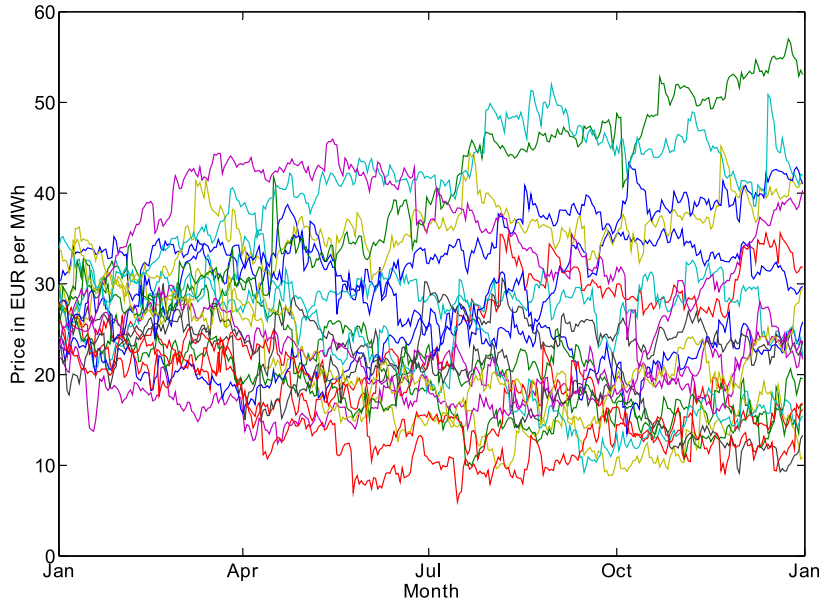
Figure 4.7.: Realizations of the gas price process for 2013.

affects the stochastic process. While the oil price and also logarithmic oil prices are not stationary the gas price is stationary after removal of seasonalities and fundamental components.

A very common model for non-stationary time series is the Brownian motion with drift applied to logarithmic prices. Drift and volatility of this process can be determined using historical data or by any estimation of the future volatility. For a stationary process, the use of an Ornstein-Uhlenbeck process or its discrete equivalent, an AR(1) process, is an appropriate simple model.

A combination of these two simple modeling approaches is given by the two factor model by Schwartz and Smith (2000). They divide the log price into two factors: one for short term variations and one for long term dynamics.

$$\psi_t = \exp\left(\chi_t + \xi_t\right)$$

with an AR(1) process χ_t and a Brownian motion ξ_t. These processes are correlated. We apply this two factor model as it considers long and short term variations. The estimation of parameters in this model is more complex. The factors are not observable in the market. Following the paper by Schwartz and Smith (2000) we apply the Kalman filter for parameter estimation.

The resulting process (ψ_t) is used to derive the process (Ψ_t) in Equation (4.2).

4.4.2. Temperature model

When modeling daily average temperature we can make use of a long history of temperature data. Here, a yearly seasonality and a linear trend can be identified. Therefore, we use a temperature model closely related to the one proposed by Benth and Benth (2007).

$$T_t = a_1 + a_2 t + a_3 \sin\left(\frac{2\pi t}{365.25}\right) + a_4 \cos\left(\frac{2\pi t}{365.25}\right) + X_t^{(T)} \qquad (4.3)$$

with $\left(X_t^{(T)}\right)$ being an AR(3) process. The model fit with respect to the deterministic part (ordinary least squares regression) and the AR(3) process is shown in Figure 4.8. The process (T_t) (see Figure 4.9) is then used to define the derived process (Λ_t) of normalized cumulated heating degree days as described in Section 4.2.

4.4.3. The residual stochastic process

The fit of normalized cumulated heating degree days, oil price formula and deterministic components to the gas price via ordinary least squares regression (see Figure 4.10) results in a residual time series. These residuals contain all unexplained, "random" deviations from the usual price behavior.

The residuals exhibit a strong autocorrelation to the first lag. Further analysis of the partial autocorrelation function reveal an ARMA(1,2) process providing a good fit (see Figure 4.11). The empirical innovations of the process show more heavy tails than a normal distribution (compare Stoll and Wiebauer (2010)). Therefore, we apply a distribution with heavy tails. The class of generalized hyperbolic distributions including the NIG distribution was introduced by Barndorff-Nielsen (1978). The normal-inverse Gaussian (NIG) distribution leads to a remarkably good fit (see Figure 4.11). Recall that a random variable X is NIG-distributed if there is a representation

$$X \stackrel{d}{=} \mu + \beta Y + \sqrt{Y} Z$$

with $Z \sim \mathcal{N}(0,1)$ and $Y \sim N^-\left(-1/2, \delta^2, \alpha^2 - \beta^2\right)$, the inverse Gaussian distribution as a special case of the generalized inverse Gaussian distribution. More details on this class of distributions can be found in the Appendix A.3.
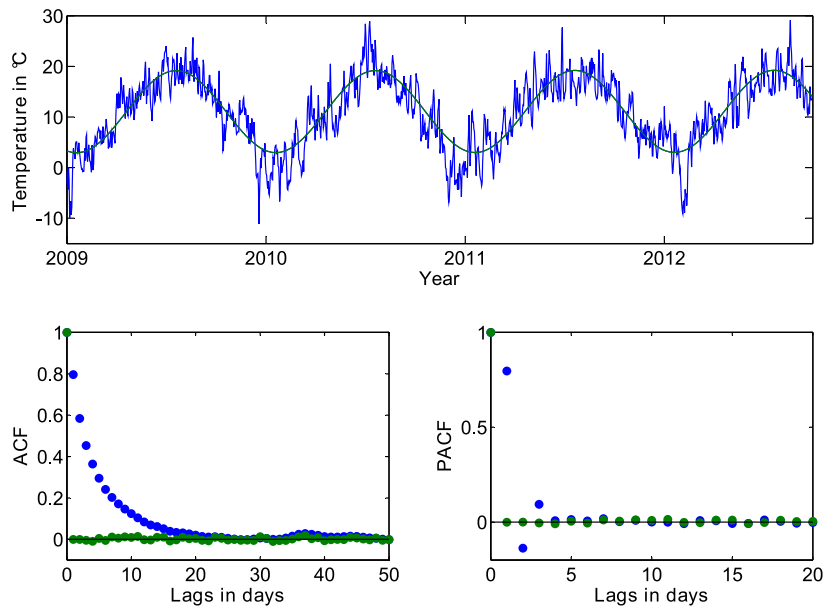
Figure 4.8.: Top: Fit of deterministic function (green line) to the histori-
cal daily average temperature (blue) in Eindhoven, Netherlands.
Bottom: Autocorrelation function (left) and partial autocorre-
lation function (right) of residual time series (blue) and innova-
tions of AR(3) process (green).

Both the distribution of the innovations and the parameters of autoregressive
processes are estimated using maximum likelihood estimation.

4.5. Conclusion

The spot price model by Stoll and Wiebauer (2010) with only temperature
as an exogenous factor is not able to explain the gas price behavior during
the last years. We have shown that adding an oil price component as another
exogenous factor remarkably improves the model fit. It is not only a good
proxy for economic influences on the price but also approximates the oil price
formulas in gas import contracts in Central European gas markets. These
fundamental reasons and the improvement of model fit give justification for
the inclusion of the model component. The resulting simulation paths from
the model are reliable.
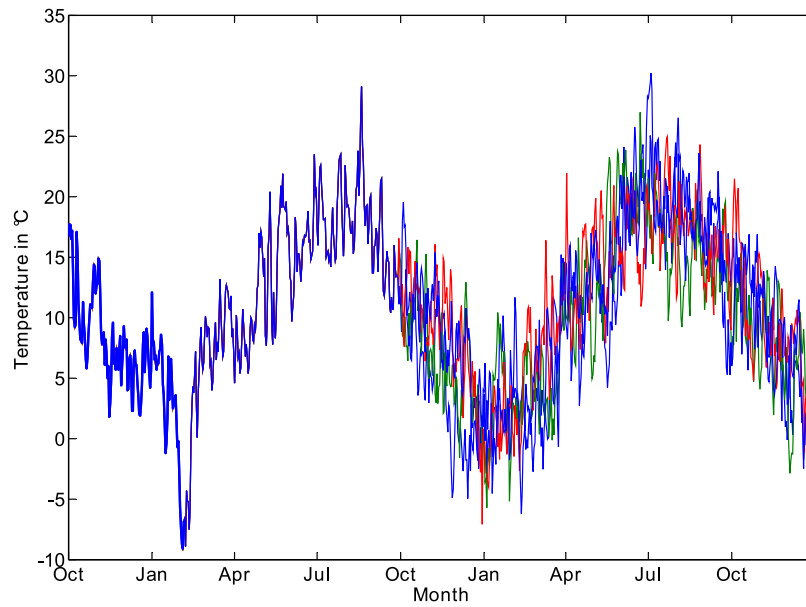
Figure 4.9.: Historical temperatures and three realizations of the temperature model.
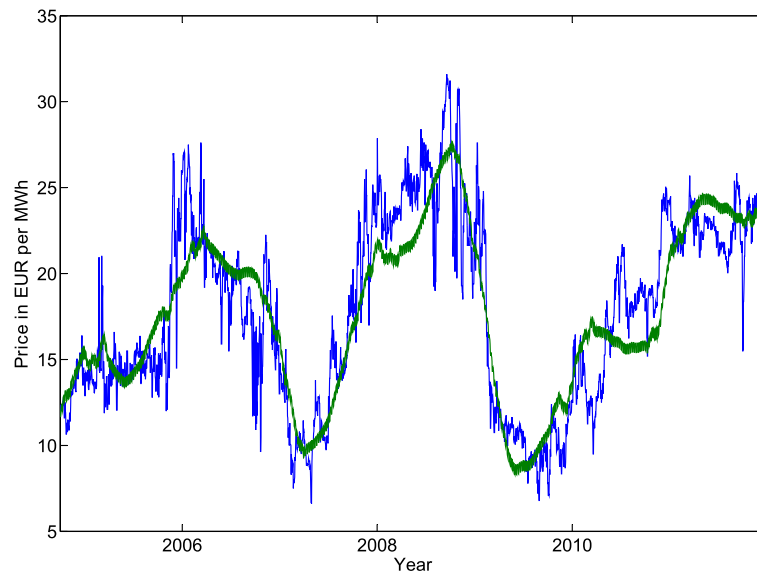
Figure 4.10.: Fit of deterministic function and exogenous components (green line) to the historical gas price (blue).
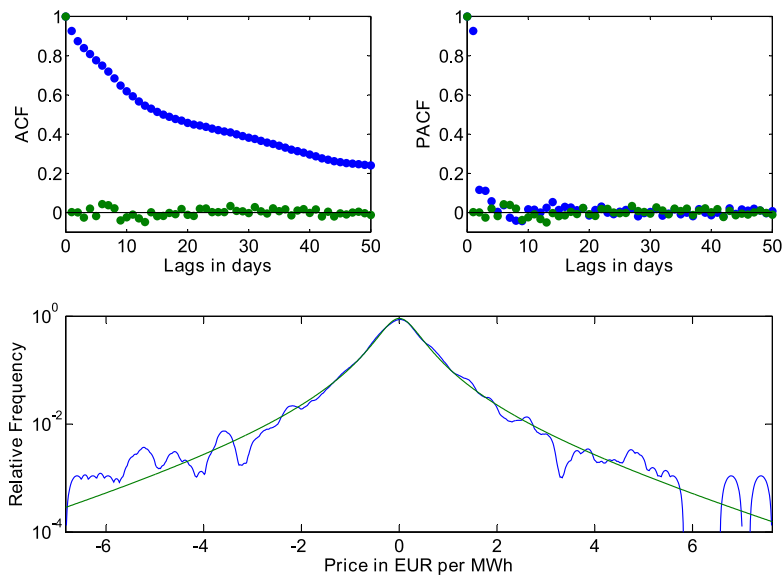
Figure 4.11.: Top: ACF (left) and PACF (right) of residual time series (blue) and innovations of ARMA(1,2) process (green). Bottom: Fit of NIG distribution (green) to empirical innovations (blue).

5. Modeling commodity prices

The simulation of electricity prices using the models introduced in Chapter 3 requires scenarios of commodity prices as input. After introduction of a gas price model in Chapter 4 we present different modeling approaches for the remaining commodities oil, coal and emission allowances in the following. In combination with gas price scenarios these models are the basis for the electricity price simulation.

The markets of coal and oil are global markets as mentioned in Chapter 2. Therefore, these prices are influenced by the state of the world economy. An increasing world economy leads to an increasing demand for coal and oil. This results in increasing prices. The other way round, the prices decrease in times of a decreasing world economy. This is the long term relationship between both commodities. For various reasons, such as supply problems due to wars or natural disasters in production areas concerning only one commodity, the prices might diverge for some time. Afterwards, their common movements are kept up.

The use of coal and oil in power plants makes the producer of power buy emission allowances. As there is a high correlation between the German economy and the world economy an increasing demand for coal and oil worldwide is usually reflected in Germany as well. Therefore, more coal and oil are used for power generation and more emission allowances are needed. This makes the price of emission allowances follow the prices of coal and oil. While the coal and oil market are global markets the market for emission allowances is more local. Thus, the German economy moving in a different direction than the world economy makes the prices of emission allowances and the fuels coal and oil diverge. Nevertheless, there are remarkable correlations between all of these variables.

The choice of future contracts, e.g. first front month or 36th front month, is critical for the correlation. Prices of future contracts further in the future are usually higher correlated. These prices are less disturbed by short term influences. Another chance to reduce short term influences is the consideration of five-day returns instead of daily returns. Correlations of daily log returns of

front month contract prices are given in Table 5.1. The modeling approaches introduced in this chapter are based on different future contracts. Hence, the correlations in the model differ as well. A model having only short term contracts as input parameters cannot reflect a long term correlation and vice versa.

	Oil	Coal	Gas	CO_2
Oil	1	0.30	0.11	0.18
Coal	0.30	1	0.35	0.20
Gas	0.11	0.35	1	0.20
CO_2	0.18	0.20	0.20	1

Table 5.1.: Empirical correlations of daily log returns of the front month prices of oil, coal, natural gas and CO_2 emission allowances from 2008-2012.

The correlations of short term contracts such as the first front month are usually lower than correlations of long term contracts. The above correlations are already significant. Therefore, a combined commodity price model is required. Independent models for each commodity would be an appreciated simplification but ignoring these correlations increases the model error. As the oil price resulting from a combined commodity price model is an exogenous factor for the gas price model in Chapter 4 there are dependencies between coal, emission allowances and gas as well. Thus, we may exclude gas from the modeling approaches as long as the gas price model in Chapter 4 is used.

As these three commodities (gas is only considered in the model in Section 5.4.3) do not exhibit any deterministic trends or seasonalities, the following models are pure stochastic approaches. In Sections 5.2 and 5.3 two well-known approaches are suggested. In Section 5.4 the concept of cointegration as introduced by Granger (1981) is described. This concept allows for a combined price model of oil, coal, gas and emission allowances.

5.1. Literature on commodity price models

Combined commodity price models focus on the common features of the commodities included. These models make a compromise of covering the dependencies and exact modeling of distinct features of the commodities. The literature provides either models for a combination of certain commodities or models for distinct commodities. We give a short overview of models for some commodity combinations and specific models for distinct commodities. To our

knowledge there is no publication on the combination of commodities needed for the model in this work.

The stylized facts of oil and coal prices are similar. Seasonalities do not play a role on both markets, the most relevant fundamental driver is the world economy. Therefore, the same models can be applied to both commodities. The possibly higher volatility of oil prices for being subject to speculations can be covered by a higher model volatility. Commonly used models are the ones proposed by Schwartz (1997) and Schwartz and Smith (2000). These stochastic models have one or two stochastic factors to model the price features. Cortazar and Schwartz (2003) include a third factor in their model to account for the term structure of oil prices. The oil price is the common example those general commodity models are applied to. Nevertheless, as explained above these models work for coal prices as well.

The trade of emission allowances started only a few years ago and the trading mechanisms are still under development (compare Section 2.5). Therefore, there are only a few publications of stochastic models so far. Daskalakis et al. (2009) extend the common Geometric Brownian motion by a jump-diffusion component. A stochastic equilibrium model is formulated by Seifert et al. (2008). Benz and Trück (2009) compare regime-switching approaches and a GARCH model for the spot price of emission allowances.

Examples of modeling approaches using cointegration are given by de Jong and Schneider (2009) and Paschke and Prokopczuk (2009). de Jong and Schneider (2009) analyze the dependencies in various European gas markets as well as the dependency of gas and power prices in the futures market. Paschke and Prokopczuk (2009) analyze the cointegration of crude oil, heating oil and gasoil.

5.2. Three-dimensional random walk

The most basic modeling approach for a discrete univariate non-stationary time series is a **random walk**

$$X_t = X_{t-1} + \varepsilon_t$$

with $\varepsilon_t \sim WN(0, \sigma^2)$. Only the parameter σ needs to be estimated. This concept may be extended to the multivariate case where dependencies between the variables are present. For the multivariate process $\mathbf{X}_t = (X_{1t}, \dots, X_{Nt})'$

$$\mathbf{X}_t = \mathbf{X}_{t-1} + \boldsymbol{\varepsilon}_t$$

with $\varepsilon_t \sim \mathcal{N}_3(\mathbf{0}, \boldsymbol{\Sigma})$ describes the multivariate random walk. The covariance matrix $\boldsymbol{\Sigma}$ contains the variances of the single variables as well as the dependencies between the variables.

The random walk is the discrete equivalent to the **Brownian motion**. If a Brownian motion is applied to logarithms of a variable X_t then the stochastic process $Y_t = \exp\left(X_t - \sigma^2 t / 2\right)$ is a **Geometric Brownian motion**. Especially, in case of price variables the (multivariate) random walk is usually applied to logarithms. This transformation ensures simulation results greater than zero. Nevertheless, the resulting multivariate Geometric Brownian motion contains the correlations between the variables.

In our case the discretized three-dimensional Geometric Brownian motion is applied to the prices of oil, coal and emission allowances. We write

$$\begin{pmatrix} \ln O_t \\ \ln C_t \\ \ln E_t \end{pmatrix} = \begin{pmatrix} \ln O_{t-1} \\ \ln C_{t-1} \\ \ln E_{t-1} \end{pmatrix} + \begin{pmatrix} \epsilon_{1t} \\ \epsilon_{2t} \\ \epsilon_{3t} \end{pmatrix}$$

with $(\epsilon_{1t}, \epsilon_{2t}, \epsilon_{3t})^T \sim \mathcal{N}_3(\mathbf{0}, \boldsymbol{\Sigma})$, the three-dimensional normal distribution with mean $\mathbf{0}$ and covariance matrix $\boldsymbol{\Sigma}$. The drift parameter μ is excluded from our considerations. Possible trends of price variables are included by means of the future contract prices as described in Section 3.5.

After including recent prices in the futures market the scenarios generated by this model represent the historical volatility as well as the expected level of the futures market (see Figure 5.1).

5.3. Correlated two factor model

A more sophisticated modeling approach in comparison to the Geometric Brownian motion was proposed by Schwartz and Smith (2000). We give a short overview of their model and present the multivariate extension to be applied to our multi-commodity situation.

The logarithm of the spot price of a commodity is decomposed into two factors

$$\ln S_t = \chi_t + \xi_t.$$

The factor (ξ_t) is called equilibrium price level and is modeled by a Brownian motion with drift. The short term deviations (χ_t) are covered by an AR(1)
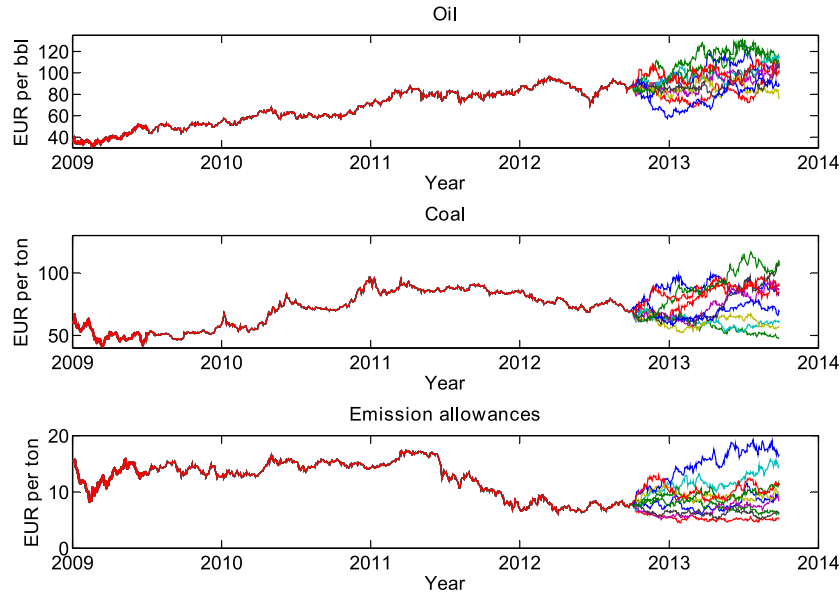
Figure 5.1.: Historical prices of oil (top), coal (center) and emission allowances (bottom) from January 2009 till September 2012. Ten realizations of the three-dimensional random walk for October 2012 till September 2013.

process, the discrete equivalent of an Ornstein-Uhlenbeck process.

$$\chi_t = \alpha\chi_{t-1} + z_{1t}$$
$$\xi_t = \mu_\xi + \xi_{t-1} + z_{2t}$$

The white noise processes (z_{1t}) and (z_{2t}) with variances σ_χ^2 and σ_ξ^2 have correlation ρ. This model setup leads to a log-normal distribution for the spot price with parameters given by Schwartz and Smith (2000).

To derive the risk-neutral model two additional parameters are included. λ_χ and λ_ξ reduce the drifts of both processes which is a standard way of setting up risk-neutral models. The model can be stated as

$$\chi_t = -\lambda_\chi + \alpha\chi_{t-1} + \varepsilon_{1t}$$
$$\xi_t = \mu_\xi - \lambda_\xi + \xi_{t-1} + \varepsilon_{2t}$$

with $\mathrm{Corr}\,(\varepsilon_{1t}, \varepsilon_{2t}) = \rho$. $(\lambda_\chi, \lambda_\xi, \mu_\xi, \alpha, \rho, \sigma_\xi, \sigma_\chi)$ are the unknown parameters of this model. The parameters are estimated using the Kalman filter. For this purpose the model is formulated in state space form. The state space form

5. Modeling commodity prices

consists of the transition equation

$$\mathbf{x}_t = \mathbf{c} + \mathbf{G}\mathbf{x}_{t-1} + \mathbf{w}_t$$

with $\mathbf{x}_t = (\chi_t, \xi_t)^T$, $E(\mathbf{w}_t) = \mathbf{0}$, $\mathrm{Var}(\mathbf{w}_t) = \mathbf{W} = \mathrm{Cov}(\chi_{\Delta t}, \xi_{\Delta_t})$ and

$$\mathbf{c} = \begin{pmatrix} -\lambda_\chi \Delta t \\ (\mu_\xi - \lambda_\xi)\,\Delta t \end{pmatrix}, \mathbf{G} = \begin{pmatrix} e^{-\kappa \Delta t} & 0 \\ 0 & 1 \end{pmatrix}.$$

Δt is the time step. $\kappa = 1 - \alpha$ is the speed of mean reversion in the $\mathrm{AR}(1)$ process (χ_t). As the variable $\mathbf{x}_t$ is not observable the measurement equation

$$\mathbf{y}_t = \mathbf{d}_t + \mathbf{F}_t\mathbf{x}_t + \mathbf{v}_t.$$

including observable futures prices is required. The log future contract prices with times to maturity $T_1, \ldots, T_n$ in the vector $\mathbf{y}_t = (\ln F_{T_1}, \ldots, \ln F_{T_n})^T$ are the basis for the estimation. For simplification we use $T_1, ..., T_n$ instead of the more precise notation $T_1(t), \ldots, T_n(t)$.

$$\mathbf{F}_t = \begin{pmatrix} e^{-\kappa T_1} & 1 \\ \vdots & \vdots \\ e^{-\kappa T_n} & 1 \end{pmatrix}$$

and $\mathbf{d}_t = (A(T_1), \ldots, A(T_n))$ with

$$
\begin{aligned}
A(T) &= (\mu_\xi - \lambda_\xi)\,T - \left(1 - e^{-\kappa T}\right)\frac{\lambda_\chi}{\kappa} \\
&+ \frac{1}{2}\left(\left(1 - e^{-2\kappa T}\right)\frac{\sigma_\chi^2}{2\kappa} + \sigma_\xi^2 T + 2\left(1 - e^{-\kappa T}\right)\frac{\rho \sigma_\chi \sigma_\xi}{\kappa}\right)
\end{aligned}
$$

are further parameters of the measurement equation. Like $(\mathbf{w}_t)$, $(\mathbf{v}_t)$ are serially uncorrelated normally distributed innovations with mean zero and covariance matrix $\mathrm{Cov}(\mathbf{v}_t) = \mathbf{V}$. $\mathbf{V}$ needs to be estimated as well.

The above is a spot price model consisting of a short term and a long term factor. A suitable method for parameter estimation is given and historical data is available. For example, prices of various monthly future contracts can be chosen in case of oil. Given this historical data short term and long term component can be separated and modeled as described above.

Extension to three-dimensional case

In the following we extend the model to a multi-commodity situation where every single commodity is modeled by the approach from above regarding the correlation structure.

The risk-neutral three-dimensional model is formulated as

$$\chi_t^i = -\lambda_\chi^i + \alpha^i \chi_{t-1}^i + \varepsilon_{1t}^i$$
$$\xi_t^i = \mu_\xi^i - \lambda_\xi^i + \xi_{t-1}^i + \varepsilon_{2t}^i$$

where $i \in \{\text{Coal, Oil, Emission allowances}\} = \{C, O, E\} = A$ and the correlations $\text{Corr}(\varepsilon_{1t}^i, \varepsilon_{2t}^i) = \rho^i$. The notations for the state space form can be derived from the notations above. The upper index i is added to denote the corresponding commodity. This means that we derive the state space form with transition equation

$$\begin{pmatrix} \mathbf{x}_t^C \\ \mathbf{x}_t^O \\ \mathbf{x}_t^E \end{pmatrix} = \begin{pmatrix} \mathbf{c}^C \\ \mathbf{c}^O \\ \mathbf{c}^E \end{pmatrix} + \begin{pmatrix} \mathbf{G}^C \\ \mathbf{G}^O \\ \mathbf{G}^E \end{pmatrix} \begin{pmatrix} \mathbf{x}_{t-1}^C \\ \mathbf{x}_{t-1}^O \\ \mathbf{x}_{t-1}^E \end{pmatrix} + \begin{pmatrix} \mathbf{w}_t^C \\ \mathbf{w}_t^O \\ \mathbf{w}_t^E \end{pmatrix}$$

and measurement equation

$$\begin{pmatrix} \mathbf{y}_t^C \\ \mathbf{y}_t^O \\ \mathbf{y}_t^E \end{pmatrix} = \begin{pmatrix} \mathbf{d}_t^C \\ \mathbf{d}_t^O \\ \mathbf{d}_t^E \end{pmatrix} + \begin{pmatrix} \mathbf{F}_t^C \\ \mathbf{F}_t^O \\ \mathbf{F}_t^E \end{pmatrix} \begin{pmatrix} \mathbf{x}_t^C \\ \mathbf{x}_t^O \\ \mathbf{x}_t^E \end{pmatrix} + \begin{pmatrix} \mathbf{v}_t^C \\ \mathbf{v}_t^O \\ \mathbf{v}_t^E \end{pmatrix}$$

where we have the obvious analogy to the form above. In the measurement equation we allow for different numbers of futures prices to be available. Even the times to maturity may differ. As a consequence all vectors and matrices in the measurement equation have $m = n^C + n^O + n^E$ rows. For example, the analogon to $\mathbf{y}_t$ is $\mathbf{y}_t^i = \left(\ln F_{T_1^i}^i, \ldots, \ln F_{T_{n^i}^i}^i \right)^T$ for $i \in A$.

The reason for a three-dimensional model instead of three independent models is given by the correlations between the variables. These correlations can be found in the errors of the transition equation. This (6×1) vector of errors from short term and long term processes of each commodity contains the dependencies between the different model components. We refer to this vector as $(\mathbf{w}_t)$ as in the one-dimensional case.

The covariance matrix of $(\mathbf{w}_t)$ is one of the unknown model parameters. Estimating $\text{Cov}(\mathbf{w}_t)$ means estimating 21 parameters (six variances and 15 covariances). On the one hand, this high number of parameters might cause numerical problems for the Kalman filter. On the other hand, not all of the 15 covariances are needed. Remembering that the six elements of $(\mathbf{w}_t)$ represent three short term and three long term factors for the three commodities we can assume several covariances to be zero. There is no fundamental reason for any short term process of a commodity to be correlated with a long term process of another commodity. Furthermore, we do not allow for any correlations between the short term processes. The common movements of commodity prices

are supposed to be covered by the correlated long term processes but short term price movements are assumed to be independent. For example, political instability in an oil producing country will not affect the coal price as strong as the oil price. Hence, short term deviations can be assumed independent. As a result of the considerations above the covariance matrix can be described as

$$\mathbf{W} = \text{Cov}\left(\mathbf{w}_t\right) = \begin{pmatrix} W_{11} & W_{12} & 0 & 0 & 0 & 0 \\ & W_{22} & 0 & W_{24} & 0 & W_{26} \\ & & W_{33} & W_{34} & 0 & 0 \\ & \vdots & & W_{44} & 0 & W_{46} \\ & & & & W_{55} & W_{56} \\ & & \dots & & & W_{66} \end{pmatrix}.$$

This choice reduces the parameters to be estimated from 21 to 12. The assumptions taken above have a fundamental justification but might not be supported by the data. This means that the estimation of all 21 parameters would not result in nine parameters being equal to zero. The estimates might be significantly different from zero by accident. Therefore, it is not possible to rely on the parameter estimation to incorporate these fundamental considerations.

A further simplification is made in the parameter estimation of $\mathbf{V} = \text{Cov}(\mathbf{v}_t)$. The errors $(\mathbf{v}_t)$ cannot be derived from the original model consisting of the short term and long term factor but are included in the state space form. The variables can be interpreted as measurement errors or errors of the model's fit to observed prices. As in Schwartz (1997) and Schwartz and Smith (2000) we assume these errors to be uncorrelated. Hence, the covariance matrix $\mathbf{V}$ is assumed to be diagonal. This results in a reduction of the number of parameters from $(m+1)m/2$ to m.

In total, the estimation of the parameters $\left(\lambda_\chi^i, \lambda_\xi^i, \mu_\xi^i, \alpha^i, \mathbf{W}, \mathbf{V}\right)$ with $i \in A$ is required for the three-dimensional model. Due to the simplifications of $\mathbf{W}$ and $\mathbf{V}$ the model has $24+m$ parameters. Each one-dimensional model contains nine parameters. As the mean reversion parameters, risk premiums and variances of the three-dimensional model are already included in the one-dimensional model the independent models can be estimated first. The parameters from this estimation can be used as starting values for the parameter estimation in the correlated model. This reduces the problem of finding adequate starting values for the correlated model to the problem of finding three sets of starting values for the independent models.

Initial estimates of mean and covariance matrix can be derived from the historical data. For the speed of mean reversion there is no heuristic approximation

but at least it is known that the parameter is in the interval $(-1, 1)$. This is guaranteed due to the decomposition into a non-stationary and a stationary term. Thus, we are able to generate a set of adequate starting values to ensure convergence of estimation by means of the Kalman filter.

After including recent prices in the futures market (compare Section 3.5) the scenarios generated by this model represent the historical volatility as well as the expected level of the futures market (see Figure 5.2).
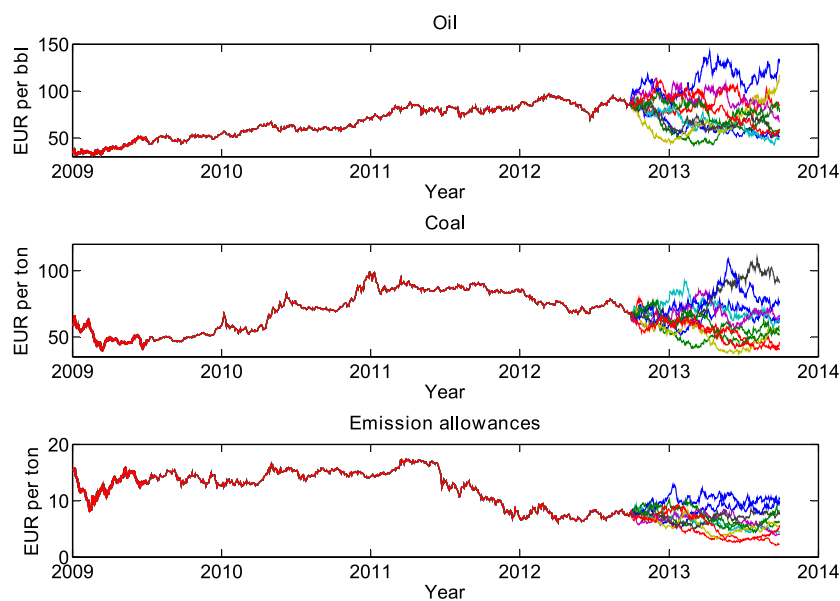
Figure 5.2.: Historical prices of oil (top), coal (center) and emission allowances (bottom) from January 2009 till September 2012. Ten realizations of the correlated two factor model for October 2012 till September 2013.

The estimated correlations between the long term factors within this model are given in Table 5.2. The comparison to Table 5.1 reveals a lower correlation between prices of oil and emission allowances. The estimated correlation of 0.02 is closer to the expectation of no correlation between those variables. As explained above, there is no fundamental reason for a strong correlation between these time series. The other two correlations are close to the empirical ones. This gives support to the model as it is capable of reproducing the correlations of the input variables.

	Oil	Coal	CO_2
Oil	1	0.36	0.02
Coal	0.36	1	0.21
CO_2	0.02	0.21	1

Table 5.2.: Estimated correlations of the long term factors in the correlated two factor model for the prices of oil, coal and CO_2 emission allowances from 2008-2012.

5.4. Cointegration

All time series can be classified with respect to the property stationarity. While mean and variance of a stationary time series are independent of the time non-stationary time series exhibit deterministic or random fluctuations of mean or variance. Linear combinations of any stationary time series are stationary as well. This result does not even require correlation between the variables. In contrast, the linear combination of non-stationary time series is usually not stationary. This can easily be seen in an example. The typical process for non-stationary time series is a Geometric Brownian motion. The difference or any other linear combination of two Geometric Brownian motions is non-stationary if their log returns have different variances. The result holds even in case of correlation 1 between the log returns of both variables. Vector processes for differenced variables do not change this situation.

The issue of stationary linear combinations of non-stationary variables is relevant as many practical examples exhibit this constellation. Such variables might contain the same non-deterministic trends but do not move to far apart. For example, prices of crude oil, gasoil and fuel oil are non-stationary. Refineries use crude oil to produce gasoil and fuel oil. As the costs for refineries are nearly constant the spread between crude oil, gasoil and fuel oil (the crack spread) is not subject to strong fluctuations. An increase of the crack spread would lead to new refineries reducing the spread and vice versa. Fluctuations of the spread, e.g. due to refinery outages, are comparably low. Thus, the crack spread is stationary. A well-known example concerning electricity prices is the clean dark spread describing the costs for a coal-fired power plant (compare Section 6.3). This spread is relevant for the valuation of such power plants. Thus, an adequate model is needed.

The solution to such problems is given by the concept of cointegration as first proposed by Granger (1981). The major result making cointegration applicable to practical situations is the Granger representation theorem given by Engle and Granger (1987). In the following we present the concept. This includes the

mathematical basics as well as adequate procedures for testing and parameter estimation. The results are based on Johansen (1995). Finally, the concept is applied to the prices of coal, oil, gas and emission allowances.

5.4.1. Definitions and properties of cointegration

In the following we consider a set of N variables $\mathbf{X}_t = (X_{1t}, \ldots, X_{Nt})^T$ with autocorrelations for each variable. Additionally, each variable is correlated with at least one other variable. This is a common situation for financial time series. Especially, the second characteristic makes the comfortable case of independent models for all variables impossible. The typical model for autocorrelated univariate time series is an autoregressive model. The multivariate equivalent is the class of **vector autoregressive processes**.

Definition 5.1 (VAR(k) process)**.** *The N-dimensional process $(\mathbf{X}_t)$ defined by*

$$\mathbf{X}_t = \mathbf{\Pi}_1 \mathbf{X}_{t-1} + \cdots + \mathbf{\Pi}_k \mathbf{X}_{t-k} + \boldsymbol{\varepsilon}_t, \quad \text{for } \mathbf{\Pi}_i \in \mathbb{R}^{N \times N}, i = 1, \ldots, k, \quad (5.1)$$

*is called **vector autoregressive process** of order k. The innovations are independent identically distributed, $\boldsymbol{\varepsilon}_t \sim \mathcal{N}_N (\mathbf{0}, \mathbf{\Omega})$. Initial values $\mathbf{X}_{-k+1}, \ldots, \mathbf{X}_0$ are given.*

The above definition can be extended by a deterministic term consisting of a constant, a linear term or indicator variables. As the time series throughout this work have mean zero when VAR(k) processes are applied the deterministic term is omitted in the definitions. The matrix $\mathbf{\Omega}$ contains variances of the components of $(\mathbf{X}_t)$ as well as their covariances.

Further characteristics of the process can be described by means of the characteristic polynomial

$$A(z) = I_N - \sum_{i=1}^{k} \mathbf{\Pi}_i z^i, \quad \text{for } z \in \mathbb{R}.$$

Due to the recursive structure of a VAR(k) process $(\mathbf{X}_t)$ can be expressed in terms of initial values and innovations.

Theorem 5.2. *A VAR(k) process can be written as*

$$\mathbf{X}_t = \sum_{s=1}^{k} \mathbf{C}_{t-s} \left(\mathbf{\Pi}_s \mathbf{X}_0 + \cdots + \mathbf{\Pi}_k \mathbf{X}_{-k+s} \right) + \sum_{j=0}^{t-1} \mathbf{C}_j \boldsymbol{\varepsilon}_{t-j}$$

5. Modeling commodity prices

where $\mathbf{C}_0 = I_N$ *and the coefficients* $\mathbf{C}_s \in \mathbb{R}^{N \times N}$ *are defined recursively by*

$$\mathbf{C}_s = \sum_{j=1}^{\min(k,s)} \mathbf{C}_{s-j}\mathbf{\Pi}_j, \quad s = 1, 2, \dots$$

The generating function $C(z) = \sum_{n=0}^{\infty} \mathbf{C}_n z^n$ *has a radius of convergence* δ. *For* $z \in \mathbb{R}$ *with* $|z| < \delta$ *the relation* $C(z) = A(z)^{-1}$ *holds.*

So far, the considered processes are unrestricted allowing for stationary processes as well as non-stationary processes. The behavior of the characteristic polynomial tells about the stationarity of $(\mathbf{X}_t)$.

Theorem 5.3. *Assume that* $|z| > 1$ *or* $z = 1$ *follows from* $|A(z)| = 0$. *Then,* $|A(1)| \neq 0$ *is a necessary and sufficient condition for the stationarity of* $(\mathbf{X}_t)$. *Furthermore, the process has the moving average representation*

$$\mathbf{X}_t = \sum_{n=0}^{\infty} \mathbf{C}_n \boldsymbol{\varepsilon}_{t-n}$$

with $C_0(z) = \sum_{n=0}^{\infty} \mathbf{C}_n z^n = A(z)^{-1}$ *converging for* $|z| < 1 + \delta$ *for some* $\delta > 0$ *and* $z \in \mathbb{R}$.

Parameters of VAR models can be estimated using the maximum likelihood method. After this estimation the stationarity of the process can be analyzed according to the conditions given in the theorem above. Usually the time series are tested for stationarity before the VAR model is estimated. If the test states non-stationarity the time series is differenced to obtain a stationary time series. Afterwards, the parameters are estimated.

Definition 5.4. *1. A stochastic process* $(\mathbf{X}_t)$ *satisfying* $\mathbf{X}_t = \sum_{i=0}^{\infty} \mathbf{C}_i \boldsymbol{\varepsilon}_{t-i}$ *is called **stationary**, denoted as* $I(0)$, *if* $\mathbf{C} = \sum_{i=0}^{\infty} \mathbf{C}_i \neq 0$.

2. A stochastic process $(\mathbf{X}_t)$ *is called **integrated** of order d, denoted as* $I(d)$, *if* $(\Delta^d \mathbf{X}_t)$ *is* $I(0)$.

This definition of integration implies that multivariate processes consisting of time series with different orders of integration are integrated. The order is the maximum of all integration orders of the components.

The major result for cointegration is the **Granger representation theorem** given by Engle and Granger (1987). In the following the theorem is presented based on Johansen (1995). This includes a different formulation of the Granger representation theorem.

Definition 5.5 (Cointegration). *A stochastic process* $\mathbf{X}_t = (X_{1t}, \ldots, X_{Nt})^T$ *is* ***cointegrated*** *of order* (d, b), *if*

(i) $X_{it} \sim I(d) \quad \forall\, i = 1, \ldots, N$, *and*

(ii) *a vector* $\boldsymbol{\beta} \in \mathbb{R}^N$ *with* $\boldsymbol{\beta}^T \mathbf{X}_t \sim I(d-b)$ *exists for* $b > 0$.

Cointegration of order (d, b) *is denoted as* $\mathbf{X}_t \sim CI(d, b)$. $\boldsymbol{\beta}$ *is called* ***cointegrating vector***. *The* ***rank of cointegration*** *is the number of linear independent cointegrating vectors.*

So far, a class of multivariate time series models (VAR) and a concept to incorporate dependencies between various variables (cointegration) have been introduced. The missing part is a method to set up multivariate models with cointegrated variables. For this purpose the VAR representation in Equation (5.1) is written in the error correction form

$$\Delta\mathbf{X}_t = \boldsymbol{\Pi}\mathbf{X}_{t-1} + \sum_{i=1}^{k-1} \boldsymbol{\Gamma}_i \Delta\mathbf{X}_{t-i} + \boldsymbol{\varepsilon}_t \qquad (5.2)$$

where $\boldsymbol{\Pi} = \sum_{i=1}^{k} \boldsymbol{\Pi}_i - I_N$ and $\boldsymbol{\Gamma}_i = -\sum_{j=i+1}^{k} \boldsymbol{\Pi}_j$. Let $\boldsymbol{\Gamma} = I_N - \sum_{i=1}^{k-1} \boldsymbol{\Gamma}_i$. Using this parametrization the connection between cointegration and autoregressive models in VAR or error correction form can be stated. For this purpose we define the matrix $\boldsymbol{\beta}_\perp \in \mathbb{R}^{N \times (N-r)}$ of full rank such that $\boldsymbol{\beta}^T \boldsymbol{\beta}_\perp = 0$. Technical details for the construction of $\boldsymbol{\beta}_\perp$ are given by Johansen (1995).

Theorem 5.6. *If* $|z| > 1$ *or* $z = 1$ *follows from* $|A(z)| = 0$, *and* $\operatorname{rank}(\boldsymbol{\Pi}) = r < N$, *then there exist matrices* $\boldsymbol{\alpha}, \boldsymbol{\beta} \in \mathbb{R}^{N \times r}$ *of rank* r *such that*

$$\boldsymbol{\Pi} = \boldsymbol{\alpha}\boldsymbol{\beta}^T.$$

A necessary and sufficient condition for the stationarity of $(\Delta\mathbf{X}_t)$ *and* $(\boldsymbol{\beta}^T\mathbf{X}_t)$ *is*

$$\left| \boldsymbol{\alpha}_\perp^T \boldsymbol{\Gamma} \boldsymbol{\beta}_\perp \right| \neq 0.$$

Then, the process $(\mathbf{X}_t)$ *can be expressed as*

$$\mathbf{X}_t = \mathbf{C} \sum_{i=1}^{t} \boldsymbol{\varepsilon}_i + C_1(L)\boldsymbol{\varepsilon}_t + \mathbf{A}$$

where $\boldsymbol{\beta}^T\mathbf{A} = 0$ *and* $\mathbf{C} = \boldsymbol{\beta}_\perp (\boldsymbol{\alpha}_\perp^T \boldsymbol{\Gamma} \boldsymbol{\beta}_\perp)^{-1} \boldsymbol{\alpha}_\perp^T \in \mathbb{R}^{N \times N}$. *Hence,* $\mathbf{X}_t \sim CI(1, 1)$ *with cointegrating vectors* $\boldsymbol{\beta}$. *For the function* $C_1(\cdot)$ *the equation*

$$A^{-1}(z) = \mathbf{C}\frac{1}{1-z} + C_1(z), \quad z \in \mathbb{R} \setminus \{1\},$$

holds. The radius of convergence for the power series $C_1(z)$ is $1 + \delta$ for some $\delta > 0$.

This is a different formulation of the well-known Granger representation theorem by Engle and Granger (1987). Johansen (1995) state the above version for practical reasons. Usually an autoregressive model is applied to data. Using the above version of the theorem it can be told from the estimated parameters whether the process is cointegrated or not. The original theorem starts with cointegration and infers the representation in VAR or error correction form.

The condition $\left|\boldsymbol{\alpha}_\perp^T \boldsymbol{\Gamma} \boldsymbol{\beta}_\perp\right| \neq 0$ is satisfied for all parameters except of a null set, if the variables are $I(1)$. As the variables in this work have orders of integration less than or equal to one this condition is usually satisfied.

A major issue for such a concept as cointegration is the estimation of parameters. A maximum likelihood method for parameter estimation and a test for cointegration are presented in the following.

5.4.2. Parameter estimation and testing

The following explanations focus on testing and parameter estimation for $I(1)$ variables.

Definition 5.7. *The model $H(r)$ of $I(1)$ variables is defined as the submodel of the VAR in Equation (5.1) where*

$$\boldsymbol{\Pi} = \boldsymbol{\alpha}\boldsymbol{\beta}^T$$

holds for $(N \times r)$ matrices $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$. The vector error correction model (5.2) turns into

$$\Delta \mathbf{X}_t = \boldsymbol{\alpha}\boldsymbol{\beta}^T \mathbf{X}_{t-1} + \sum_{i=1}^{k-1} \boldsymbol{\Gamma}_i \Delta \mathbf{X}_{t-i} + \boldsymbol{\varepsilon}_t$$

with unrestricted parameters $(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\Gamma}_1, \ldots, \boldsymbol{\Gamma}_{k-1}, \boldsymbol{\Omega})$.

Using this definition models with different ranks of cointegration can be seen as nested models since

$$H(0) \subset H(1) \subset \cdots \subset H(N)$$

holds. $H(0)$ implies $\boldsymbol{\Pi} = 0$ which corresponds to the usual VAR model in differences. In contrast $H(N)$ corresponds to the usual VAR model in VECM form.

A $H(r)$ model in VECM form can be rewritten as

$$\mathbf{Z}_{0t} = \boldsymbol{\alpha}\boldsymbol{\beta}^T\mathbf{Z}_{1t} + \mathbf{R}\mathbf{Z}_{2t} + \boldsymbol{\varepsilon}_t$$

with the notations $\mathbf{Z}_{0t} = \Delta\mathbf{X}_t \in \mathbb{R}^N, \mathbf{Z}_{1t} = \mathbf{X}_{t-1} \in \mathbb{R}^N, \mathbf{R} = (\boldsymbol{\Gamma}_1, \ldots, \boldsymbol{\Gamma}_{k-1}) \in \mathbb{R}^{N \times N(k-1)}$ and $\mathbf{Z}_{2t} = \left(\Delta\mathbf{X}_{t-1}^T, \ldots, \Delta\mathbf{X}_{t-k+1}^T\right)^T$. $\mathbf{R}$ is a matrix of parameters corresponding to the vector $\mathbf{Z}_{2t} \in \mathbb{R}^{N(k-1)}$. For this model the log-likelihood function neglecting a constant can be stated as

$$\log L\left(\mathbf{R}, \boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\Omega}\right) = -\frac{1}{2}n\log|\boldsymbol{\Omega}|$$
$$-\frac{1}{2}\sum_{t=1}^{n}\left(\mathbf{Z}_{0t} - \boldsymbol{\alpha}\boldsymbol{\beta}^T\mathbf{Z}_{1t} - \mathbf{R}\mathbf{Z}_{2t}\right)^T\boldsymbol{\Omega}^{-1}\left(\mathbf{Z}_{0t} - \boldsymbol{\alpha}\boldsymbol{\beta}^T\mathbf{Z}_{1t} - \mathbf{R}\mathbf{Z}_{2t}\right).$$

The first order condition for the estimation of $\mathbf{R}$ is

$$\sum_{t=1}^{n}\left(\mathbf{Z}_{0t} - \boldsymbol{\alpha}\boldsymbol{\beta}^T\mathbf{Z}_{1t} - \widehat{\mathbf{R}}\mathbf{Z}_{2t}\right)\mathbf{Z}_{2t}^T = 0.$$

The definition

$$\mathbf{M}_{ij} = \frac{1}{n}\sum_{t=1}^{n}\mathbf{Z}_{it}\mathbf{Z}_{jt}^T, \quad i,j = 0,1,2,$$

with the property $\mathbf{M}_{ij} = \mathbf{M}_{ji}^T$ allows for the reformulation of the first order condition above:

$$\mathbf{M}_{02} = \boldsymbol{\alpha}\boldsymbol{\beta}^T\mathbf{M}_{12} + \widehat{\mathbf{R}}\mathbf{M}_{22}$$
$$\Leftrightarrow \quad \widehat{\mathbf{R}}\left(\boldsymbol{\alpha}, \boldsymbol{\beta}\right) = \mathbf{M}_{02}\mathbf{M}_{22}^{-1} - \boldsymbol{\alpha}\boldsymbol{\beta}^T\mathbf{M}_{12}\mathbf{M}_{22}^{-1}.$$

As a consequence the residuals obtained by regressing $(\mathbf{Z}_{0t})$ and $(\mathbf{Z}_{1t})$ on $(\mathbf{Z}_{2t})$ can be defined as

$$\mathbf{R}_{0t} = \mathbf{Z}_{0t} - \mathbf{M}_{02}\mathbf{M}_{22}^{-1}\mathbf{Z}_{2t},$$
$$\mathbf{R}_{1t} = \mathbf{Z}_{1t} - \mathbf{M}_{12}\mathbf{M}_{22}^{-1}\mathbf{Z}_{2t}.$$

This leads to the concentrated likelihood function

$$\log L\left(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\Omega}\right) = -\frac{1}{2}n\log|\boldsymbol{\Omega}| - \frac{1}{2}\sum_{t=1}^{n}\left(\mathbf{R}_{0t} - \boldsymbol{\alpha}\boldsymbol{\beta}^T\mathbf{R}_{1t}\right)^T\boldsymbol{\Omega}^{-1}\left(\mathbf{R}_{0t} - \boldsymbol{\alpha}\boldsymbol{\beta}^T\mathbf{R}_{1t}\right).$$

The regression equation

$$\mathbf{R}_{0t} = \boldsymbol{\alpha}\boldsymbol{\beta}^T\mathbf{R}_{1t} + \widetilde{\boldsymbol{\varepsilon}}_t$$

5. Modeling commodity prices

would lead to the same likelihood function. This is a **reduced rank regression** as described by Anderson (1951). With the definition

$$\mathbf{S}_{ij} = \frac{1}{n}\sum_{t=1}^{n}\mathbf{R}_{it}\mathbf{R}_{jt}^{T} = \mathbf{M}_{ij} - \mathbf{M}_{i2}\mathbf{M}_{22}^{-1}\mathbf{M}_{2j}, \quad i,j = 0,1,$$

estimators for $\boldsymbol{\alpha}$ and $\boldsymbol{\Omega}$ can be obtained. For fixed $\boldsymbol{\beta}$ the regression equation above is linear and the estimators are

$$\widehat{\boldsymbol{\alpha}}(\boldsymbol{\beta}) = \mathbf{S}_{01}\boldsymbol{\beta}\left(\boldsymbol{\beta}^{T}\mathbf{S}_{11}\boldsymbol{\beta}\right)^{-1} \quad \text{and}$$
$$\widehat{\boldsymbol{\Omega}}(\boldsymbol{\beta}) = \mathbf{S}_{00} - \mathbf{S}_{01}\boldsymbol{\beta}\left(\boldsymbol{\beta}^{T}\mathbf{S}_{11}\boldsymbol{\beta}\right)^{-1}\boldsymbol{\beta}^{T}\mathbf{S}_{10} = \mathbf{S}_{00} - \widehat{\boldsymbol{\alpha}}(\boldsymbol{\beta})\left(\boldsymbol{\beta}^{T}\mathbf{S}_{11}\boldsymbol{\beta}\right)\widehat{\boldsymbol{\alpha}}(\boldsymbol{\beta})^{T}.$$

The estimators for $\boldsymbol{\alpha}, \boldsymbol{\Omega}$ and $\mathbf{R}$ deduced above depend on $\boldsymbol{\beta}$. Replacing $\boldsymbol{\alpha}$ and $\boldsymbol{\Omega}$ by $\widehat{\boldsymbol{\alpha}}(\boldsymbol{\beta})$ and $\widehat{\boldsymbol{\Omega}}(\boldsymbol{\beta})$ in the concentrated likelihood function results in a likelihood function only depending on $\boldsymbol{\beta}$. With some more rather technical considerations the maximum likelihood estimator for $\boldsymbol{\beta}$ can be derived.

Theorem 5.8. *Given the hypothesis*

$$H(r) : \boldsymbol{\Pi} = \boldsymbol{\alpha}\boldsymbol{\beta}^{T},$$

the maximum likelihood estimator of $\boldsymbol{\beta}$ is the result of the following procedure. The equation

$$\left|\lambda\mathbf{S}_{11} - \mathbf{S}_{10}\mathbf{S}_{00}^{-1}\mathbf{S}_{01}\right| = 0$$

is solved for the eigenvalues $1 > \widehat{\lambda}_1 > \cdots > \widehat{\lambda}_N > 0$ with the corresponding eigenvectors $\widehat{\mathbf{V}} = (\widehat{\mathbf{v}}_1,\ldots,\widehat{\mathbf{v}}_N)$. The eigenvectors are normalized by $\widehat{\mathbf{V}}^{T}\mathbf{S}_{11}\widehat{\mathbf{V}} = I_N$. The cointegrating relations $\boldsymbol{\beta}$ result from the eigenvectors

$$\widehat{\boldsymbol{\beta}} = (\widehat{\mathbf{v}}_1,\ldots,\widehat{\mathbf{v}}_r).$$

This estimator of $\boldsymbol{\beta}$ is inserted in the equations above to derive estimates of $\boldsymbol{\alpha}, \boldsymbol{\Omega}$ and $\mathbf{R}$. The maximized likelihood function for the model is given by

$$L_{\max}^{-2/T}(H(r)) = |\mathbf{S}_{00}|\prod_{i=1}^{r}\left(1 - \widehat{\lambda}_i\right).$$

The likelihood ratio test statistic $Q(H(r)|H(N))$ for $H(r)$ in $H(N)$ is given by

$$-2\log Q\left(H(r)|H(N)\right) = -n\sum_{i=r+1}^{N}\log\left(1 - \widehat{\lambda}_i\right).$$

The statistic changes to

$$-2\log Q\left(H(r)|H(r+1)\right) = -n\log\left(1 - \widehat{\lambda}_{r+1}\right)$$

for testing $H(r)$ in $H(r+1)$. Under the assumption $\mathrm{rank}\,(\mathbf{\Pi}) = r$ *the asymptotic distribution is*

$$-2\log Q\left(H(r)|H(N)\right) \xrightarrow{w} \mathrm{tr}\left\{\int_0^1 (d\mathbf{B})\mathbf{B}^T \left[\int_0^1 \mathbf{B}\mathbf{B}^T du\right]^{-1} \int_0^1 \mathbf{B}(d\mathbf{B})^T\right\}$$

where $\mathbf{B}$ *is a* $N-r$ *dimensional Brownian motion.*

A more detailed deduction of this theorem, the proof and simulated values of the distribution can be found in Johansen (1995).

The above theorem and the previous deduction introduce a procedure to estimate all parameters for the cointegrated model with a given rank of cointegration $r \in \{0, \ldots, N\}$. A last open question to be answered is the determination of r.

The rank of cointegration r takes on the values $0, 1, \ldots, N$. Due to the nested structure of the models with different ranks of cointegration a sequence of tests can be run. The test statistic $Q_r = -2\log Q\left(H(r)|H(N)\right)$ is compared with its quantile c_r. If $Q_r < c_r$, then $\widehat{r} = r$. If $Q_r \geq c_r$ then the test statistic and the quantile are compared for $r+1$. The quantiles c_r result from simulated values of the distribution of the test statistic as mentioned above. After the procedure is carried out for $r = 0, \ldots, N$ the highest r satisfying the above condition is known. Thus, the parameters of the model $H(\widehat{r})$ can be estimated.

After introduction of cointegration including a test for cointegration, parameter estimation and determination of the rank of cointegration the concept may be applied. In the following we present the application to the prices of oil, coal, gas and emission allowances.

5.4.3. Application to commodity prices

The application of cointegration procedures to real data implies various difficulties.

- The available period of data might be too short to identify a cointegrating relationship. Cointegration describes a common long term movement that might not be obvious in a short data set.

- The available period of data might contain too many "disturbances" by exceptional events to identify a cointegrating relationship. A war in oil

producing countries leads the oil price away from its equilibrium price described by the cointegration.

- The chosen products might be too volatile to identify a cointegrating relationship. Contracts on the future markets with long maturity are less volatile than short term products.

As our cointegrated commodity price model is supposed to be an input factor for the spot price of electricity we are interested in cointegration on "spot" products. This means front month contracts in our case. As mentioned above, the front month contracts might be too volatile. If we use long term future contracts instead there is a better chance to identify a strong cointegration. To use this, another process linking long term future contracts to front month contracts is needed. In order to avoid this additional complexity of the model we stay with prices of front month contracts.

Based on the data used in this work the front month prices of oil, coal and emission allowances are not cointegrated. The test by Johansen (see Section 5.4.2) cannot state cointegration at a reasonable level of confidence. This result is not surprising if we consider Figure 5.3. The prices of oil and coal have a common trend but the prices of emission allowances are independent. Inclusion of natural gas as a fourth variable changes the situation. The prices of gas and emission allowances show a similar (though mirrored) behavior.

The cointegration test states this set of variables to be cointegrated of rank one at a significance level of 0.05. Cointegration of rank three can be stated for the same data set at a significance level of 0.1. If log prices are used instead the cointegration of rank one is identified at the same significance level. The cointegration of rank three cannot be stated anymore. The drawback of a model based on original prices is the chance of negative prices in simulations. Especially, prices of emission allowances are close to zero which immediately leads to negative simulation results. Therefore, we decide for the cointegration of rank one based on log prices.

	Oil	Coal	Gas	CO_2
Oil	1	0.36	0.00	0.10
Coal	0.36	1	0.57	0.38
Gas	0.00	0.57	1	0.03
CO_2	0.10	0.38	0.03	1

Table 5.3.: Correlations of 1000 realizations of prices of oil, coal, natural gas and emission allowances based on the cointegration approach.
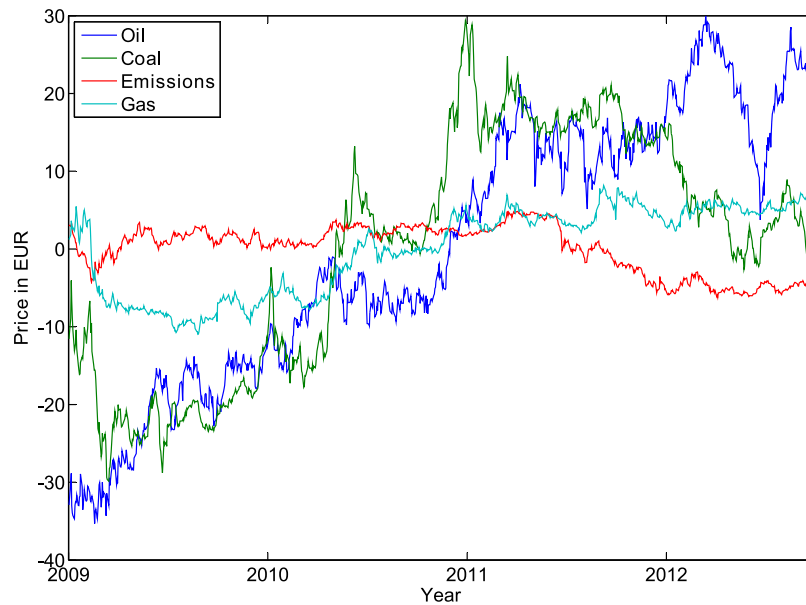
Figure 5.3.: Historical front month contract prices of oil (blue, Euro per barrel), coal (green, Euro per ton), emission allowances (red, Euro per ton) and gas (light blue, Euro per MWh) from January 2009 till September 2012. The mean is subtracted.

The simulated correlations of the cointegrated model given in Table 5.3 deviate from the empirical ones given in Table 5.1. The greatest deviation is observable in the correlation of coal and gas prices: 0.57 (cointegration) to 0.35 (empirical). Not even this deviation is big enough to challenge the approach. The concept of cointegration describes a stronger relationship than the (linear) correlation of variables. As this relationship is statistically significant it is more important to include the cointegration than to match the empirical correlations. Two realizations of the cointegration model are given in Figure 5.4. The strong connection of coal and oil prices within the model makes both variables move together. Gas and emission allowances show the typical fluctuations within a certain price range.
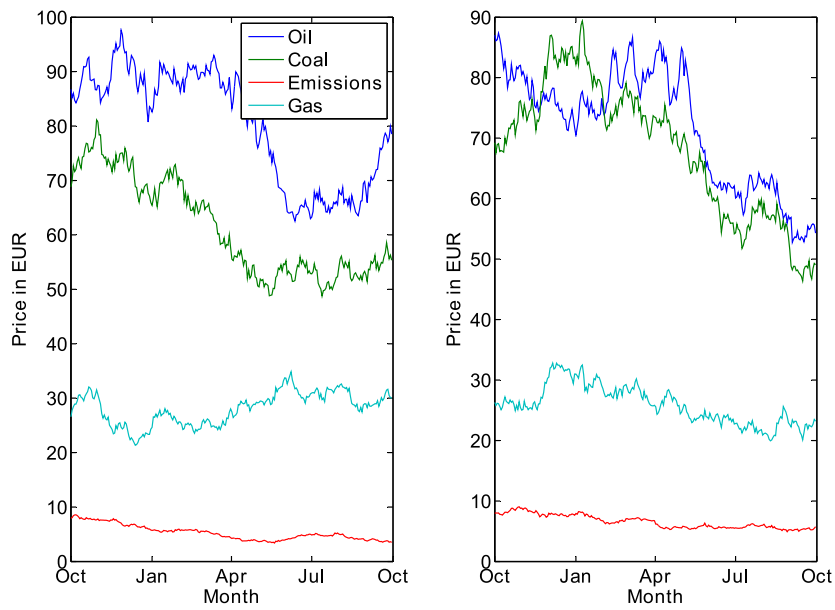
Figure 5.4.: Two realizations of the cointegration approach for oil (blue), coal (green), emission allowances (red) and natural gas (light blue). The time period from October 2012 till September 2013 is simulated.

6. Discussion of electricity price models

In the previous chapters a new spot price model for electricity based on various fundamental input variables is introduced. Regarding the load and the commodity prices several alternative modeling approaches are presented. In this chapter we give a discussion of all approaches presented above with respect to the resulting electricity prices. This includes the comparison to existing models from the literature. Before starting the comparison a short overview of the presented models is given.

6.1. Overview of presented models

The spot price process is defined as

$$S_t = g\left(l_t, G_t, C_t, E_t, t\right) + X_t^{(S)}$$

which was defined more general in Equation (3.2). The function g can be obtained by applying the model to the whole data at once and by applying it to each of the 24 hourly time series. We distinguish these approaches in our analysis.

Model E1: The model

$$
\begin{aligned}
g\left(l_t, G_t, C_t, E_t, t\right) = \ & a_0 + a_1 \mathbf{1}_{Sat}(t) + a_2 \mathbf{1}_{Sun}(t) + a_3 l_t + a_4 G_t \mathbf{1}_A(t) \\
& + a_5 C_t + a_6 E_t + a_7 l_t G_t + a_8 l_t C_t + a_9 l_t E_t
\end{aligned}
$$

is applied to the whole data set (compare Equation (3.5)). $\mathbf{1}_A(t)$ describes the hours from 7 a.m. to midnight.

Model E2: The model

$$
\begin{aligned}
g\left(l_t, G_t, C_t, E_t, t\right) = \ & a_0(t) + a_1(t) \mathbf{1}_{We}(t) \\
& + a_2(t) l_t + a_3(t) G_t + a_4(t) C_t + a_5(t) E_t \\
& + a_6(t) l_t G_t + a_7(t) l_t C_t + a_8(t) l_t E_t
\end{aligned}
$$

is applied to the 24 hourly time series (compare Equation (3.6)). The indicator variables for Saturday and Sunday are aggregated to one indicator variable for the whole weekend.

There are two approaches to obtain the residual load process (l_t).

Model L1: The grid load (see Equation (3.8)) is modeled as

$$L_t = \widehat{L}_t + X_t^{(L)} + Y_t^{(L)}.$$

The renewables, being subtracted from the grid load, are obtained by means of a historical simulation with varying forecasts of the future capacities as described in Section 3.4.1.

Model L2: In analogy to the grid load, the residual load (see Equation (3.9)) is directly modeled as

$$l_t = \widehat{l}_t + X_t^{(l)}.$$

For the commodity prices we consider the following three model alternatives.

Model F1: The prices of oil, coal and emission allowances follow a three-dimensional random walk (see Section 5.2)

$$\begin{pmatrix} \ln O_t \\ \ln C_t \\ \ln E_t \end{pmatrix} = \begin{pmatrix} \ln O_{t-1} \\ \ln C_{t-1} \\ \ln E_{t-1} \end{pmatrix} + \begin{pmatrix} \epsilon_{1t} \\ \epsilon_{2t} \\ \epsilon_{3t} \end{pmatrix}.$$

The gas price model with normalized cumulated heating degree days (Λ_t) and oil price component (Ψ_t) as proposed in Equation (4.2)

$$G_t = \widehat{G}_t + \alpha_1 \Lambda_t + \alpha_2 \Psi_t + X_t^{(G)}$$

is used.

Model F2: The prices of oil, coal and emission allowances are modeled by the correlated two factor model described in Section 5.3. The gas price model is the same as in **F1**.

Model F3: The log prices of oil, coal, emission allowances and gas are cointegrated of rank one. The model is simulated using the VAR representation in Equation (5.1).

For details about the stochastic processes we refer to the corresponding chapters where the calibration of these processes is presented. This overview focusses on the relationships between the model components. In the following

the spot price models are compared with respect to the underlying load and commodity price models.

6.2. Analysis of model features

A comprehensive discussion of the above models with respect to statistical criteria is not possible for several reasons.

- Typical criteria for model selection such as the AIC criterion are not applicable as likelihood functions for the models are not available.

- The regression models **E1**, **E2**, **L1** and **L2** can be compared according to their R^2 or adjusted R^2. Nevertheless, the R^2 cannot tell whether the combination of **E1** and **F1** provides a "better" model than the combination **E1** and **F2** as the commodity price models are no regression models where the coefficient of determination can be calculated.

- The data contains structural changes due to the increase of renewables and the decrease of nuclear capacities. Therefore, it is not advisable to divide the data used in this work into two sets: one for model calibration and one for model validation. The data used for model validation would consist of the most recent data, i.e. the data containing the effects of changed nuclear and renewable capacities. Thus, the model cannot learn about these effects from the remaining data.

For these reasons a final decision about the best model cannot be based on a statistical analysis. In the following, we discuss the models with respect to various aspects.

Comparison of R^2

The models **E1** and **E2** can be compared according to the goodness-of-fit R^2. This measure can be calculated for some other models as well. Table 6.1 gives an overview of some models. The value belonging to SMaPS refers to the goodness-of-fit of the model introduced by Burger et al. (2004). This model is one of the first fundamental spot price models (compare Section 3.2 and 3.3). Not even the use of residual load instead of grid load remarkably increases the goodness-of-fit of the SMaPS model. The fuel-adjusted heat-

6. Discussion of electricity price models

	E1	**E2**	SMaPS	Andersson et al.
R^2	0.69	0.71	0.35	0.79

Table 6.1.: In-sample comparison of different models with respect to R^2.

rate model proposed by Andersson et al. (2013) exceeds the R^2 of the other models. $R^2 = 0.79$ is stated in their paper based on data from January 2010 till December 2010. Fitting a model to this data leads to an increased R^2 due to the smaller number of observations. Furthermore, influences of the economical crisis 2008/2009, the decrease of nuclear capacities and the increase of renewables do not affect this data.

Stylized facts

The proposed models are supposed to exhibit the stylized facts given in Section 3.1.

- All models compared above have the load as an input variable. Therefore, the **seasonalities** of the load are reflected by the resulting spot price processes.

- The SMaPS model cannot reproduce **negative prices** due to the multiplicative structure of the model. The fuel-adjusted heat-rate model is based on a dual-exponential model which allows for negative prices as well. Our models have an additive structure and can generate scenarios including negative prices.

- The SARIMA process of the SMaPS model relies on normal distributed innovations. The residuals of the fuel-adjusted heat-rate model are white noise of normal distribution. Although the dual-exponential function strongly increases it is not guaranteed that **price spikes** occur. Thus, these models do not simulate price spikes. Our models use a SARIMA process with t-distributed innovations. Due to the low number of degrees of freedom this distribution has heavy tails and is capable of generating price spikes.

- The SARIMA processes of our models and the SMaPS model are **mean-reverting** like the white noise process in the fuel-adjusted heat-rate model.

- The white noise of the model by Andersson et al. (2013) does not consider

the autocorrelations of the residuals. This feature is covered by the SARIMA process included in our model.

Input variables

The SMaPS model relies on the grid load adjusted by average availability of power plants. Apart from pricing electricity derivatives the model can be applied for pricing of retail power contracts (see Chapter 7). Our models **E1** and **E2** are capable of risk-adequate pricing on the retail market, if the load model **L1** is used. Adequate valuation of conventional power plants is not possible due to the missing commodity prices in the SMaPS model. The fuel adjustment of Andersson et al. (2013) considers the prices of coal and emission allowances. Natural gas or oil prices are neglected. Thus, the model can be used for the valuation of coal-fired power plants but not for valuation of gas-fired power plants or pricing on the retail market. In addition to the residual load, the prices of coal, gas and emission allowances are included in our model. Altogether, this allows for comprehensive valuations of power plants and risk-adequate pricing on the retail market. The model can easily be extended by an oil price component for valuations of oil price related assets or derivatives.

Long term uncertainty

Burger et al. (2004) included a long term process (random walk with drift) in their model to account for the uncertainty of price development on a long term horizon. Such a factor is not mentioned by Andersson et al. (2013) but for risk management purposes this uncertainty plays a major role. Reasons for changes of electricity price level on a long term horizon are changing commodity prices, changes of generation capacities and changes of demand. Most of these reasons are already considered in our models. The non-stationary commodity price models reflect the uncertainty of commodity price levels. In contrast to model **L2**, the grid load model in model **L1** includes a long term factor to account for possible changes of the load level for economic reasons. The models for renewables have capacity forecasts as a major component. Except of capacity changes of conventional power plants this source of long term price uncertainty is modeled as well. Altogether, the major part of sources of long term uncertainty is covered by the model components. Another stochastic long term factor is unnecessary.

Load model

Model **L1** is favored over model **L2** as the use for retail pricing as presented in Chapter 7 is not possible with model **L2**. Furthermore, expectations about future capacities of renewables can only be included in model **L1**. Generation by renewables is the major driver of the residual load in the years to come.

Choosing model **L1** gives the chance to achieve some further improvements with more sophisticated models for generation by renewables. The historical simulation used so far is a simple approach that needs to be replaced by more reliable models.

6.3. Out-of-sample model validation

In the discussion above we validated the models with respect to required features and input variables. Furthermore, the comparison with two fundamental spot price models from the literature gives evidence for the quality of the model. A preference for either of the models **E1** and **E2** is not justified so far. The differences between these models are small. The increased R^2 of model **E2** is achieved by an increased number of parameters. Both models give reliable results so that both alternatives might be used.

A decision with stronger impact is the choice of the commodity price model. The models for the non-stationary commodity prices oil, coal and emission allowances are pure stochastic. Therefore, we cannot measure the in-sample goodness-of-fit. Comparison of mean (obtained from the prices of recent future prices) and variance (increasing with time) with historical prices is not meaningful.

The most relevant issue concerning commodity prices is the issue of dependencies. The dependencies on electricity as well as the dependencies among themselves need to be covered by the model. The relationship between electricity and commodity prices is determined by the choice of the polynomial approximation. Thus, the dependencies among the commodities themselves determined by the commodity price model need to be analyzed. We examine this issue by comparing the simulated **clean dark spread** to the historical clean dark spread. The spread is defined as

$$\text{CDS}_t = S_t - \frac{a \cdot C_t}{\eta} - \frac{b \cdot E_t}{\eta}.$$

a is needed for the conversion from tons to MWh depending on the quality of coal in use. We decide for an average value of $a = 1/7$. The same argumentation leads to the factor b describing the emissions intensity per MWh of generated electricity. For this calculation we set $b = 0.33$. The efficiency η describes the percentage of energy content of coal being transformed into electricity. This percentage strongly depends on the power plant being used. We assume an average of $\eta = 0.35$. The exact choice of these three factors depends on the power plant being valuated. For comparison of historical and simulated spreads any choice is possible. Nevertheless, these average values give realistic spreads.

For an out-of-sample comparison we simulate the clean dark spread on monthly prices, i.e. the average of simulated hourly or daily prices is taken as the monthly price. In this example the clean dark spread for January 2013 is calculated. The historical data used for model calibration ends in September 2012. Based on this data the simulated month is the fourth front month. Therefore, the clean dark spread of historical prices of the fourth front month is used for the comparison. As these prices are not used within the model calibration it is an out-of-sample validation of the model.

The clean dark spreads in Figure 6.1 are centered at the mean of the historical clean dark spread. These results are obtained by using the commodity price models **F1**-**F3** in combination with the spot price model **E1**. Model **E2** gives similar results.

Model	5%	95%	Range
Random Walk	-4.5	17.7	22.2
Two factor	-5.7	18.3	24.0
Cointegration	-2.7	16.1	18.8
Historical	-1.3	15.4	16.7

Table 6.2.: Quantiles of simulated and historical clean dark spreads.

According to the visualization in Figure 6.1 the model **F3** is the best approximation of the historical spread. This is supported by the quantiles given in Table 6.2. The range between 0.05 and 0.95 quantile reveals the close fit of the cointegrated model to the historical range. The differences to the other models are remarkable. Though a close fit to the historical spreads is requested the simulations are supposed to exceed the historical range of the spread. Both is guaranteed by the cointegrated commodity price model. The other models exceed the historical range too far.
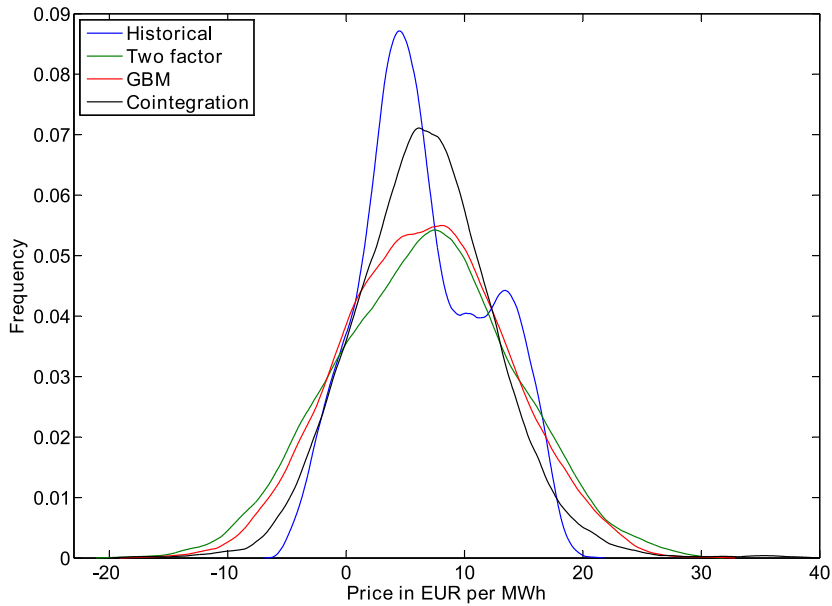
Figure 6.1.: 1000 simulations of the clean dark spreads for January 2013 resulting from model **F1** (red), **F2** (green) and **F3** (black) compared to the historical clean dark spread of the fourth front month (blue).

Electricity price scenarios

According to the above argumentation it is advisable to use any of the merit order curve approximations **E1** or **E2**, the grid load model with distinct models for the renewable generation **L1** and the cointegrated commodity price model **F3**. This combination of model components leads to the widest range of possible applications. One model for "all" valuations leads to consistent results which is of particular interest for risk management issues. For this purpose the inclusion of the oil price in model **E1** or **E2** is advisable.

In Figure 6.2 realizations of our preferred model combination are presented. Especially, the scenarios for one week in June reveal a wide range of prices due to the strong influence of renewables. The lower price level on weekends can be observed as well.

The scenarios do not reveal a structure as obvious as suggested by the consideration of an HPFC (compare Section 7.2.1). The expectations given by an HPFC are still valid but due to stochastic influences most realizations of the

Figure 6.2.: Simulated spot prices of electricity for 2013 (top). Four scenarios for one week in June 2013 (bottom). All scenarios are shifted to a price forward curve based on future contracts from 2012/09/28.

model will not reveal these structures. Grid load, generation of renewables, commodity prices and the short term process of the spot price cause stochastic price behavior.

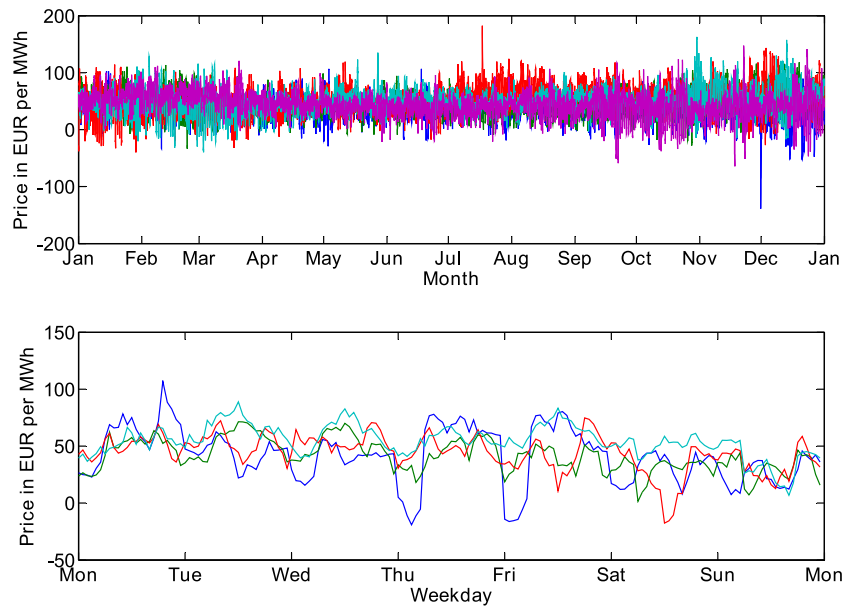# 7. Pricing of retail power contracts

In the following we give a detailed description of risk factors in retail power contracts and their pricing. It is published in Burger and Müller (2011). Section 7.3 is adapted for the spot price model introduced in Chapter 3. The example gives evidence for the new spot price model being applicable to retail pricing issues.

7.1. Introduction

Electricity retail markets have changed dramatically since the establishment of wholesale markets. Before the markets were liberalized, retail prices were cost-based and were reviewed by a regulator. Since the introduction of the first wholesale electricity markets, retail prices have been deduced from published wholesale market prices.

If electricity is sold to an end customer and the sold volume depends on the consumption of the customer, we refer to this market as a retail market. This includes business-to-business and business-to-consumer sales. In this work, for retail contracts, the volume risk and all associated costs (such as balancing power) are borne by the supplier. In practice, some end customers have contracts that are derived from wholesale contracts. A modular computation of risk premiums allows the application of the following pricing concepts for these customers. There are two common types of contracts on the retail market: fixed-price contracts and indexed contracts. The supply price for fixed-price contracts is known when the supply contract is concluded. Indexed contracts contain a specified number of fixing possibilities where the customer has the chance to diversify the price by fixing the price over several dates. Fixed-price contracts can be regarded as a special case of an indexed contract with only one fixing possibility.

7. Pricing of retail power contracts

We consider full-service contracts on the German market with a contract horizon of one year unless otherwise specified. In contrast with the retail market, the wholesale market includes standardized over-the-counter contracts or takes place at an energy exchange such as the EEX or Nord Pool in Scandinavia.

Retail electricity contracts differ from standardized wholesale products in several respects. The most significant difference is the uncertain volume of the contracts. While the volume of a forward or future contract is fixed, the volume of a retail contract depends on the consumption of the customer. This uncertainty is not an option in the usual sense of financial products because it is not exercised in a market-oriented fashion. While the subject of the pricing of electricity on wholesale markets has been widely researched (see, for example, Schwartz (1997), Eydeland and Wolyniec (2003), Geman (2005), Weron (2006), Burger et al. (2007)), the pricing of electricity retail products is less commonly considered. Despite this, the pricing of retail electricity is essential for suppliers, and calculating the risks is not a straightforward task. Difficulties encountered with the pricing of electricity retail contracts include the lack of publicly available data and the necessity of a consolidated knowledge of operational market details.

In the literature there are only a few works that name the risk factors involved in retail contracts: Although, for example, Boroumand and Zachmann (2012), Lemming (2003), Ojanen (2002) and Unger (2002) describe the sources of risk, they do not address the issue of pricing the main risk factors, instead using a risk management-based approach. Comprehensive approaches for retail contract pricing including risk premiums have been proposed by several authors. For example, Keppo and Räsänen (1999) use stochastic processes for consumer consumption and energy price. Combining these processes, a future value of a customer's consumption pattern is derived and used to price specific call options. These call options lead to a value for the electricity tariff of an end customer. In contrast to this option-based approach, Dhaliwal et al. (2001) introduce a customer preference index to indicate a range of acceptable retail prices. The range of market prices is covered by a fuzzy set approach. These two components together lead to a retail price customers will be likely to accept.

A so-called breakeven price is calculated by Eakin and Faruqui (2000): The breakeven price is specified as the price that does not lead to an expected profit or loss for the supplier. As log-normal distributions are assumed for price and customer load and a correlation between these processes is used, the breakeven price contains the risks of a retail contract. Karandikar et al. (2007) and Prokopczuk et al. (2007) present some retail pricing methods using the risk-adjusted return on capital (RAROC) and capital asset pricing model

(CAPM) approaches.

A major issue for retailers is the minimization of procurement costs, i.e., the reduction of settlement risk. Buying electricity with fixed quantities on the wholesale market and selling flexible quantities to customers exposes retailers to the risk of increased costs due to unfavorable procurement or changing market prices. Optimization approaches for the determination of optimal procurement quantities, the type of contracts to be used in procurement and strategies for adjustments within the run of time are presented by various authors. Karandikar et al. (2010), Deng et al. (2005), Bunn et al. (2010), Bunn et al. (2011), Horowitz et al. (2004) and Deng and Oren (2006) all differ in the hedging instruments that they consider.

Similar ideas are presented by Arroyo et al. (2007), Balakrishnan et al. (2006) and Hatami et al. (2009). In these papers the reduction of procurement costs and settlement risk is treated in one optimization together with the determination of a retail price on the customer side.

One common aspect of major energy utilities having generation assets and retail customers is not reflected in these approaches. Usually the spread between fuel costs and electricity price is logged to reduce the risk on the generation side to a minimum. On the retail market these utilities would act like any other retailer: energy is purchased on the futures market and sold to customers. Therefore, the price for retail customers must not be determined in a combined optimization of procurement and customer side.

We give a complete overview about the different types of risks in retail contracts. For most types of risk a possible method of determining an adequate premium is presented. We focus on the uncertainty of the contract volume and use combined stochastic models for load and price. These models are based on the models described in Chapter 3.

After this introduction, we explain the decomposition of the total price into several price components in Section 7.2. This includes the analysis of the various sources of risk and choice of adequate risk measures. The extension of our spot price models to combined models for price and load is explained in Section 7.3. An application to customer data is used for comparison. The explanations focus on full-service contracts but the methodology for other contract types can be derived.

7.2. Price components

We decompose the retail prices into three components:

$$P(t) = F(t) + R(t) + M$$

which reflect the hourly energy price $F(t)$ deduced from the price on the wholesale market, a possibly time-dependent risk premium $R(t)$ and a sales margin M. The latter component is the result of a management decision of the supplier and is not the subject of further analysis in this work. Under a mathematical point of view, the other components contain a greater potential for further research. In what follows, the deduction of an hourly price forward curve (HPFC) from the wholesale market prices (see Section 7.2.1), the sources of risk within retail contracts (see Section 7.2.2) and some risk measures for the calculation of risk premiums (see Section 7.2.3) are presented.

7.2.1. Hourly Price Forward Curve

A supplier entering the retail market needs to calculate a fair price of energy in order to compete with other market participants. The fair price of energy, as the basic price component, covers expected costs of future delivery of energy on a specified time granularity. In this work this (finest) granularity is one hour caused by the fact that this is the finest granularity of spot products in many countries. The statements can be adapted to any other market situation. Forward and future contracts often have a large delivery period of a year, a quarter or a month. So, a method for calculating a forward curve on an hourly basis needs to be found. Some approaches for the calculation of an HPFC are proposed by Fleten and Lemming (2003), Koekebakker and Ollmar (2005) and Burger et al. (2007).

An example of an HPFC for one year is given in Figure 7.1. It is the result of a simple regression approach based on historical spot prices. This implies the assumption that forward contract prices are expected spot prices. This example reveals a typical characteristic of forward contract prices: These prices give a step function whose steps are transferred to the HPFC. Although some of these steps will realize, it is a more realistic assumption to expect smoother transitions between intervals of the forward contract prices. Therefore, a smoothing method, as proposed by Benth et al. (2007), might be included in the calculation of the HPFC.

After calculating one price for each hour of a year, the basic price for the
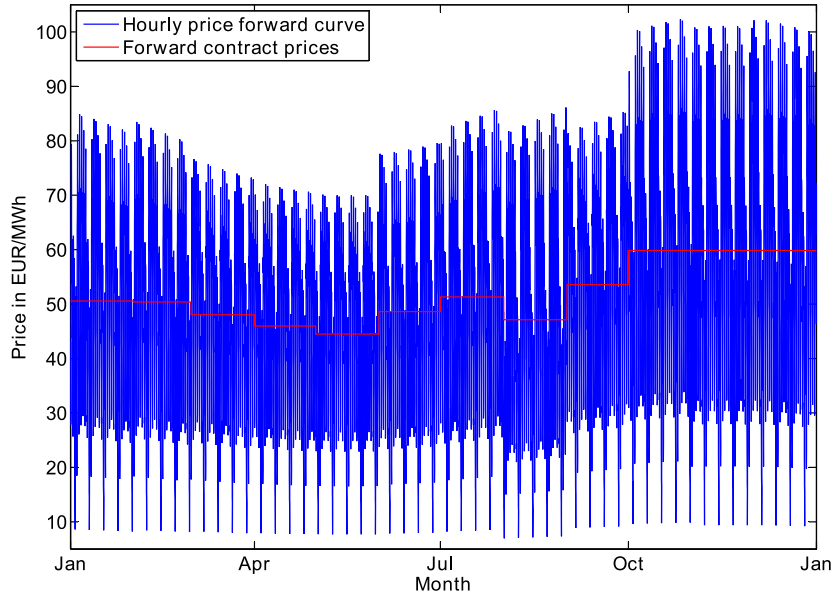
Figure 7.1.: Example HPFC for 2010 deduced from monthly and quarterly forward contract prices from the EEX. A regression model is used for the calculation of hourly weights from historical spot prices. Holidays are neglected within this example.

customer is deduced from the HPFC. Usually, in practice, a customer is charged either one fixed price or one peak and one off-peak price. These peak and off-peak times might differ from definitions at an exchange. The decision is left to the supplier. These prices result from averaging the HPFC over the corresponding hours:

$$\text{basic price} = \frac{\sum_{t=t_1}^{t_2} \widehat{L}_t F(t)}{\sum_{t=t_1}^{t_2} \widehat{L}_t} \tag{7.1}$$

with the load forecast $\widehat{L}_t$ for $t \in [t_1, t_2]$, which represents the interval of delivery.

7.2.2. Risk factors

In addition to the costs of delivering energy to the customer, a retail power contract contains various sources of further costs to the supplier. Some of these

factors cause losses on average, while others increase the probability of losses, but do not contribute to the losses on average. In this section we describe the most important risk factors with respect to full-service retail contracts.

Price validity period

The process of concluding a retail power contract starts with a request for a quotation by the customer. The supplier calculates the price quotation, or, more precisely the basic price, on the basis of the latest forward contract prices. As soon as this price quotation is submitted, the customer can choose to either accept or refuse the offer within a certain period of time (the so-called **price validity period**). The length of the price validity period ranges from a few hours to several days, or, in some cases, even some weeks, depending on the supplier and the customer. If the forward contract prices are increasing, the customer concludes the contract based on lower forward prices. If, however, forward prices decrease, the customer demands a new offer based on the actual lower prices. In a financial sense, the customer is the holder of a call option on the market price. Since it is unusual to charge an upfront premium for a price quotation, this option is free of charge for the customer. Including an option premium in the retail price only means increasing the strike price of the call option, but this is the only possibility for the supplier, who has to bear the resulting costs. In theory, as the writer of the option, the supplier has to receive a premium for giving this optionality. This premium can be determined using an option-pricing approach (Black-Scholes) or the method described by Bartelj et al. (2010), although not all customers take advantage of this optionality. In some cases the decision regarding execution of the option is determined by duration of decision processes rather than by market prices.

Credit Risk

As soon as a retail contract is signed, the supplier begins sourcing energy based on a load forecast for the customer. Purchasing the forward contracts traded on the wholesale market for hedging purposes reduces the price risk for the supplier. This procedure occupies the risk of the counterpart, i.e., the customer, defaulting before payment of delivered energy. In case of a defaulting customer, the utility will realize a loss of the delivered but unpaid energy. The energy that has not been already delivered can be sold by the supplier again, but it may not be able to obtain its purchase price. To insure against this **credit risk**, the supplier will charge the customer a premium for bearing this

risk. The most important components for evaluation of credit risk are the **exposure at default** (EAD) and the **probability of default** (PD). The EAD is the mark-to-market of the energy amount sold to the customer. The determination of the PD of a customer is a more complex task for the supplier. In case of large companies, rating agencies analyze their business activities and classify the companies. This allows a PD to be calculated. As suppliers have a portfolio of retail customers, including smaller companies without a rating from any of the agencies, an internal rating for the estimation of PD is required. When setting up an internal rating system, calibration of the internal system on published ratings is essential. This means that the internal system is calibrated so that external ratings are reproduced as far as possible, if they are available. The expected losses due to credit risk within a retail contract can be quantified using EAD and PD.

Price-volume correlation

If the spot price is higher than the normal price level, this is usually the consequence of the grid load being higher than normal. As the grid load is the sum of all customer loads, there is a high probability of individual customer loads being above the normal level. Customers are therefore likely to deviate from the supplier's forecast in the same direction as the spot price. There is a **price-volume correlation** with the following price effect. As soon as a customer asks for the price quotation it submits data on its past consumption. This data is used to calculate a load forecast for the delivery period. If the quotation is accepted, the supplier will begin procurement based on this forecast. Usually the realized load will deviate from the forecast. If the demand exceeds the previous forecast, the hedged quantity is not sufficient to cover the demand, and further quantities need to be purchased on the spot market. Due to the described price-volume correlation, high spot prices are more probable in this case. This leads to increased sourcing costs for the supplier. Conversely, surplus quantities resulting from a new, lower forecast have to be placed on the market. These quantities are likely to be sold at low prices due to the price-volume correlation. An approach using stochastic dynamic programming to determine an optimal load forecast is given by Balakrishnan et al. (2004).

Individual volume risk

In addition to these correlated deviations from forecasts, changes in customer electricity demand might occur for a variety of reasons, such as the extension

of the customer's machine park, or an increase or decrease in the customer's production. These incidents can be reduced to individual customers and do not influence the behavior of all customers. Therefore, this so-called **individual volume risk** is not systematically correlated with the price.

Hourly price profile

As described in Section 7.2.1, the basic price is based on the HPFC, which is an estimation of the future hourly price profile. Within the time to delivery, the hourly price profile might change. Since market liquidity for hourly profiles is low, hedging this risk is often not possible. Therefore, the **hourly price profile risk** is another component of the total risk. On average, the HPFC should match the realized price profile, so that the risk does not lead to any costs. As soon as deviations from the HPFC are realized, additional costs occur due to the unhedged quantities. Models for the calculation of a risk premium on hourly price profile risk, individual volume risk and price-volume correlation is introduced in Section 7.3.

Costs of balancing power

While the demand of any customer differs from the final load forecast, the transmission system operator has to balance these fluctuations and charge the supplier for costs of balancing power. Burger et al. (2007) present an approach for the determination of **balancing power costs** from historical load and load forecast data. A model based on combined seasonal autoregressive integrated moving average (SARIMA) and Markov processes for balancing power market prices is given by Olsson and Söder (2008). An example of German imbalance energy prices on a quarter-hourly basis is given in Figure 7.2.

Operational risk

Across the whole process – from giving a price quotation to the delivery of energy – there are many possibilities for further failure that are aggregated as **operational risk**. The duration of the sourcing process and the quality of load forecast are only two possible sources for negative influences on the total result. Due to the variety of sources, there is no comprehensive approach for quantification of operational risk in retail contracts.
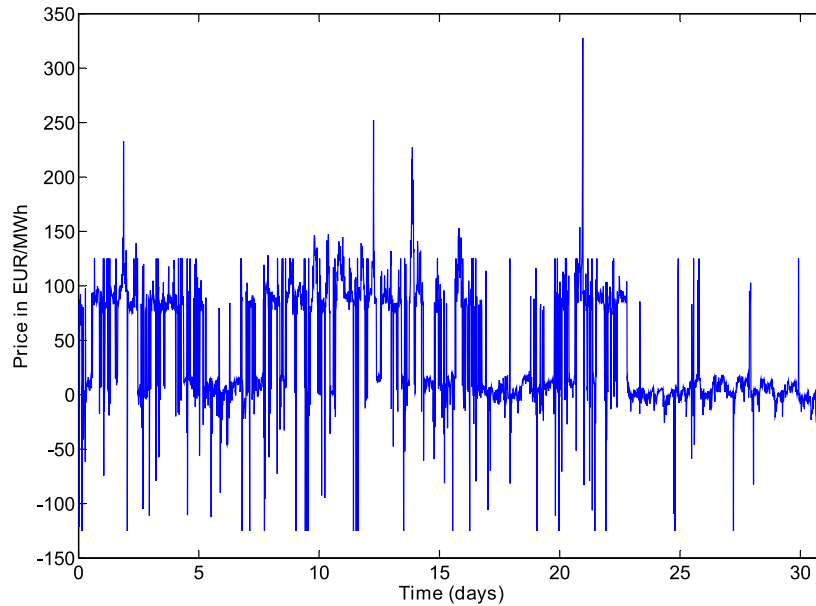
Figure 7.2.: German imbalance energy prices in July 2010.

The risk factors in this overview can be classified into two groups: systematic and unsystematic risk factors. While unsystematic risk factors have a zero mean, systematic risk factors cause additional costs on average. Both groups contribute to the total risk as there are fluctuations around the mean value. Therefore, customers are not only charged for the expected costs but also for an additional risk premium. This premium covers the possibility of losses due to the underlying risk factors. The classification of risk factors, as shown in Table

Systematic risk	Unsystematic risk
Price validity period	Hourly price profile risk
Credit risk	Individual volume risk
Price-volume correlation	
Balancing power	
Operational risk	

Table 7.1.: Classification of risk factors in retail contracts as systematic or unsystematic risk factors.

7.1 is fairly theoretical, since a strict differentiation within the calculation of a risk premium is not always applicable.

While the above-mentioned risk factors are typical for common full-service

contracts, there are further types of contracts that reduce some risks. Indexed contracts, for example, contain a specified number of fixing possibilities, i.e., the customer has the chance to diversify the price by fixing the price over several dates. There is a risk reduction for the customer, who does not fix the whole quantity at the wrong moment. Nevertheless, common full-service contracts with fixed prices are still widely used and need to be priced with respect to the risk. An overview of risk measures for quantification of risk in electricity retail markets is given in the following section.

7.2.3. Risk-adequate return on capital

While charging the customer for the basic energy price and the expected costs resulting from the above-mentioned risk factors usually secures a supplier's earnings on average, this is not the case in every scenario, nor even in the majority of them. Therefore, the price of a retail contract has to contain a risk component as a fair premium for taking on the risk, just as participants in a financial market ask for a premium to take a risk. In order to cover the risk of further costs, an adequate risk measure is needed. The concept of charging customers for a risk premium is comparable with insurance companies charging insurance premiums. A wide range of concepts for calculating insurance premiums can be found in Goovaerts et al. (1984). We now present the concept of **RAROC** as a method for achieving a risk premium. An application of the RAROC approach to electricity markets is given by Prokopczuk et al. (2007). A more general discussion can be found in Hallerbach (2004) and Stoughton and Zechner (2007).

The central idea is that (equity) capital has to be allocated for risks. Most companies have clear concepts for the return on their (equity) capital. As (equity) capital is a scarce resource, they only enter contracts if a certain return on the allocated capital, the so-called **hurdle rate**, μ, is achieved. The hurdle rate is the financial objective of each project within a company and may be influenced by different factors, such as the costs of allocated capital in case of funding by loans, or the expected return of the shareholders. These are only two possible approaches for the determination of a hurdle rate among others, some of which might theoretically be more advanced. As the focus is on the calculation of risk premiums, the hurdle rate is assumed to be given. Prokopczuk et al. (2007) give examples of approaches for determining the hurdle rate. The RAROC approach describes the relation between profitability and riskiness of a contract:

$$\text{RAROC} = \frac{\text{expected return}}{\text{economic capital}} \overset{!}{\geq} \mu.$$

The economic capital, as the capital that needs to be allocated to cover risk even in worst-case scenarios, is determined by a risk measure based on the distribution of costs of a retail contract. The aggregated economic capital of all contracts is supposed to ensure a company's survival in worst-case scenarios. Given a certain hurdle rate, the expected return can be determined by the RAROC approach via

$$\text{expected return} \geq \text{economic capital} \cdot \mu.$$

A common risk measure for determining the economic capital is the value-at-risk (VaR) at a given level of confidence α. Jorion (2006) gives a detailed discussion of VaR. Using an adequate risk measure in combination with the RAROC approach leads to an adequate risk premium. After that, the price can be composed of the basic energy price, the risk premium and, finally, the sales margin. This theoretical price might differ from the price charged on the retail market due to competition between suppliers.

7.3. Correlated price-load models

As mentioned in Section 7.2.2, the pricing of price-load correlation, individual volume risk and hourly price profile risk can be carried out simultaneously using one model. Stochastic price and load scenarios with the correct correlation need to be generated by the model in order to represent the fluctuations in both components. In the literature, some spot price models using grid load as a fundamental driver have been proposed (see, for example, Coulon and Howison (2009), Jermakyan and Pirrong (2008) and Aïd et al. (2009)). These models generate stochastic price and grid load scenarios but methods for the deduction of customer load scenarios are not covered. We propose such a method to extend the spot price model presented in Chapter 3 for retail pricing purposes.

7.3.1. A customer load model

The spot price as modeled in this work contains the residual load as a fundamental driver. As the grid load is the basis for the residual load the dependency of spot price and grid load is covered by the proposed model. In order to price customer load fluctuations we need to incorporate a customer load model. On the one hand, the customer load is supposed to be correlated to the grid load. On the other hand, uncorrelated deviations from customer load forecast and

the grid load need to be incorporated. Therefore, we propose to model the customer load in analogy to the grid load by a load forecast and a SARIMA-process for short-term deviations (compare Section 3.4.1). In addition to these components the grid load as a regressor introduces the correlation to the grid load into the model:

$$C_t = \exp\left\{aL_t + \widehat{C}_t + X_t^{(C)}\right\}, \quad a \in \mathbb{R}. \tag{7.2}$$

In analogy to the model in Equation (3.8) the short term process $\left(X_t^{(C)}\right)$ is chosen to be a SARIMA-process with a seasonality of 24 hours and possibly non-normally distributed innovations. The load forecast consists of a system of similar-days to capture all seasonalities within the customer load. The model is chosen to be multiplicative due to the condition of positive load. Especially when pricing customers with a comparably low load level negative load scenarios are generated by an additive model. This model is a general approach in order to cover all different types of customers. Therefore, order and parameters of the SARIMA-process as well as the distribution of the innovations strongly depend on the customer under consideration. We analyze three customers representing different classes of customers:

1. A municipal utility delivering households and industrial companies in a local area.

2. An industrial company 'A' running its business without any structural changes.

3. A cyclical industrial company 'B' being sensitive to the situation of the economy, i.e. being affected by the crisis in 2009.

Load profiles of these customers for 2008 and 2009 can be found in Figure 7.3. For comparison these customers are scaled to an average yearly consumption of 100 GWh. The daily, weekly and yearly patterns of all customers are covered by the deterministic component of the model, $\widehat{C}_t$. Together with the grid load as a regressor a major part of the customer load can be explained (compare Table 7.2). The intention of including the grid load in the model is the description of correlations between customer load, grid load and price. As in Section 7.2.2 stated, all customers are likely to deviate from the load forecast in the same direction as the price in the same time. Empirical evidence for this correlation causing a risk for retailers is given in Table 7.3. The importance of the grid load as a regressor reveals the fact that e.g. the goodness-of-fit of the model for company B decreases to 0.71 if the grid load is removed from the model. This is due to B's strong dependency on the economy. The economic situation is reflected in the consumption of energy. Therefore the grid load
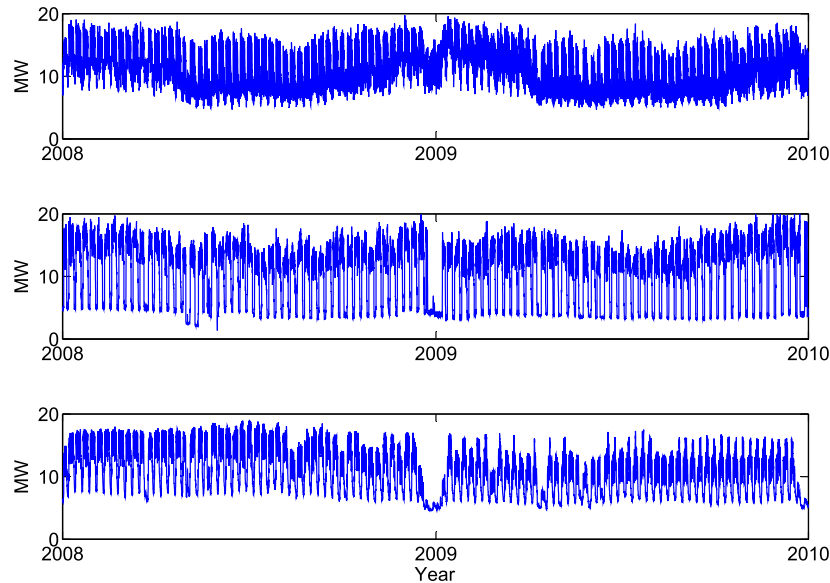
Figure 7.3.: Exemplary load profiles of three customers in the years 2008 till 2009. Top: Municipal utility. Middle: Industrial company A. Bottom: Industrial company B underlying economical changes.

can explain the decrease in B's load in 2009 and therefore improve the forecast quality.

After removing the seasonalities and economical influences by the regression a stochastic process for the residual time series needs to be fitted. As shown in Figure 7.4 the residual time series contain autocorrelations with lags of some hours and some days. These correlations can be covered by a SARIMA-process. The innovations of these SARIMA-processes are not normally distributed (see Figure 7.5). In all three cases a Student's t-distribution with additional scaling parameter provides a better fit to the empirical data. Empirical work on a wide range of customers has proven that in case of 'normal' customers without major

Customer	R^2
Municipal utility	0.90
Industrial company A	0.87
Industrial company B	0.86

Table 7.2.: Goodness-of-fit of the customer load model applied to three customers.

Customer	Grid load	Spot price
Municipal utility	0.86	0.61
Industrial company A	0.65	0.31
Industrial company B	0.72	0.45

Table 7.3.: Correlations of three customers to grid load and spot price.

changes in demand structure the forecast quality is high. As a consequence, the stochastic deviations from this forecast are rather small and, more important, stationary so that the use of a SARIMA-process is justified. Nevertheless, there are customers increasing the machine park, running bad in their business or changing structure of their demand for other reasons. These customers will not satisfy the condition of stationarity as the forecast quality will be low. Therefore, this model should not be used in these cases. Under the aspect of risk management, these customers would add a high risk to the retailer's portfolio: Structural changes in future times have to be expected but cannot be forecasted and hedged. Therefore, a higher risk premium is required.

There is no long term process included in the customer load model due to the lack of customer data on a long term horizon. Long term fluctuations for economic reasons are indirectly incorporated via the grid load component which contains a long term component (see Section 3.4.1). In the following we present an example using the customer load model and the spot price model introduced in Chapter 3.

7.3.2. Example

Having introduced the model in detail we want to give a practical example of the model for risk-adequate pricing using the RAROC-approach. Given an individual customer - see Figure 7.6 for the virtual load and corresponding scenarios - the model is used for calculation of a risk premium for the hourly price profile risk, individual volume risk and price-volume correlation.

Following our decomposition of the price of a retail power contract in Section 7.2 we need a basic price as a first step. In this example we derive a basic price of $B = 50.14$ EUR/MWh from the HPFC for the year 2013 (compare Figure 7.1) by means of Equation (7.1). In addition to this price the customer is charged for the risk premium. Within the calculation of the risk premium using the RAROC-approach the distribution of costs is needed. We obtain this (empirical) distribution (see Figure 7.7) as a result of a Monte Carlo simulation using scenarios from the extended SMaPS model described above.
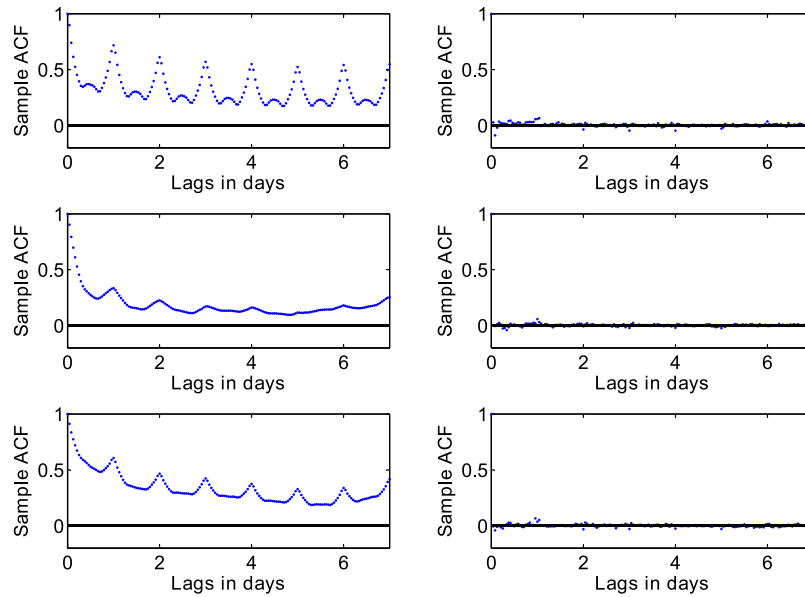
Figure 7.4.: Autocorrelation function for the first 7 days of residual time series (left) and innovations of the SARIMA-process (right) of a municipal utility (top), a normal industrial company (middle) and a cyclical industrial company (bottom).

Apart from the expected costs, M, the RAROC-approach is used for determination of a return on the allocated capital for the risks within the retail contract. The risk beyond the mean is measured by the 99%-quantile of the cost distribution and the risk premium consists of a return on the difference between quantile and mean:

$$
\begin{aligned}
\text{Total price} \ &= \ \text{Expected costs} + \text{Hurdle rate} \cdot \text{Risk} \\
&= \ M + \mu \cdot (99\%\text{-quantile} - M) \\
&= \ 50.19 + 0.15 \cdot (51.83 - 50.19) \approx 50.44
\end{aligned}
$$

where $\mu = 0.15$ is the assumed hurdle rate. This means the customer within this example is charged 50.44 EUR/MWh plus an additional sales margin and risk premiums for further risk factors. Compared to the basic price of 50.14 EUR/MWh a risk premium of 0.30 EUR/MWh is included in the total price. The mean of the distribution of costs is higher than the basic price due to expected costs from various risk factors. This result is similar to the one presented in Burger and Müller (2011) where the SMaPS model is used for the calculation.
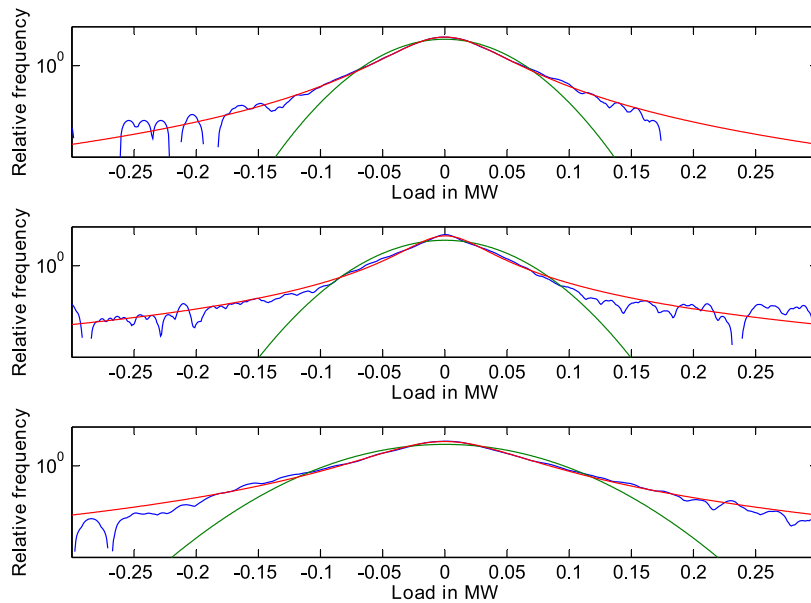
Figure 7.5.: Fit of Student's t-distribution (red) and normal distribution (green) to kernel density of empirical innovations (blue) for municipal utility (top), industrial company 'A' (middle) and industrial company 'B' (bottom) on a logarithmic y-axis.

7.4. Conclusion

From a supplier's point of view, having the ability to compete on the retail power market requires the determination of an adequate price for the delivery of energy within common full-service contracts. Since these contracts include a high degree of flexibility for customers, the supplier faces two challenges:

1. calculation of a fair price of energy;

2. determination of a premium for the flexibility.

The fair price of energy is deduced from forward contract prices on the wholesale market by breaking down these prices into an hourly granularity. This step includes estimation of the seasonal structure within the forward contract intervals. The estimation of price behavior on a holiday day (e.g. Christmas Eve) is a particularly difficult challenge for the supplier.
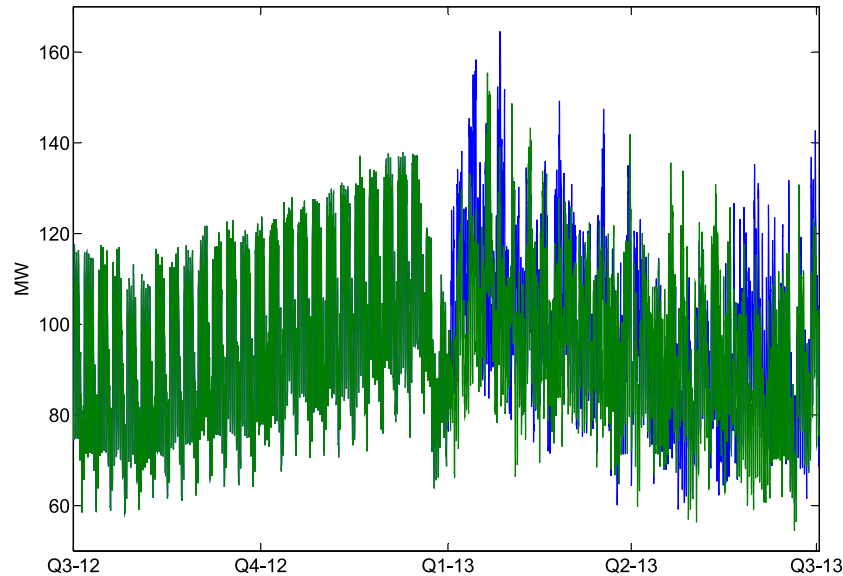
Figure 7.6.: A virtual customer load, 1 July 2012 - 31 December 2012, and two corresponding scenarios, 1 January 2013 - 30 June 2013.

On top of these average costs of delivering energy, the customer is charged for a risk premium. This premium is the price of the flexibility, which poses a risk for the supplier. The total risk can be divided into risk factors related to fluctuations of price and load (such as price validity period, price-volume correlation, individual volume risk, hourly price profile risk and costs of balancing power) and risks that are not specific for trading on electricity markets (such as credit risk and operational risk). These risk factors cause expected costs as well as an increased probability of further costs that threaten the supplier's solvency.

Due to the diverse nature of the risk factors, there is no global approach that covers all risk factors. Therefore, individual approaches have to be applied to the risk factors in combination with individual risk measures. We have proposed a correlated price-load model as an extension of the electricity price model introduced in Chapter 3 to price the hourly price profile risk, price-volume correlation and individual volume risk simultaneously. These risk factors have the highest potential for the generation of unexpected costs. Using the load dependent spot price model in combination with the deduction of the customer load from the grid load by a regression model, a cost distribution due to these risk factors can be simulated. The application of an adequate
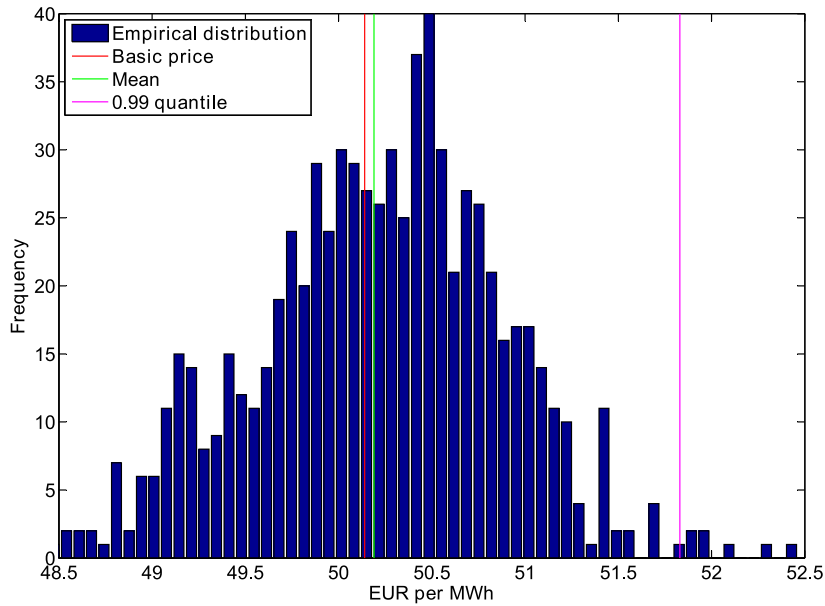
Figure 7.7.: The empirical distribution of costs based on 750 correlated price and load scenarios for year 2013. The basic price, the simulated mean price and the 99%-quantile are the basis for the risk premium.

risk measure to this distribution leads to the economic capital, which is used in the RAROC-approach.

The result of calculating an hourly price forward curve and risk premiums for all mentioned risk factors is a risk-adequate price of a retail power contract. This price, or this price plus a sales margin, is not necessarily the price on the retail market. For competitive reasons, deviations from this theoretical price might be necessary. Nevertheless, risk-adequate pricing is essential for the supplier in order to gain knowledge about the risks within the retail contracts and, therefore, the retail portfolio.

8. Conclusion

In this thesis we introduce a new model for the spot price of electricity. In this model, residual load and prices of coal, gas and emission allowances as major drivers of the electricity price are incorporated. This leads to a model exhibiting the same stylized facts as historical spot prices: seasonalities, spikes, mean reversion and negative prices. Moreover, the inclusion of load and commodity prices makes the model applicable to many valuation issues: Electricity derivatives, power plants and retail contracts. To our knowledge it is the first model including all these factors and, thus, providing a wide range of applications.

Stochastic models for the underlying variables are presented as well. The residual load model consists of distinct models for grid load and generation of renewables. As the grid load induces seasonalities to the electricity price these seasonal variations are covered by the model. Furthermore, economic load fluctuations, e.g. due to an economic crisis, are considered. Renewables are modeled by means of a generation forecast resulting from expectations about future installed capacities and a stochastic process to cover weather influences.

For the gas price we introduce a spot price model relying on temperature and oil price as fundamental factors. The temperature is used to derive a component approximating the influence of the filling level of all gas storages in the market. In addition to this factor describing the supply side there is the oil price component reflecting the state of the world economy and influencing the demand for gas.

Finally, the prices of coal, oil and emission allowances are either modeled by a three-dimensional random walk or by a correlated two factor model. Both approaches consider correlations between those commodities and result in correlated non-stationary simulations. As an alternative we introduce a cointegrated model for prices of gas, coal, oil and emission allowances. This model describes stronger relationships than correlation models for the commodities. The stationary linear combination of all commodities identified in the cointegrated model suggests the existence of a common trend moving these commodity prices together.

8. Conclusion

The modeling framework summarized above is capable of generating realistic simulation results. The application to pricing of a retail contract gives evidence for this. Adequate prices for a full-service contract with its various sources of risk can be derived. Especially, the inclusion of load as an input factor makes the model applicable to such issues.

A major topic for further research is the inclusion of capacity changes in such a model. The strong increase of installed capacities of renewables as well as changes of the conventional power generation mix will lead to further changes of price behavior. Thus, these effects need to be considered.

The cointegrated model for commodity prices as our preferred model might be extended as well. So far, only one factor covering short term fluctuations is considered. By means of a second factor describing cointegrated long term commodity price behavior the model can be improved. As a consequence a better fit to the expected volatility structure on the market could be achieved.

Altogether, the models introduced in this work provide a good fit to historical data and, thus, reproduce all relevant features. The resulting simulations are reliable and can be used for a wide range of applications. With the possible extensions mentioned above, the model framework is applicable in future as well.

A. Mathematical appendix

Some of the basic concepts used in the thesis are provided in this chapter. It is not a comprehensive overview of all methods but gives some information on the most relevant issues including indications to further literature.

A.1. Stochastic processes

Throughout the thesis we set up time series models in discrete time. All models contain at least one stochastic process. Definitions and characteristics of these processes are given in this section. The explanations follow Brockwell and Davis (1987).

A.1.1. Definitions and properties

Definition A.1 (Stochastic process). *A family of random variables $\{X_t, t \in \mathbb{T}\}$ on a probability space $(\Omega, \mathcal{F}, P)$ is called a **stochastic process**. The function $t \mapsto X_t(\omega)$ for $\omega \in \Omega$ is called **realization** of the stochastic process. Further notations are path and scenario.*

The stochastic processes are observable at any point of time in an index set $\mathbb{T}$. Processes with a countable index set are called **discrete processes**. If the process takes values in $\mathbb{R}$ it is denoted as real-valued. As price time series consist of real-valued prices at certain points in time, we restrict our considerations to discrete, real-valued stochastic processes. The indication of the index set is omitted as it is usually $\mathbb{N}$ or $\mathbb{Z}$. The short notations (X_t) for the stochastic process $\{X_t, t \in \mathbb{T}\}$ and (x_t) for a realization of the process are used within this work.

The following definition extends some well-known features of random variables to stochastic processes.

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Definition A.2. *Let (X_t) be a stochastic process.*

(i)
$$\mu_t := E(X_t)$$
*is the **mean** of (X_t).*

(ii)
$$\sigma_t^2 := E\left[(X_t - \mu_t)^2\right] = E(X_t^2) - \mu_t^2$$
*is the **variance** of (X_t).*

(iii)
$$\gamma(r,s) := E\left[(X_r - \mu_r)(X_s - \mu_s)\right] = E(X_r X_s) - \mu_r \mu_s$$
*is the **autocovariance** of (X_t).*

(iv)
$$\rho(r,s) := \frac{\gamma(r,s)}{\sigma_r \sigma_s}$$
*is the **autocorrelation** of (X_t).*

These characteristics are necessary for the definition of stationarity.

Definition A.3 (Stationarity)**.** *A stochastic process (X_t) is called **stationary**, if*

(i) $E|X_t|^2 < \infty$ *for all t,*

(ii) $\mu_t = m$ *for all t and some $m \in \mathbb{R}$ and*

(iii) $\gamma(r,s) = \gamma(r+t, s+t)$ *for all t*

hold.

Despite different notations in the literature (weak stationarity, covariance stationarity) we refer to this definition as stationarity throughout the thesis.

Definition A.4 (White noise process)**.** *A stochastic process (ε_t) with*

(i) $E(\varepsilon_t) = 0$,

(ii) $Var(\varepsilon_t) = E(\varepsilon_t^2) = \sigma^2$ *and*

(iii) $E(\varepsilon_t \varepsilon_s) = Cov(\varepsilon_t, \varepsilon_s) = 0, \quad$ *for all* $t \neq s,$

*is called a **white noise process**, written* $\varepsilon_t \sim WN(0, \sigma^2)$.

Due to this definition, a white noise process is stationary. These definitions are the basis for the class of autoregressive moving average (ARMA) processes.

Definition A.5 (ARMA process)**.** (X_t) *is called an **ARMA process**, if* (X_t) *is stationary and*

$$X_t - \sum_{k=1}^{p} \phi_k X_{t-k} = \varepsilon_t + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j}, \tag{A.1}$$

holds for all t. $\varepsilon_t \sim WN(0, \sigma^2)$ *and* $\phi_k, \theta_j \in \mathbb{R}$ *for all* $k = 1, \ldots, p,\ j = 1, \ldots, q.$ (p, q) *is the order of the process.* (ε_t) *are the innovations of the process.*

The representation in Equation (A.1) is equivalent to

$$\phi(L)X_t = \theta(L)\varepsilon_t \tag{A.2}$$

where $\phi(z) = 1 - \phi_1 z - \ldots - \phi_p z^p$ and $\theta(z) = 1 + \theta_1 z + \ldots + \theta_q z^q$. The lag operator L fulfills $L^j X_t = X_{t-j}$ for $j \in \mathbb{Z}$.

By means of the polynomials in Equation (A.2) some important properties for the parameter estimation can be defined.

Definition A.6. *Let* (X_t) *with* $\phi(L)X_t = \theta(L)\varepsilon_t$ *be an ARMA(p,q) process.*

(i) If the polynomials $\phi(\cdot)$ *and* $\theta(\cdot)$ *have no common zeros and* $\phi(z) \neq 0$ *holds for all* $z \in \mathbb{C}$ *with* $|z| \leq 1$, *then* (X_t) *is called **causal**.*

(ii) If $\pi_j, j \in \mathbb{N}$, *exist, so that* $\sum_{j=0}^{\infty} |\pi_j| < \infty$ *and* $\varepsilon_t = \sum_{j=0}^{\infty} \pi_j X_{t-j}$ *holds, then the ARMA process is called **invertible**.*

Any causal, invertible ARMA process can optionally be formulated as an $AR(\infty)$ or a $MA(\infty)$ process. The formulation as a $MA(\infty)$ process is based on Wold's decomposition.

Theorem A.7 (Wold's decomposition)**.** *A process* (X_t) *with variance* $\sigma^2 > 0$ *can be decomposed into*

$$X_t = \sum_{k=0}^{\infty} \psi_k \varepsilon_{t-k} + V_t$$

where

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(i) $\psi_0 = 1$ *and* $\sum_{k=0}^{\infty} \psi_j^2 < \infty$,

(ii) $\varepsilon_t \sim WN(0, \sigma^2)$,

(iii) $\varepsilon_t \in M_n = \overline{sp}\{X_t, -\infty < t \leq n\}$ $\forall t$,

(iv) $E(V_t \varepsilon_s) = 0 \ \forall t, s$,

(v) $V_t \in M_{-\infty} = \bigcap_{n=-\infty}^{\infty} M_n$ *and*

(vi) V_t *is a deterministic component.*

$\overline{sp}\{A\}$ denotes the closed linear span of a set A. Usually the deterministic component V_t is zero as seasonal components were subtracted from the process before.

A.1.2. Parameter estimation of ARMA processes

Given a data set and an assumption of the model order the parameters of the ARMA process need to be estimated. The following method for the estimation of the ARMA parameters $\mathbf{b} = (\phi_1, \ldots, \phi_p, \theta_1, \ldots, \theta_q)^T$ and the variance of the white noise process σ^2 is implemented in various software packages.

The parameter estimation for the ARMA process $\mathbf{x}_n = (x_1, \ldots, x_n)^T$ is based on the maximization of the likelihood function

$$L(\mathbf{b}, \sigma^2 | \mathbf{x}_n) = \left(2\pi\sigma^2\right)^{-n/2} |\mathbf{G}_n(\mathbf{b})|^{-1/2} \exp\left(-\frac{1}{2\sigma^2}\mathbf{x}_n^T \mathbf{G}_n^{-1}(\mathbf{b})\mathbf{x}_n\right).$$

This function and the corresponding log-likelihood function

$$l(\mathbf{b}, \sigma^2 | \mathbf{x}_n) = -\frac{n}{2}\ln\left(2\pi\sigma^2\right) - \frac{1}{2}\ln\left(|\mathbf{G}_n(\mathbf{b})|\right) - \frac{1}{2\sigma^2}\mathbf{x}_n^T \mathbf{G}_n^{-1}(\mathbf{b})\mathbf{x}_n \qquad \text{(A.3)}$$

rely on the density of the normal distribution. $\mathbf{\Gamma}_n(\mathbf{b})$ is the covariance matrix of $\mathbf{x}_n$ and $\mathbf{G}_n(\mathbf{b}) = \sigma^{-2}\mathbf{\Gamma}_n(\mathbf{b})$ holds.

From the maximization of l with constant $\mathbf{b}$ we obtain

$$\widehat{\sigma}_n^2 = \frac{1}{n}\mathbf{x}_n^T \mathbf{G}_n^{-1}(\mathbf{b})\mathbf{x}_n$$

as an estimator of the variance of the innovations. As a consequence, we can rewrite the maximization in Equation (A.3) as the minimization of

$$g(\mathbf{b}) := \ln\left(\mathbf{x}_n^T \mathbf{G}_n^{-1}(\mathbf{b})\mathbf{x}_n/n\right) + \frac{1}{n}\ln\left(|\mathbf{G}_n(\mathbf{b})|\right).$$

Therefore,

$$\widehat{\mathbf{b}} = \operatorname*{argmin}_{\mathbf{b}} g(\mathbf{b})$$

is an estimator of the parameters of $\mathbf{x}_n$. This method is called maximum likelihood estimation, if the process is assumed to be normally distributed. The estimator is asymptotically efficient, unbiased and consistent.

These properties of the estimator can be extended to ARMA processes with non-normally distributed innovations.

Theorem A.8. *Let* (X_t) *be causal and* $\phi(L)X_t = \theta(L)\varepsilon_t$ *an invertible ARMA process with* $\varepsilon_t \sim WN(0, \sigma^2)$. *Then*

(i) $\widehat{\mathbf{b}} \to \mathbf{b}$ *a.s.*

(ii) $\widehat{\sigma}_n^2 \to \sigma^2$ *a.s.*

(iii) $\widehat{\mathbf{b}}$ *is asymptotically normally distributed. The parameters depend on* $\mathbf{b}$.

This result goes back to Hannan (1973). We use the formulation of Brockwell and Davis (1987). The proof requires further results including the representation of ARMA processes through spectral densities.

Due to the asymptotical normal distribution the estimator is unbiased and efficient even in case of non-normally distributed innovations. This method is denoted as quasi-maximum likelihood estimation.

A.1.3. Test for stationarity

A stationary data set is the basis for the application of an ARMA process. In order to test for stationarity we introduce the Dickey-Fuller-test (DF-test) and its augmentation, the Augmented Dickey-Fuller-test (ADF-test) as proposed by Dickey and Said (1984).

The DF-test is based on the regression approach

$$X_t = \rho X_{t-1} + \varepsilon_t$$

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with iid $\varepsilon_t \sim N(0, \sigma^2)$. A constant term can be included in the approach. As the data sets within this thesis have zero mean, the constant term is not considered. The coefficient ρ is estimated by the least squares method

$$\widehat{\rho} = \frac{\sum_{t=1}^{T} X_{t-1} X_t}{\sum_{t=1}^{T} X_{t-1}^2}.$$

In case $\rho = 1$ the process is a non-stationary random walk. If $|\rho| < 1$ holds, then the time series is stationary. This leads to the hypothesis

$$H_0 : \rho = 1$$

versus the alternate hypothesis

$$H_1 : |\rho| < 1.$$

As the test statistics are not valid in the case $|\rho| = 1$ the differenced situation $\Delta X_t = (\rho-1)X_{t-1} + \varepsilon_t$ is considered. The test is now stated as the hypothesis

$$H_0 : b = 0$$

versus the alternate hypothesis

$$H_1 : b < 0$$

with $b = \rho - 1$. Dickey and Fuller numerically calculated critical values for the test statistics of $\widehat{b}$.

So far, the method is restricted to AR(1) processes. For many applications higher orders are needed. A first augmentation is the consideration of AR(p) processes $(p \geq 2)$, the **ADF-test**. Using the lag operator as in Section A.1.1 for the AR(p) process

$$\phi(L)X_t = \varepsilon_t$$

leads to the hypothesis

$$H_0 : \gamma := 1 - \rho = 0 \quad \text{versus} \quad H_1 : -2 < \gamma < 0$$

where $\rho = \phi_1 + \ldots + \phi_p$. The test statistics and critical values are identically equal to the case $p = 1$. Dickey and Said (1984) give a further augmentation to ARMA(p,q) processes for arbitrary (p, q). They show that the same test statistics are applicable. Tables of critical values are given e.g. by Hamilton (1994).

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A.2. Bessel function

The differential equation

$$x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + x\frac{\mathrm{d}y}{\mathrm{d}x} + (x^2 - \alpha^2)y = 0, \qquad \alpha \in \mathbb{C},$$

is called **Bessel's differential equation** of order α. Usually $\alpha \in \mathbb{Z}$ holds. The equation is classified as a second-order linear ordinary differential equation with two independent solutions. These solutions are called **Bessel function of the first kind**, J_α, and **Bessel function of the second kind** Y_α.

If the arguments of the Bessel functions are imaginary, then the Bessel functions are called **modified Bessel functions** of the first and second kind, I_α and K_α. These are linear independent solutions of the differential equation

$$x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + x\frac{\mathrm{d}y}{\mathrm{d}x} - (x^2 - \alpha^2)y = 0, \qquad \alpha \in \mathbb{C}.$$

Dependent on the application there are various integral representations for the modified Bessel function of the second kind, for example

$$K_\alpha(x) = \frac{1}{2}e^{-\frac{1}{2}\alpha\pi i}\int_{-\infty}^{\infty} e^{-ix\sinh(t) - \alpha t}\mathrm{d}t.$$

For further details on Bessel functions see Watson (1995) and Abramowitz and Stegun (1965).

The modified Bessel function of the second kind is also denoted as modified Bessel function of the third kind, modified Hankel function or MacDonald function.

A.3. Probability distributions

We give definitions of the most relevant non-standard probability distributions that are used within this thesis. The definitions can be found in Cont and Tankov (2004) or Embrechts et al. (2005).

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A.3.1. Student's t-distribution

Definition A.9 (Student's t-distribution)**.** *A random variable X has a **Student's t-distribution** with parameter $\nu > 0$, if its density is*

$$f(x) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\,\Gamma\left(\frac{\nu}{2}\right)}\left(1 + \frac{x^2}{\nu}\right)^{-\frac{\nu+1}{2}}.$$

$\Gamma(\cdot)$ *describes the gamma function.*

Random numbers can be generated according to the algorithms presented by Devroye (1986).

A.3.2. Inverse Gaussian distribution

Definition A.10 (Inverse Gaussian distribution)**.** *A random variable X has an **inverse Gaussian distribution** with parameters λ and μ, if its density is*

$$f(x) = \sqrt{\frac{\lambda}{2\pi x^3}} \cdot \exp\left(-\frac{\lambda(x-\mu)^2}{2\mu^2 x}\right), \qquad x > 0.$$

An algorithm for generation of random numbers from this distribution is given by Haas et al. (1976).

A.3.3. Gamma distribution

Definition A.11 (Gamma distribution)**.** *A random variable X has a **gamma distribution** with parameters $a > 0$ and $b > 0$, if its density is*

$$f(x) = \frac{1}{b^a \Gamma(a)} x^{a-1} \exp\left(-\frac{x}{b}\right), \qquad x > 0.$$

We write $X \sim \Gamma(a, b)$.

As a second parametrization the use of $1/b$ instead of b is common.

The gamma distribution has a useful feature for simulation purposes.

$$b \cdot X \sim \Gamma(a, b) \quad \text{for } X \sim \Gamma(a, 1) \quad \text{and } b > 0$$

gives us the possibility to generate $\Gamma(a, 1)$-distributed random numbers as a basis for any other choice of parameters. For this purpose we can use the rejection method. In case of $a \geq 1$ we use a Student's t-distribution with two degrees of freedom. In case of $0 < a \leq 1$ the Weibull distribution gives the decision about acceptance or rejection. Detailed descriptions of the corresponding simulation algorithms are given by Devroye (1986).

A.3.4. Generalized inverse Gaussian distribution

The **generalized inverse Gaussian** (GIG) distribution is used within the definition of the generalized hyperbolic distributions in Section A.3.5.

Definition A.12 (GIG distribution). *A random variable X with density function*

$$f(x) = \frac{\chi^{-\lambda} \left(\sqrt{\chi\psi}\right)^{\lambda}}{2K_\lambda \left(\sqrt{\chi\psi}\right)} x^{\lambda-1} \exp\left(-\frac{1}{2}\left(\chi x^{-1} + \psi x\right)\right), \qquad x > 0,$$

follows a GIG distribution with parameters λ, χ, ψ ($X \sim N^-(\lambda, \chi, \psi)$). K_λ describes the modified Bessel function of the second kind with index λ (see Section A.2). The parameters need to satisfy

$$\begin{aligned} \lambda < 0 &\Rightarrow \chi > 0, \quad \psi \geq 0, \\ \lambda = 0 &\Rightarrow \chi > 0, \quad \psi > 0, \\ \lambda > 0 &\Rightarrow \chi \geq 0 \quad \psi > 0. \end{aligned}$$

This definition includes the inverse Gaussian distribution ($\lambda = -0.5$) and the gamma distribution ($\chi = 0$) as special cases. As our applications only rely on these special cases we do not give a simulation algorithm for the GIG distribution.

A.3.5. Generalized hyperbolic distributions

The class of **generalized hyperbolic** (GH) distributions was introduced by Barndorff-Nielsen (1978). We refer to the parametrization as given by Cont and Tankov (2004). Prause (1999) gives an overview of further equivalent parametrizations.

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Definition A.13 (GH distribution). *Let the random variables $X \sim N(0,1)$ and $Y \sim N^-(\lambda, \delta^2, \alpha^2 - \beta^2)$ be independent and*

$$Z \stackrel{d}{=} \mu + \beta Y + \sqrt{Y} X$$

the normal mean-variance mixture. Z has a GH distribution with parameters $\lambda, \alpha, \beta, \delta, \mu$, written $Z \sim GH(\lambda, \alpha, \beta, \delta, \mu))$. Its density function is given by

$$f(x) = C \cdot \left(\delta^2 + (x - \mu)^2 \right)^{\frac{\lambda}{2} - \frac{1}{4}} K_{\lambda - \frac{1}{2}} \left(\alpha \sqrt{\delta^2 + (x - \mu)^2} \right) e^{\beta(x - \mu)}$$

where

$$C = \frac{(\alpha^2 - \beta^2)^{\lambda/2}}{\sqrt{2\pi} \alpha^{\lambda - 1/2} \delta^\lambda K_\lambda \left(\delta \sqrt{\alpha^2 - \beta^2} \right)}.$$

Apart from $\alpha > 0$ there are further restrictions on the parameters resulting from the restrictions on the parameters of the GIG distribution (see Section A.3.4):

$$\begin{aligned} \lambda < 0 &\Rightarrow \delta > 0, \quad |\beta| \leq \alpha, \\ \lambda = 0 &\Rightarrow \delta > 0, \quad |\beta| < \alpha \text{ and} \\ \lambda > 0 &\Rightarrow \delta \geq 0, \quad |\beta| < \alpha. \end{aligned}$$

Although there are interactions, each parameter has major impact on single features of the distribution.

- λ: shape of the distribution,

- α: heaviness of the tails,

- β: skewness,

- δ: kurtosis,

- μ: location.

Amongst others there are some well-known special cases of the GH distribution.

- In case of $\lambda = -0.5$ we have the normal inverse Gaussian (NIG) distribution, written $NIG(\alpha, \beta, \delta, \mu))$, with density function

$$f(x) = \frac{\alpha \delta K_1 \left(\alpha \sqrt{\delta^2 + (x - \mu)^2} \right)}{\pi \sqrt{\delta^2 + (x - \mu)^2}} \cdot \exp \left(\delta \sqrt{\alpha^2 - \beta^2} + \beta (x - \mu) \right).$$

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- If $\delta = 0, \lambda > 0$, we get

$$f(x) = \frac{(\alpha^2 - \beta^2)^\lambda}{\sqrt{\pi} \, (2\alpha)^{\lambda - 1/2} \, \Gamma(\lambda)} \cdot |x - \mu|^{\lambda - 1/2} \cdot K_{\lambda - 1/2} \left(\alpha |x - \mu| \right) e^{\beta(x - \mu)},$$

  with the gamma function Γ. In this case, f describes the density of the variance gamma distribution, written $VG(\lambda, \alpha, \beta, \mu)$.

- If $\lambda < 0$ and $\alpha = \beta = \mu = 0$, we obtain the well-known Student's t-distribution (compare Section A.3.1).

Parameters of any GH distribution can be estimated using the maximum likelihood method.

The simulation algorithm of GH random numbers is the result of the representation as a normal mean-variance mixture. Normal random numbers and GIG random numbers are required. If NIG or VG random numbers are needed, the generation of GIG random numbers reduces to the generation of inverse Gaussian or gamma distributed random numbers. These algorithms are given in Section A.3.2 and A.3.3.

B. Programming

The models presented in this work are implemented using the mathematical software Matlab. As far as available the provided functions were used. We give a short overview of the most important non-trivial functions used. Detailed descriptions can be found at MathWorks (2013).

- **regress**: The function regresses a variable on a set of explaining variables. The coefficients are obtained by a ordinary least squares regression. Confidence intervals as well as some statistical measures are provided. The fundamental components of the spot price models are linear with respect to the parameters. Therefore, the parameters are estimated via linear regression.

- **mle**: This function allows for the maximum likelihood estimation of parameters of a probability distribution. Various standard distributions including Student's t-distribution are implemented. The parameters of user-defined distributions such as truncated distributions can be estimated as well. The truncated Student's t-distribution including a scale parameter is fitted via maximum likelihood estimation.

- **fminsearch**: This function does an unconstrained minimization of a multivariate function. Various numerical parameters concerning convergence and number of steps can be changed. The algorithm does not need any numerical or analytical derivatives. Therefore it can be applied for the optimization problems within this work. This includes the fit of generalized hyperbolic distributions and the parameter estimation via Kalman filter.

- **adftest**: The implementation of the ADF-test for unit roots.

- **autocorr**: The function provides the autocorrelation function of a time series. It is used for the determination of model order of the various SARIMA processes.

- **arima**: This function of the econometrics toolbox creates a model of

the ARIMA class. Using some optional input arguments this function can define seasonal ARIMA models. The corresponding parameters are obtained by the function **estimate**. In addition to the estimation method the class has a simulation method **simulate**.

- **ksdensity**: Given empirical data this function estimates the density using a smoothing kernel. Various kernels are provided. The kernel densities plotted within this work are based on the Epanechnikov kernel.

- **jcitest**: This function tests for cointegration of a set of variables. Test statistics, critical values, rank of cointegration and parameters are calculated.

Apart from these implemented functions further functions are needed. Especially the load forecasts with similar days require functions dealing with the calendar. A more complex functionality is required for the estimation of the multivariate two factor model by Schwartz and Smith (2000). As mentioned above the Kalman filter is used in this model. As the algorithm was extended to the multivariate case we implemented the Kalman filter without the corresponding function provided by Matlab.

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